Matching

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Match agents to item.

- public housing
- dorm assignment
- school choice
- course allocation
- organ transplant
- food banks

Goals:

- high value
- respect priority
- stability of soln
- minimizing waste
- transparent
- easy to optimize
- minimize regret

Model

- set of N agents
- set of I items
- defn Prefs $\succ_i$: strict total orders over I, $a \succ_i b$
- defn util $v_i : I \to [0, \infty]$ where $v_i(a) > v_i(b)$ iff $a \succ_i b$

Ex.

items / agents

	a	b	c	d
1	100	3	2	1
2	100	99	2	1
3	100	99	18	1
4	100	99	98	97

defn An allocation x

- $x_{ia} = 1$ if i gets a
- $x_{ia} = 0$ if i doesn't get a

feasible if $x_{ia} = x_{ja} = 1$ then $i = j$

defn An alloc x is ex post Pareto Eff (PE) if there is no alloc. y s.t.

$$\sum_{a \in I} v_{ia} y_{ia} \geq \sum_{a \in I} v_{ia} x_{ia}$$

$\forall$ all $i \in N$, strict for some i.

Mechanisms

def. Map inputs ($\succ_i$) to alloc. x

def. Strategyprf if each agent's alloc. is optimized by reporting her true value.

def. PE if equil alloc is PE

Serial Dictatorship

Pick an ordering of agents.
For $i = 1$ to n,

- let a be i's favorite remaining item
- set $x_{ia} = 1$ and remove a

Claim ex post PE + strategyproof

defn A mech. has "equal-treatment-of-equals" if identical agents get the same alloc. in expectation.

Random Serial dictatorship (RSD)

- run serial dictatorship w/random order.

Claim Ex post PE, strategyprf, equal-treatment-of-equals

def a randomized alloc is a set of allocs $\{x^1, \dots, x^k\}$ and a convex comb. of them $\{q_1, \dots, q^k\}$

def. A lottery is p_{ia} s.t.

$$\sum_{i=1}^{n} p_{ia} = 1 \quad \forall a$$

$$\sum_{a \in I} p_{ia} = 1 \quad \forall i$$

$v_{1a} = 100$, $v_{1b} = 3$, $v_{1c} = 2$, $v_{1d} = 1$

pref b w/prob. 1 vs a w/prob. 1/100 ?

defn (risk neutral) util of lottery = exp. util of outcome

$\sum_{i\in I} p_{ia} = 1 \ \forall a$

$\sum_{a\in I} p_{ia} = 1 \ \forall i$

util of lottery = exp. util of outcome

Assume risk neutrality.

Ex. RSD lottery $p_{ia} = 1/4$

- $E[V_i] = \sum_{a\in I} v_{ia} p_{ia} \approx 25_i$

defn. A lottery is ex ante PE if there is no lottery q that someone prefers and no one is harmed.

claim. ex ante PE $\rightarrow$ ex post PE

Q.? Is RSD ex ante PE?

Ex. 8 agents, 8 items

Values	a	2 copies b
4 copies of 1	100	1
4 copies of 2	100	99

RSD:

$E[V_1^{RSD}] \approx \frac{1}{8}\times 100 = 12.5$

$E[V_2^{RSD}] \approx \frac{3}{8}\times 100 = 37.5$

ALT: give a to a type 1

" b " 2

$E[V_1^{ALT}] \approx 25$

$E[V_2^{ALT}] \approx 49.5$

Two-sided matching

Agents to agents.

- job markets, NRMP
- school choice
- marriage markets

Goals

- PE
- no justified envy (stability)
- strategyproof

- set of men M
- set of women W
- prefs $a \succ_x b$: strict, total
- matching $\mu : M \to W$
  - $\mu(m)$ is m's match
  - $\mu(w)$ is w's match
  - (m, w) if $\mu(m) = w$

- defn PE: μ is PE if $\nexists$ ν s.t.
  - $\nu(x) \succeq_x \mu(x) \ \forall x$
  - $\nu(x) \succ_x \mu(x)$ for some x

defn. μ is stable if

- IR: x prefers $\mu(x)$ to being single
- no pair (m, w) s.t. $w \succ_m \mu(m)$ and $m \succ_w \mu(w)$

Goal: find PE stable μ

$m_1 : w_1 \succ w_2 \succ w_3$ $\qquad$ $w_1 : m_3 \succ m_2 \succ m_1$

$m_2 : w_2 \succ w_3 \succ w_1$ $\qquad$ $w_2 : m_1 \succ m_3 \succ m_2$

$m_3 : w_2 \succ w_3 \succ w_1$ $\qquad$ $w_3 : m_1 \succ m_3 \succ m_2$

Claim Stable $\Rightarrow$ PE

Deferred Acceptance

- Let $\mu(m) = m \ \forall m$
- Let $S = \{m : \mu(m) = m\}$

While $\exists m \in S$

a) m applies to favorite w who has not yet rejected him

Claim Stable $\Rightarrow$ PE

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m-DA on example

1) $m_1 \to w_1$: $\mu = \{(m_1, w_1)\}$
2) $m_2 \to w_2$: $\mu = \{(m_1, w_1), (m_2, w_2)\}$
3) $m_3 \to w_2$: $\mu = \{(m_1, w_1), (m_3, w_2)\}$
4) $m_2 \to w_3$: $\mu = \{(m_1, w_1), (m_2, w_3), (m_3, w_2)\}$

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w-DA on example

1) $w_1 \to m_3$: $\mu = \{(w_1, m_3)\}$
2) $w_2 \to m_1$: $\mu = \{(w_1, m_3), (w_2, m_1)\}$
3) $w_3 \to m_3$: $\mu = \{(w_2, m_1), (w_3, m_3)\}$
4) $w_1 \to m_2$: $\mu = \{(w_1, m_2), (w_2, m_1), (w_3, m_3)\}$

a) m applies to favorite w who has not yet rejected him
b) let $m' = \mu(w)$. If $m \succ_w m'$,
- $S := S \cup m'$
- $\mu(w) = m$

Thm DA is stable.

Prf. (sketch)

1) DA terminate
2) Stability: men's options get worse, women's " better

Every person prefers outcome when they propose!

Claim: Unique man-opt / woman-pessimal SM + m-DA finds it (in fact, form a lattice)

Claim: m-DA is strategyprf for men but not for women