ADFOCS: Market Design
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Problem Set #3

1. Consider selling a single item to two bidders. Bidder 1's value is drawn uniformly from the interval $[0, 1]$. Bidder 2's value is drawn uniformly from the interval $[0, 2]$.

   - Describe the allocation and payments of the revenue-optimal auction. You may express your answer graphically.
   - Find all value profiles for which the resulting allocation differs from the efficient allocation. You may again express your answer graphically.

2. Again consider selling a single item to two bidders. Bidder 1's value is drawn uniformly from the interval $[0, 1]$, and bidder 2's value is drawn uniformly from the interval $[0, 2]$.

   - Consider the second-price auction with monopoly reserves (i.e., reserve of $1/2$ and 1 for each bidder respectively). Find all value profiles for which the resulting allocation differs from the allocation of the optimal mechanism from part (a). You may express your answer graphically.
   - Calculate the expected revenue of the second-price auction with monopoly reserves.
   - Suppose that instead of running a second-price auction, we instead offer a take-it-or-leave it price to bidder 1, then if she rejects we offer a take-it-or-leave-it price to bidder 2. Compute the optimal prices to offer, and calculate the expected revenue.

3. Again consider selling a single item to two bidders. Suppose both bidders have values drawn uniformly from the interval $[0, 1]$. Consider the following two auctions: A) second-price with no reserve; B) second price with reserve $1/2$ only for bidder 1. That is, bidder 2 wins if $b_1 < 1/2$; otherwise the bidder with the higher bid wins.

   - What payments make auction B truthful?
   - Which auction has higher expected revenue?[1]

[1] You do not have to calculate the expected revenues of the two auctions to answer this question; see if you can find an easier way!