Socially Fair Clustering

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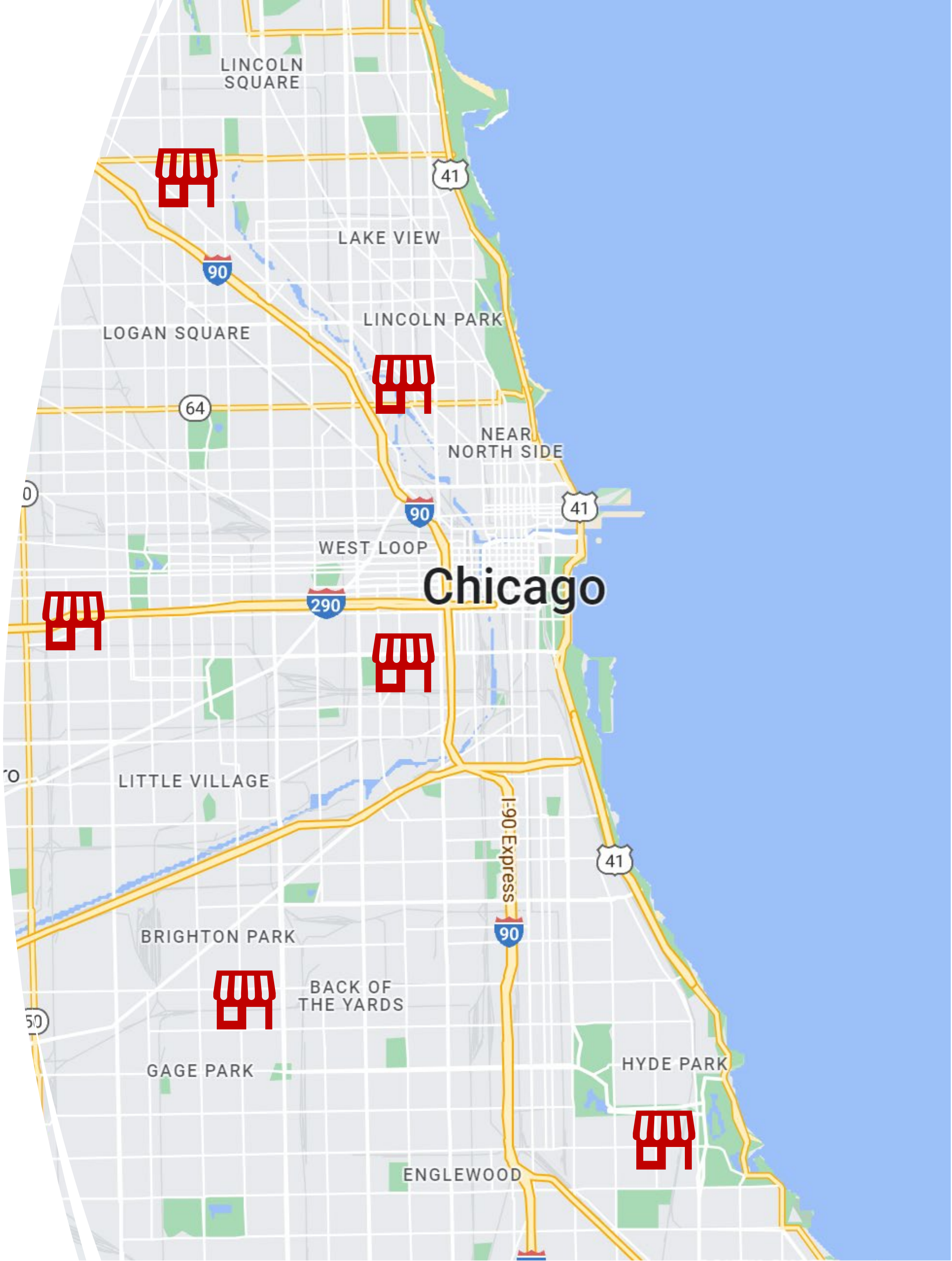
Fairness in Facility Location

Choose locations for stores/hospitals/fire stations/etc so as to minimize the average distance from people to these facilities.

+ fair for minority groups



Fair clustering

Given:

- a set of points X and a distance function d on X.
- a list of groups $G_1, \dots, G_\ell \subset X$

Centers and clustering:

A set of centers $\{c_1, \dots, c_k\}$ defines the Voronoi clustering: cluster C_i consists of the points that are closer to c_i than to other centers

Cost function:

Let

$$\mathrm{cost}(j, C) = \frac{1}{|G_j|} \sum_{u \in G_j} d(u, C)^p.$$

$$\mathrm{cost}(C) = \max_{1 \le j \le \ell} \mathrm{cost}(j, C)$$

Known Results for k-medians and k-means

k-medians:

$6\frac{2}{3}$	Charikar, Guha, Tardos, Shmoys '02
2.675	Byrka, Pensyl, Rybicki, Srinivasan, Trinh '14

k-means:

6.357	Ahmadian, Norouzi-Fard, Svensson, Ward '17

Known Results

In the context of socially fair clustering, the problem was introduced by

Abbasi, Bhaskara, and Venkatasubramanian (2021) for $p = 1{,}2$

Ghadiri, Samadi, and Vempala (2021) for p = 2

They gave

- an $O(\ell)$ approximation algorithm
- a matching integrality gap of $\Omega(\ell)$
- a bicriteria approximation algorithm

Anthony, Goyal, Gupta, and Nagarajan (2010) studied the problem in the context of “robust clustering” and gave an $O(\log n + \log \ell)$ approximation algorithm for $p = 1$.

