# Information Acquisition in Matching Markets: The Role of Price Discovery

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# Matching Markets

Agents exert significant effort investigating potential placements and learning preferences.

# Matching Markets

## Find High Schools ▲

When you apply to high school in New York City, you have a wide range of choices. We offer more than 700 programs at over 400 high schools. There are two paths to high school admissions.

- **High School.** You can apply to 12 high school programs with your high school application. Submit your application online or through a school counselor.
- **Specialized High Schools.** The specialized high schools use a separate admissions process. Students test or audition to apply.

# Matching Markets



These resources can help you find schools for your high school application:

1. **MySchools.** Explore NYC public schools and programs, find choices for your application, and apply—all in one place. Use MySchools ⬀ or contact schools directly to find the most up-to-date information about schools' academic offerings, activities, sports, and more.
2. **2020 NYC High School Admissions Guide.** Find high schools and programs by borough. Learn how programs make offers and what makes a balanced application. Scroll to the Documents section of this page to view or download this guide.
3. **Diversity in Our Schools.** Learn about pilot diversity initiatives that high schools across the city are participating in to increase diversity within their schools.

# Information Acquisition

Marketplace forms match *and* guides information acquisition.

Questions:

How can we think about markets with information acquisition?

Can market design help marketplaces reach good outcomes?

Focus on one-sided uncertainty (e.g., college admissions).

# Matching with Information Acquisition

How can we think about markets with information acquisition?

Model has information acquisition costs for uniformed side.

- Students pay to learn own independent private preferences; colleges have known priorities.

- Can rationalize student behaviors.

# Matching with Information Acquisition

How can we think about markets with information acquisition?

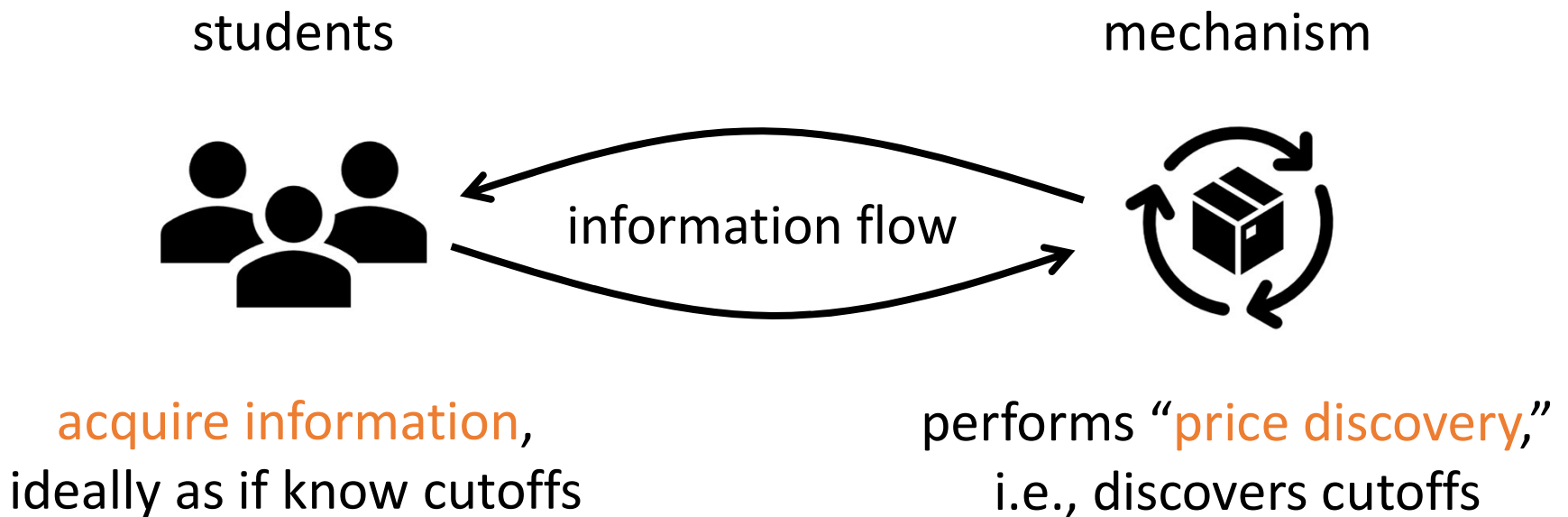Model has information acquisition costs for uniformed side.

