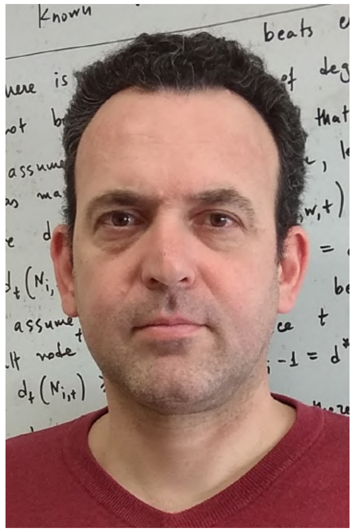
Optimizing over Serial Dictatorships



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Motivation

<u>SERIAL DICTATORSHIP:</u> an *action sequence* $(\sigma_1, \sigma_2, \ldots, \sigma_n)$ of *n* agents, where each agent picks the best available option at her turn

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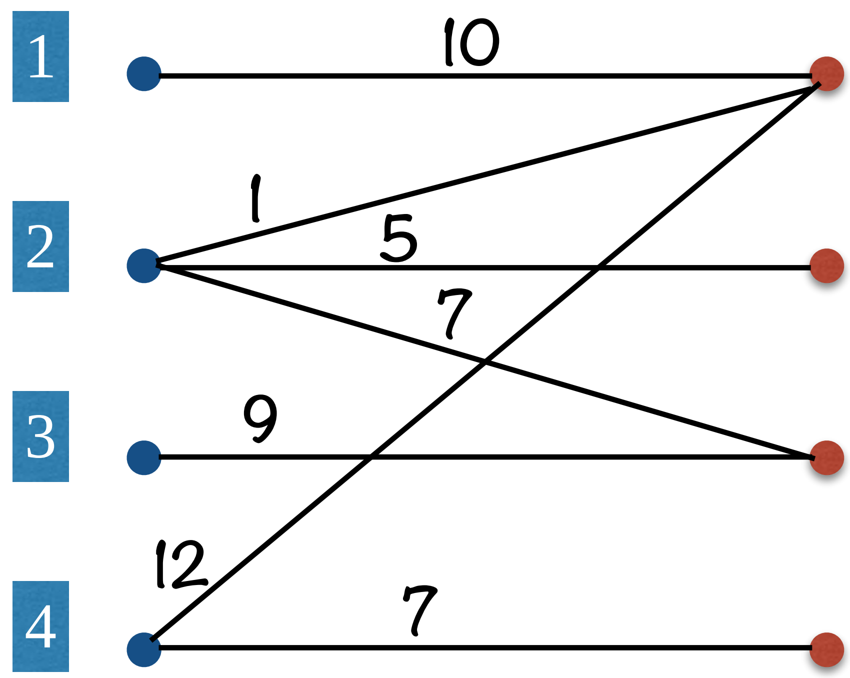

remaining edge weights = 0

Complete weighted bipartite graph

Goal: ***Maximum-weight matching***

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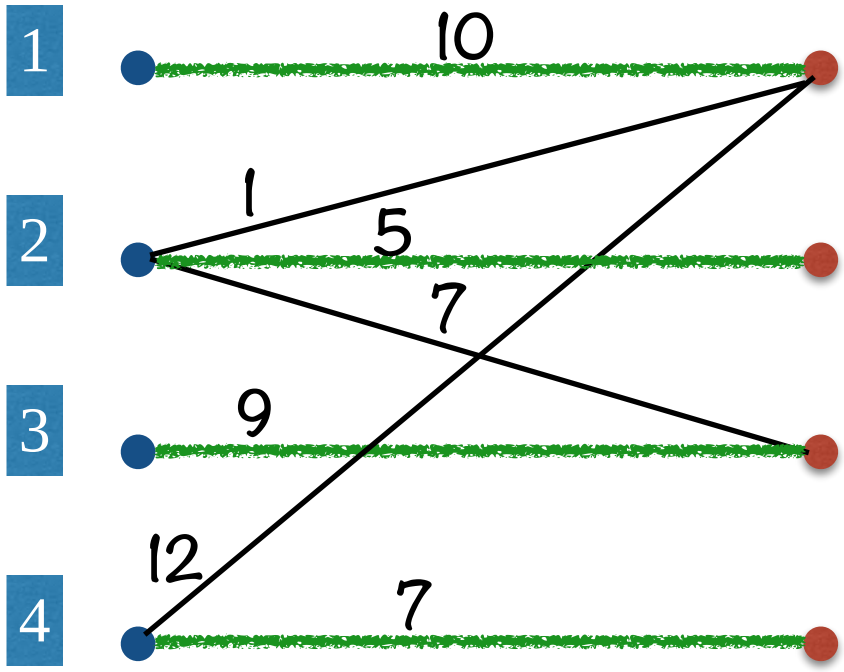