Known Results

Bhattacharya, Chalermsook, Mehlhorn, and Neumann (2014):
The problem doesn't admit a better than $O\left(\frac{\log \ell}{\log\log \ell}\right)$ approximation unless $NP \subset \bigcap DTIME(2^{n^\delta})$.

M, Vakilian (2021): There is an $O\left(\frac{\log \ell}{\log\log \ell}\right)$ approximation algorithm for every p (the constant in $O(\cdot)$ depends on p).

Our Setting

Original setting:

$$\mathrm{cost}(j, C) = \frac{1}{|G_j|} \sum_{u \in G_j} d(u, C)^p$$

More general setting:

$$\mathrm{cost}(j, C) = \sum_{u \in X} w_j(u)\, d(u, C)^p$$

In particular, we may let

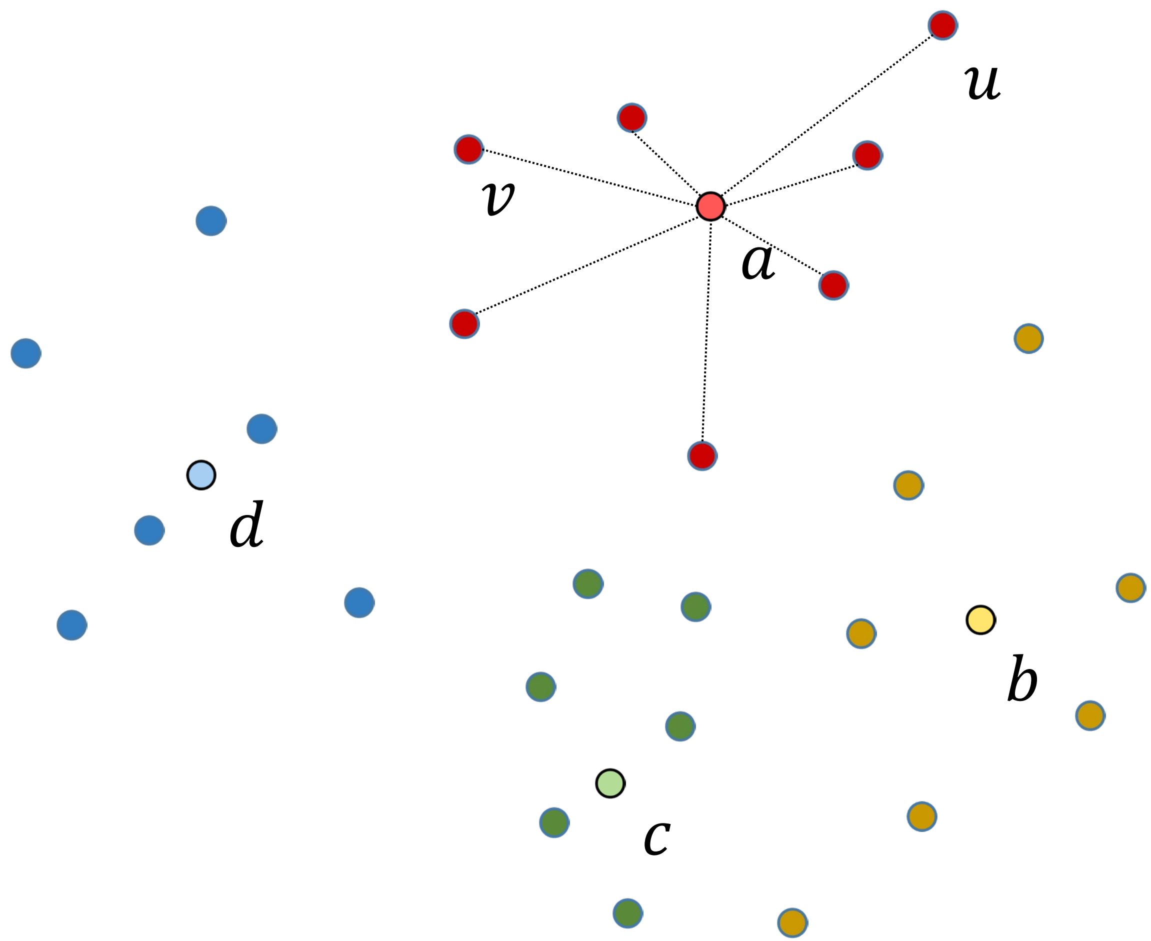
$$w_j(u) = \frac{1}{|G_j|} \text{ if } u \in G_j \text{ and } \ldots = 0, \text{otherwise}$$

Basic LP Relaxation

LP variables

x_{uv} is the indicator variable of the event that u is assigned to center v

y_v is the indicator variable of the event that v is a center



$$y_a = y_b = y_c = y_d = 1$$
$$y_u = y_v = \cdots = 0$$

$$x_{ua} = x_{vb} = \cdots = 1$$
$$x_{ub} = x_{uc} = x_{ud} = 0$$

Basic LP Relaxation

minimize z

s.t.

$z \geq \sum_{uv} w_j(u) d(u,v) \cdot x_{uv}$ for all $j = 1, \ldots, \ell$

$\sum_v x_{uv} = 1$ every u is assigned to some center

$\sum_v y_v \leq k$ there are at most k centers

$x_{uv} \leq y_v$ u is assigned to center v, only if v is a center

$x_{uv}, y_v \geq 0$