Outcomes consist of assignment and information.

- Stability with respect to endogenous information acquisition.

- Regret-free stability: optimal information acquisition,
  i.e., each student acquires information as if "last to market."

- Regret-free stable outcomes characterized by cutoffs.

# Matching with Information Acquisition

Can market design help marketplaces reach good outcomes?

Marketplaces induce information flows.

students                                                    mechanism



information flow

acquire information,                          performs "price discovery,"
ideally as if know cutoffs                    i.e., discovers cutoffs

# Matching with Information Acquisition

Can market design help marketplaces reach good outcomes?

Marketplaces induce information flows.

students          information deadlock          mechanism

info          flow

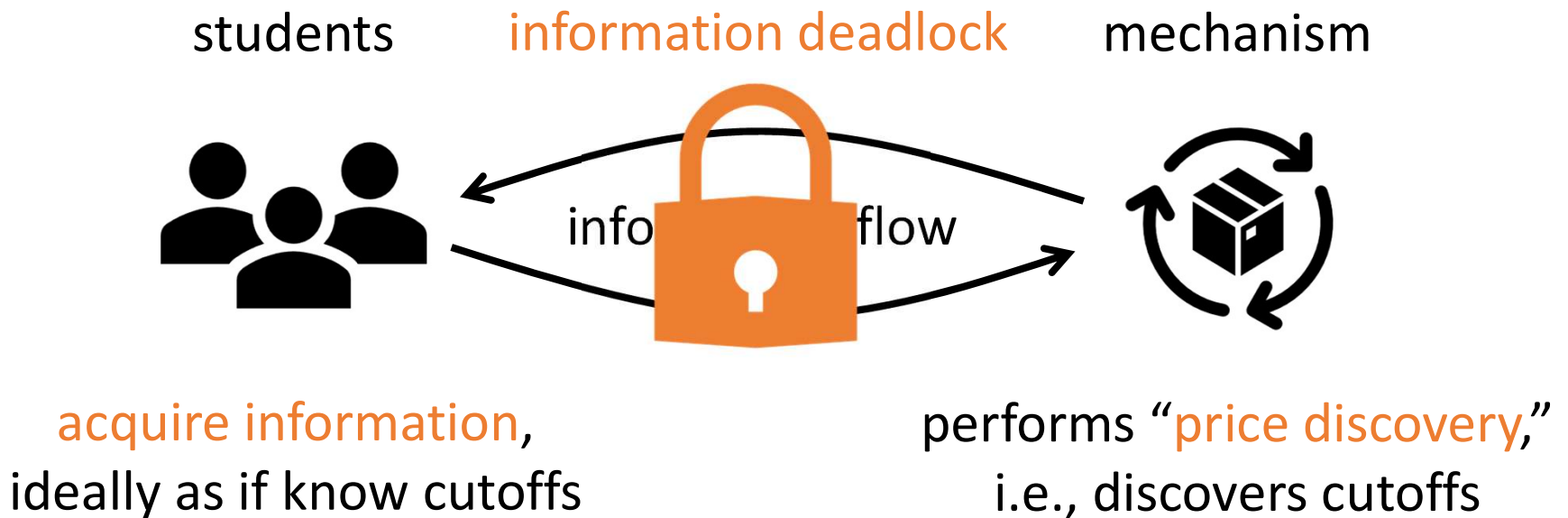acquire information,
ideally as if know cutoffs

performs "price discovery,"
i.e., discovers cutoffs

# Matching with Information Acquisition

Can market design help marketplaces reach good outcomes?

Marketplaces induce information flows.

Design can circumvent information deadlocks by leveraging additional sources of information.

- Use historical data or market surveys to calculate cutoffs
- Current systems emphasize these aspects as well as the choice of matching algorithm.

# Matching with Information Acquisition

How can we think about markets with information acquisition?

Model has information acquisition costs for uniformed side.

Outcomes consist of assignment and information.

Can market design help marketplaces reach good outcomes?

Marketplaces induce information flows.

Design can circumvent information deadlocks by leveraging additional sources of information.