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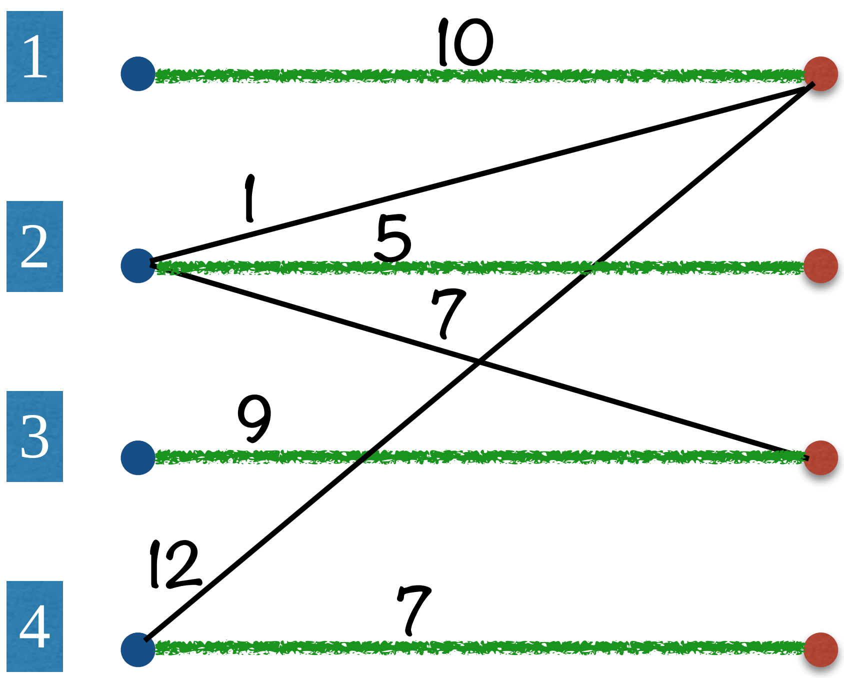

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Action sequence: 1 4 3 2 produces the maximum-weight matching

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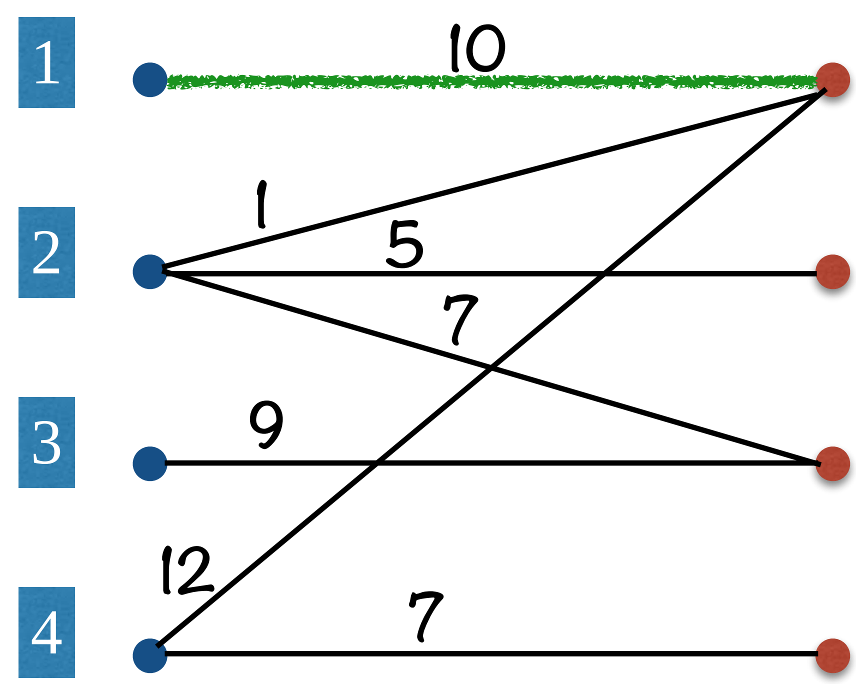

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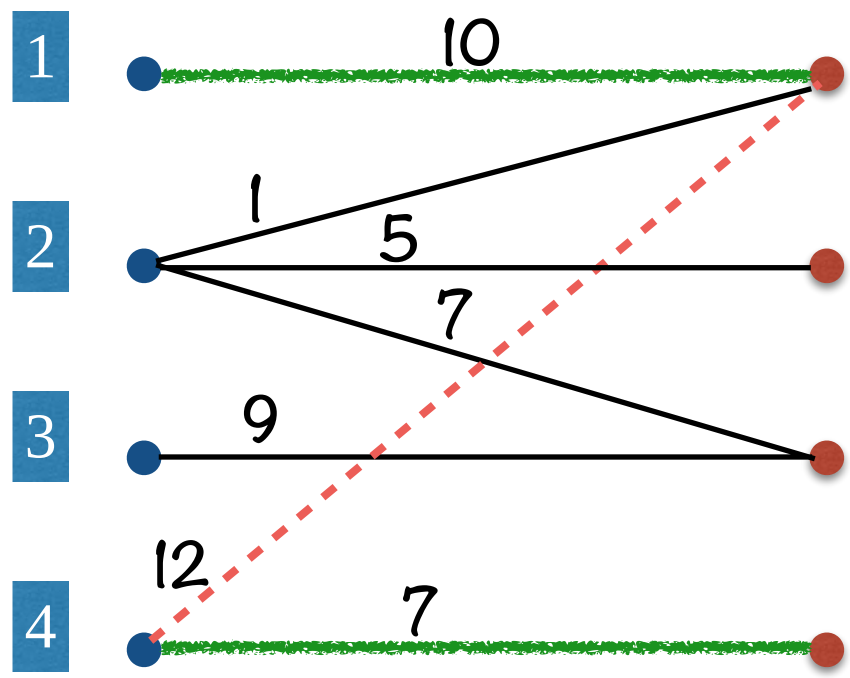

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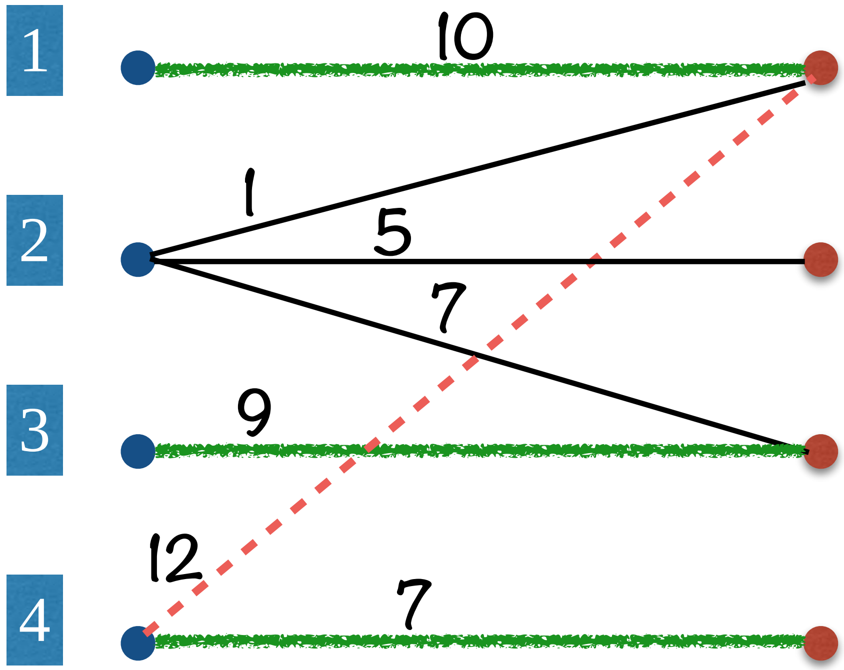