# Related Literature

**Empirical results on importance of information in school choice.**

[Grenet-He-Kübler '19], [Kapor-Neilson-Zimmerman '16], [Luflade '17], [Narita '16]

**Importance of stability in market design.**

[Roth '81], [Roth '94], [Roth-Xing '94], [Roth '02]

**Stability with partially informed agents.**

[Aziz-Biró-Gaspers-de Haan-Mattei-Rastegari '16], [Bikhchandani '17], [Chakraborty-Citanna-Ostrovsky '10], [Liu-Mailath-Postlewaite-Samuelson '14], [Chen-He '19]

**Market design with information acquisition.**

[Azevedo-Pennock-Waggoner-Weyl '19], [Bergemann-Välimäki '02], [Kadam '15], [Kleinberg-Waggoner-Weyl '16], [Lee-Schwarz '09], [Rastegari-Condon-Immorlica-Leyton Brown '14]

# Model

# Full-Information Model

Agents: finite set $C$ of $n$ colleges, continuum $\Omega$ of students.

- College $i$ has quota $q_i$

- Student $\omega$ has priority $r_i^\omega$ and value $v_i^\omega$ at college $i$

Economy: $E = (C, \Omega, \eta, q)$ for measure $\eta$ over students $\Omega$.

Outcomes: matching $\mu: \Omega \rightarrow C$ of students to colleges.

- Feasible if no college exceeds its quota ($\mu^{-1}(i) \leq q_i$).

- Stable if no student wants to switch to a college at which her priority exceeds that of an admitted student.

# Stable Matchings in Full-Information Model

A matching $\mu$ induces cutoffs $\boldsymbol{P}$, budget set $B^\omega(\boldsymbol{P}) = \{i \mid r_i^\omega \geq P_i\}$.

Stable if every student matched to favorite college in budget set:
$$\forall\, \omega, \qquad \mu(\omega) = \mathrm{argmax}\big(v_i^\omega \mid i \in B^\omega(\boldsymbol{P})\big).$$
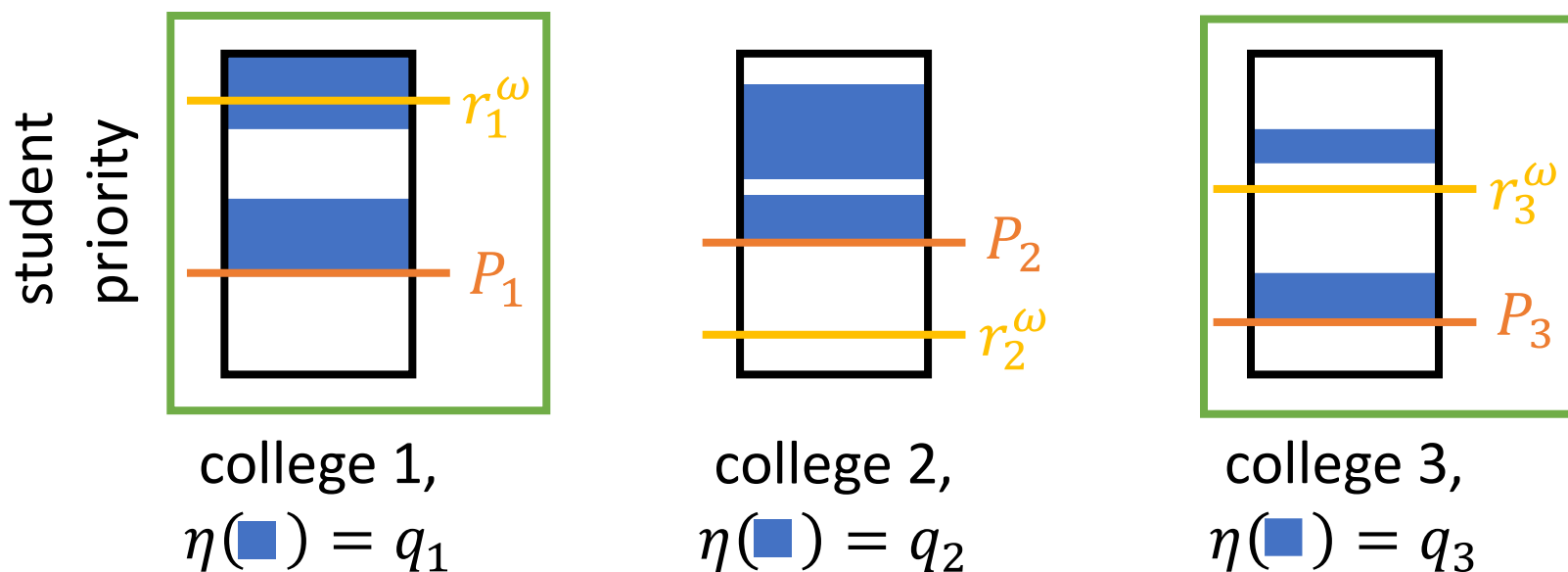
student priority