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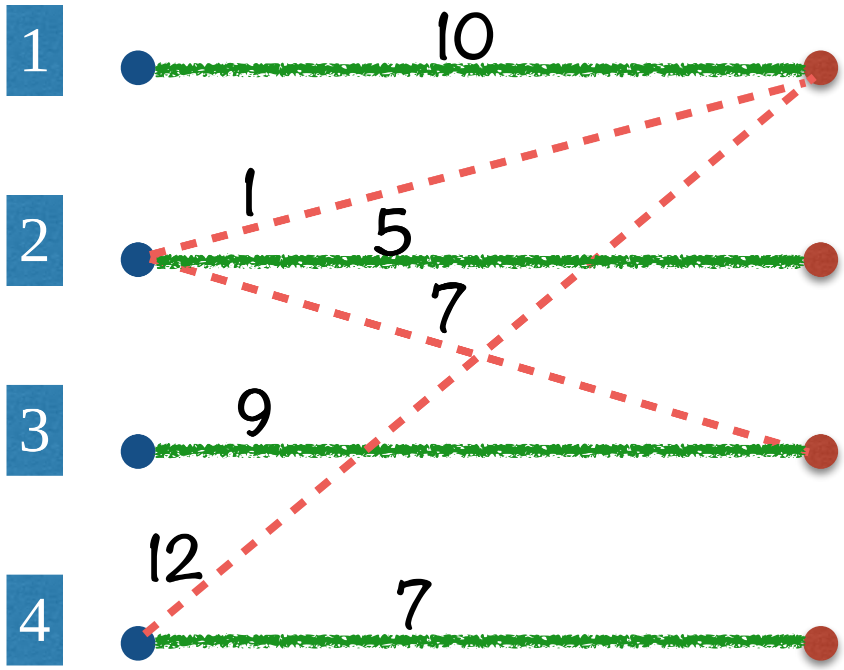

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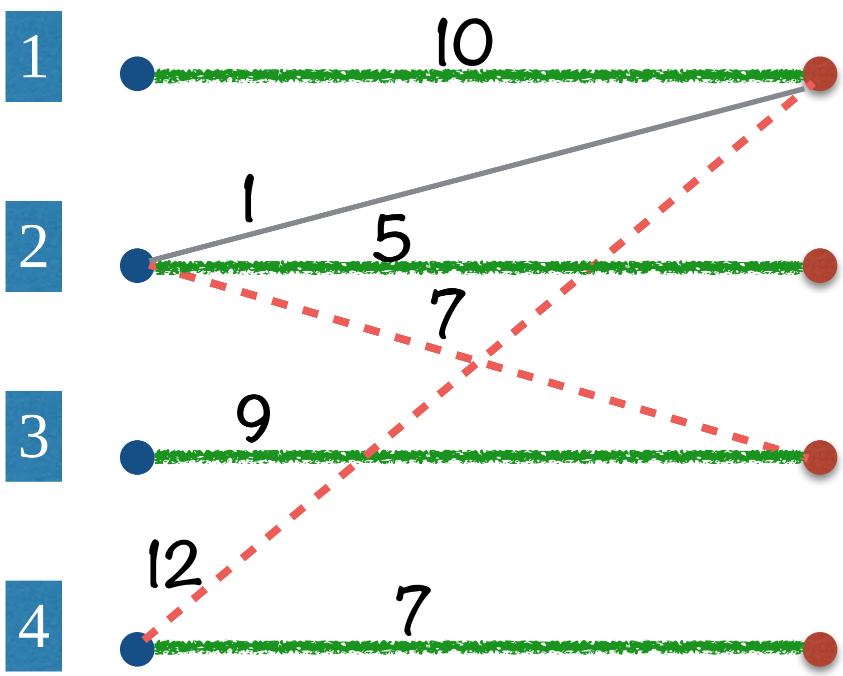

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Theorem: Any *max-weight matching* in a complete weighted bipartite graph, can *always* be induced by an *action sequence* of *n* agents.

SERIAL DICTATORSHIP: an *action sequence* $(\sigma_1, \sigma_2, \ldots, \sigma_n)$ of *n* agents,
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General Query model

- A set $\{1, 2, \ldots, n\}$ of *n* entities
- *Monotone* valuation functions, $v_i : \mathcal{S} \to \mathbb{R}_+$ for all $i \in [n]$
  ($\mathcal{S}$ is the set of all *ordered* subsets of $[n] \setminus \{i\}$)

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Goal: Understand the query complexity (# value queries required) of finding an action sequence σ that optimizes $\sum_{i \in [n]} v_i(\sigma^i)$, where σ^i : prefix of i in σ

For $\sigma = (1432)$, the sum is $v_1(\phi) + v_4(1) + v_3(14) + v_2(143)$

General Query model

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- any *randomized* algorithm that makes at most $\mathcal{O}(poly(n))$ value queries has an *approximation ratio* of at least $n^{\left(\frac{\log\log n}{\log n}\right)}$.