college 1,
$\eta(\blacksquare) = q_1$

college 2,
$\eta(\blacksquare) = q_2$

college 3,
$\eta(\blacksquare) = q_3$

# Incomplete-Information Model

Agents: finite set $C$ of $n$ colleges, continuum $\Omega$ of students.

- College $i$ has quota $q_i$, student $\omega$ has known priority $r_i^\omega$ at $i$.

- Students have incomplete information about their independent private values.

# Students



a student $\omega \in \Omega$
with values $v^\omega$

inspection state of student

initial information of student

priorities $r^\omega$

priors $F^\omega$ over values

signals $\Pi^\omega$ and their costs $c^\omega$

information acquired over time

set $\chi^\omega$ of acquired signals and
their realizations $\pi(v^\omega)$ for $\pi \in \chi^\omega$

student $\omega = \left( r^\omega, F^\omega, \Pi^\omega, c^\omega, \chi^\omega, \{\pi(v^\omega)\}_{\pi \in \chi^\omega}; v^\omega \right) \in \Omega$

inspection state $\theta(\omega) = \left( r^\omega, F^\omega, \Pi^\omega, c^\omega, \chi^\omega, \{\pi(v^\omega)\}_{\pi \in \chi^\omega} \right) \in \Theta$

# Example: Pandora's Box Students

college 1,

$$F_1^\omega = \begin{cases} 8 & 1/2 \\ 0 & \text{oth.} \end{cases}$$

college 2,

$$F_2^\omega = \begin{cases} 7 & 1/2 \\ 0 & \text{oth.} \end{cases}$$

college 3,

$$F_3^\omega = \begin{cases} 6 & 1/2 \\ 0 & \text{oth.} \end{cases}$$

Signals $\Pi^\omega$: pay cost $c_i^\omega = 2$
to learn value $v_i^\omega$ at college $i$.

student $\omega$

# Incomplete Information Model

**Agents:** finite set $C$ of $n$ colleges, continuum $\Omega$ of students.

**Economy:** $E = (C, \Omega, \eta, q)$ for measure $\eta$ over students $\Omega_0$.

- zero-measure indifferences and complete preferences

- more students than seats at colleges

- require $F$ and $\eta$ to be consistent

# Incomplete Information Model

Agents: finite set $C$ of $n$ colleges, continuum of students.

Economy: $E = (C, \Omega, \eta, q)$ for measure $\eta$ over students $\Omega$.

Outcomes: $(\mu, \chi)$ include matching and acquired information.
- Information $\chi: \Omega_0 \to \Pi$ specifies inspections of each student.
- Matching $\mu: \Omega \to C \cup \phi$ of students to colleges.
- Feasible if satisfies quotas and inspection rules.

# Stable Outcomes

An outcome $(\mu, \chi)$ includes matching and information gathered.
- induces cutoffs $\boldsymbol{P}$,
- budget set $B^\omega(\boldsymbol{P}) = \{i \mid r_i^\omega \geq P_i\}$.

Stable if every student $\omega$,

1. matched to favorite inspected college in budget set:
$$\mu(\omega) = \mathrm{argmax}\left(v_i^\omega \mid i \in B^\omega(\boldsymbol{P}) \cap f(\chi^\omega)\right),$$

2. and student $\omega$ would not like to acquire more information, according to optimal inspection rule.

# Example: Pandora's Box Inspection Rule

Utility for budget set $B^\omega$, inspections $\chi^\omega$:

$$\max\{v_i^\omega \mid i \in B^\omega \cap \chi^\omega\} - \sum_{i \in \chi^\omega} c_i^\omega.$$

Theorem [Weitzman '79]. Define the index $\underline{v}_i^\omega$ to be:

$$E_{v_i \sim F_i^\omega}\left[\max\{0, v_i - \underline{v}_i^\omega\}\right] = c_i^\omega.$$

Then the optimal inspection rule is to sequentially inspect colleges in $B^\omega$ in decreasing order of index, stopping if realized value is higher than next index.

# Example: Pandora's Box Inspection Rule

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Main idea.
- $E_{v_i \sim F_i^\omega}\left[\max\{0, v_i - \underline{v}_i^\omega\}\right]$ is extra value you get by opening box $i$ if you already have access to an option of value $\underline{v}_i^\omega$;
- hence you want to open the box only if this gain is at least $c_i^\omega$.

# Stable Outcomes

An outcome $(\mu, \chi)$ is stable if for every student $\omega$,

1. matched to favorite inspected college in budget set:
$$\mu(\omega) = \mathrm{argmax}\big(v_i^\omega \mid i \in B^\omega(\boldsymbol{P}) \cap \chi^\omega\big),$$

2. and student $\omega$ would not like to acquire more information, according to optimal inspection rule.

An outcome $(\mu, \chi)$ is regret-free stable if it is stable and for every student $\omega$, inspections $\chi^\omega$ are optimal given budget set $B^\omega(\boldsymbol{P})$.

# Example: Pandora's Box Outcomes

college 1,
$F_1^\omega = \{8, 1/2\}$,
index $\underline{v}_1^\omega = 4$

college 2,
$F_2^\omega = \{7, 1/2\}$,
index $\underline{v}_2^\omega = 3$

college 3,
$F_3^\omega = \{6, 1/2\}$,
index $\underline{v}_1^\omega = 2$

student $\omega$,
costs $\boldsymbol{c}^\omega = 2$

If budget set $B^\omega = \{1,2,3\}$, inspect college 1 first and then college 2 only if $v_1^\omega = 0$.

If budget set $B^\omega = \{2,3\}$, inspect college 2 first.

# Model Features

Theorem. Regret-free stable outcomes exist and form a lattice.
(proven through connection to market equilibria; see paper)

Rationalizes some student behaviors.
- Students may choose to remain partially informed.
- Students may find it costly to rank colleges in advance.
- Admitted students may want to delay inspections.

# Information Flows

# Marketplaces

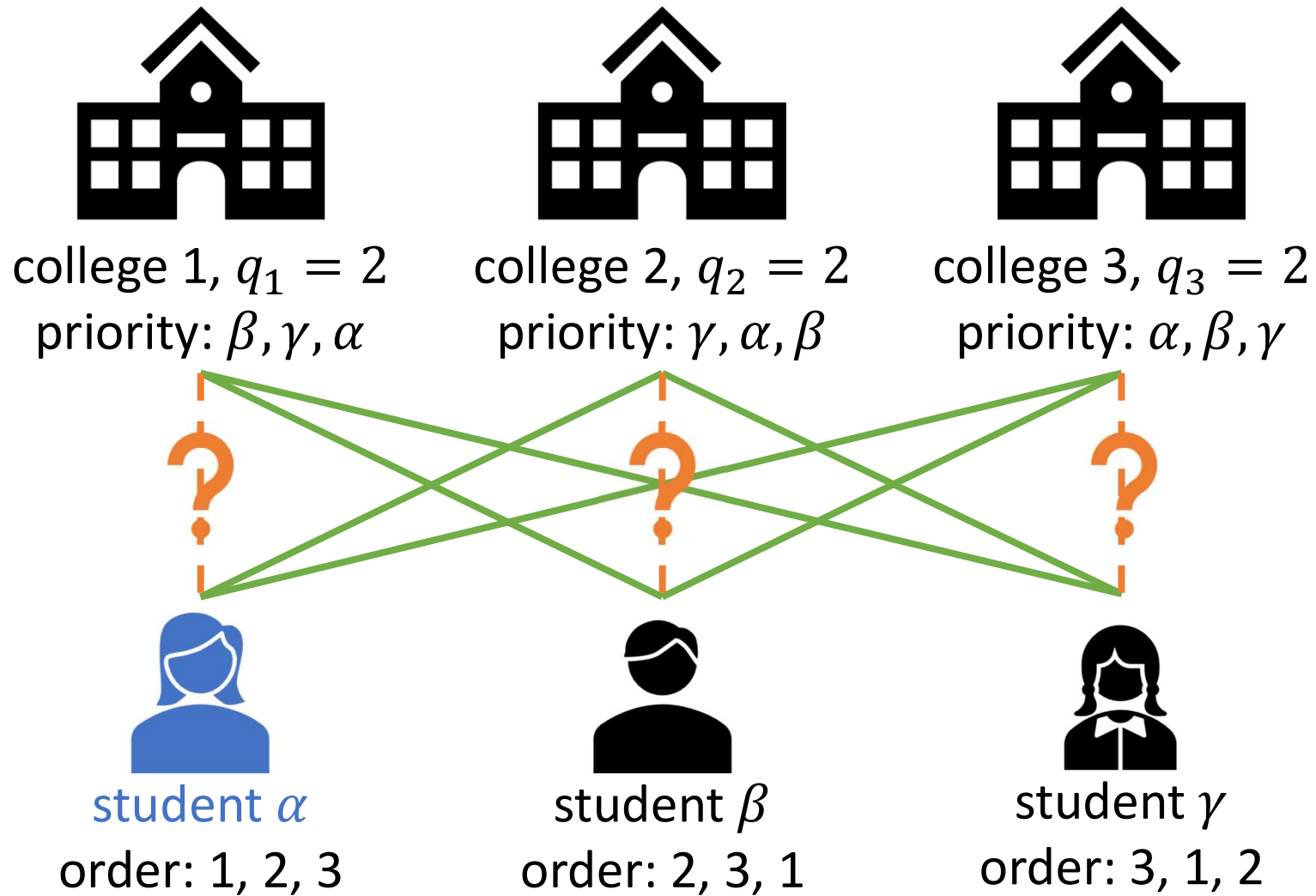students                                    mechanism
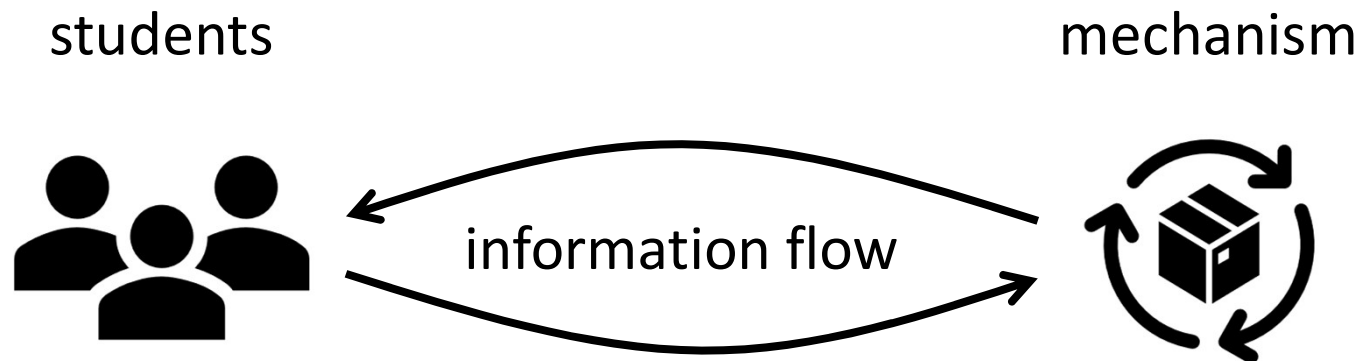


information flow

Example: Iterative college-proposing deferred acceptance
- Colleges propose to top students who have not rejected them
- Students tentatively accept "best" offer received so far

# Example: Information Deadlock in ICPDA



college 1, $q_1 = 2$
priority: $\beta, \gamma, \alpha$

college 2, $q_2 = 2$
priority: $\gamma, \alpha, \beta$

college 3, $q_3 = 2$
priority: $\alpha, \beta, \gamma$

student $\alpha$
order: 1, 2, 3

student $\beta$
order: 2, 3, 1

student $\gamma$
order: 3, 1, 2

# Impossibility Result

Theorem.  No communication process that maintains aggregate uncertainty and has no external information guarantees regret-free stable outcomes.

Proof. Embed information deadlock example in an economy.

Remark. Proof shows a constant fraction of students may be required to acquire information sub-optimally.

# Possibility Result

If cutoffs are known, then simply posting them results in regret-free stable outcomes.

Step 1. Estimate cutoffs.

- Historical data: approximately correct if discrete economy sampled from stationary continuum

- Market surveys: induces regret for surveyed students

- Boostrapping "free information" students: requires structural assumption on demand

Step 2. Post estimates and deal with errors.

- For example, perturb quotas to absorb extra demand

# Information Flows in Practice

Australia. Post cutoffs using historical demand, commit to them.

## 2020 GUIDE TO ADMISSION CRITERIA FOR DOMESTIC STUDENTS

Below is a guide to the Australian Tertiary Admission Rank (ATAR) and International Baccalaureate (IB) scores for admission in 2020. For most courses, the scores are guaranteed, except where marked with an asterisk*. The asterisked scores are an indicative score for what you will need for admission in 2020. All published scores are correct at the time of print and subject to change. For the most up-to-date information on ATARs, visit

– sydney.edu.au/sydney-atar

With more than 400 areas of study to choose from, we offer an incredible breadth and depth of courses.

| Course name | ATAR/IB | Duration in years |
|---|---|---|
| **Architecture, design and planning** | | |
| ● B Architecture and Environments | 85/31 | 3 |
| ■ B Design Computing | 80/28 | 3 |

| Course name | ATAR/IB | Duration in years |
|---|---|---|
| **Business** | | |
| ◆ B Commerce | 95/36 | 3 |
| ▲ B Commerce/B Advanced Studies | 95/36 | 4 |

# Information Flows in Practice

Australia. Post cutoffs using historical demand, commit to them.

Everything you need to know about guaranteed ATARs

**Why is the University publishing a guaranteed ATAR?** −

> Our transparency around the ATAR allows you to put your preferred course as your first preference with confidence.
>
> This means you have all the information needed to make an informed decision about your course preferences.

**Who is eligible to gain entry through the guaranteed ATAR?** +

**Does the guaranteed ATAR apply to all courses?** +

# Information Flows in Practice

Israel. Post cutoffs with "gray zone", followed by "ICPDA".

בדיקה לפי הנתונים הבאים:

| | |
|---|---|
| **105** | ממוצע בגרות מותאם |
| **630** | ציון פסיכומטרי |
| **628** | ציון התאמה לפי בגרות+פסיכומטרי |

| ציונך (🕯️) מצוי בטווח של<br>■קבלה ■חסר החלטה ■דחיה | פירוט | הרשמה | שנה<br>קודמת | החלטה | תכנית לימודים | פקולטה |
|---|---|---|---|---|---|---|
|  | פרטים | פתוחה | בדיון<br>פרטני | | 0321 - פיזיקה ואסטרונומיה- חד חוגי | מדעים<br>מדוייקים |
|  | פרטים | פתוחה | דחיה | דחיה | 0321 - פיזיקה חד-חוגי-תכנית מחקרית<br>למצטיינים | |
|  | פרטים | סגורה | בדיון<br>פרטני | | 0321 - פיזיקה ואסטרונומיה תכנית משולבת עם<br>לביולוגיה | |
|  | פרטים | פתוחה | בדיון<br>פרטני | | 0321 - פיזיקה ואסטרונומיה דו חוגי | |
|  | פרטים | פתוחה | בדיון<br>פרטני | | 0321 - פיזיקה ואסטרונומיה דו חוגי-פיזיקה כימיה<br>דו-חוגי | |
|  | פרטים | פתוחה | דחיה | דחיה | 0323 - מסלול משולב מתמטיקה - פיזיקה חד<br>חוגי | |

# Conclusion

A model of matching that captures the importance of information flows.

Regret-free stable outcomes where students acquire information optimally as if they were "last-to-market."

Can design help markets reach good outcomes efficiently?

Yes, but only by leveraging additional sources of information, something widely used in practice currently.

# Takeaway

Market designers should pay careful attention to information flows in marketplaces, as well as the underlying matching algorithm.

Information aspects can be as or more important than algorithmic aspects!