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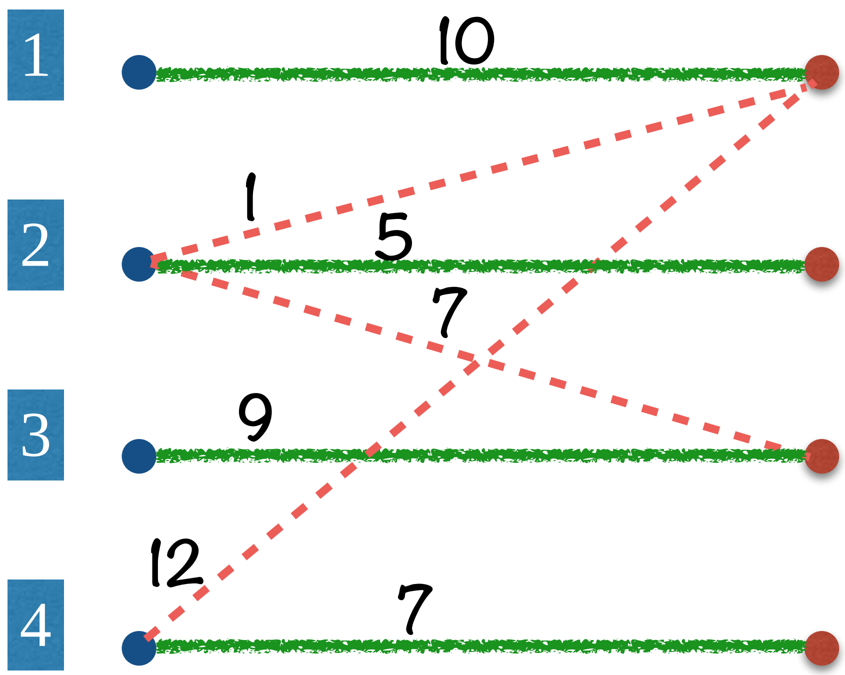
Specific Problems

Maximum weight matching:

$v_i(S) =$ value of maximum-valued item available for i, after agents in S have picked their items.



$\sigma =$ 1 4 3 2

Goal: Find an action sequence σ that maximizes the social welfare, $SW(\sigma) = \sum_{i \in [n]} v_i(\sigma^i)$ and understand its relation with the overall maximum social welfare.

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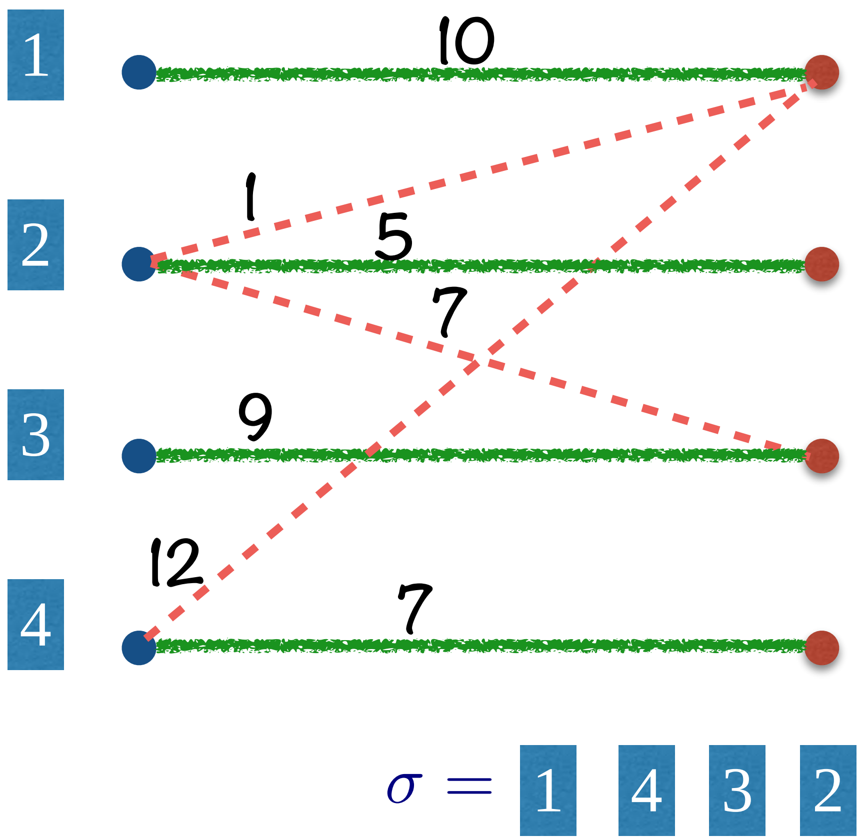

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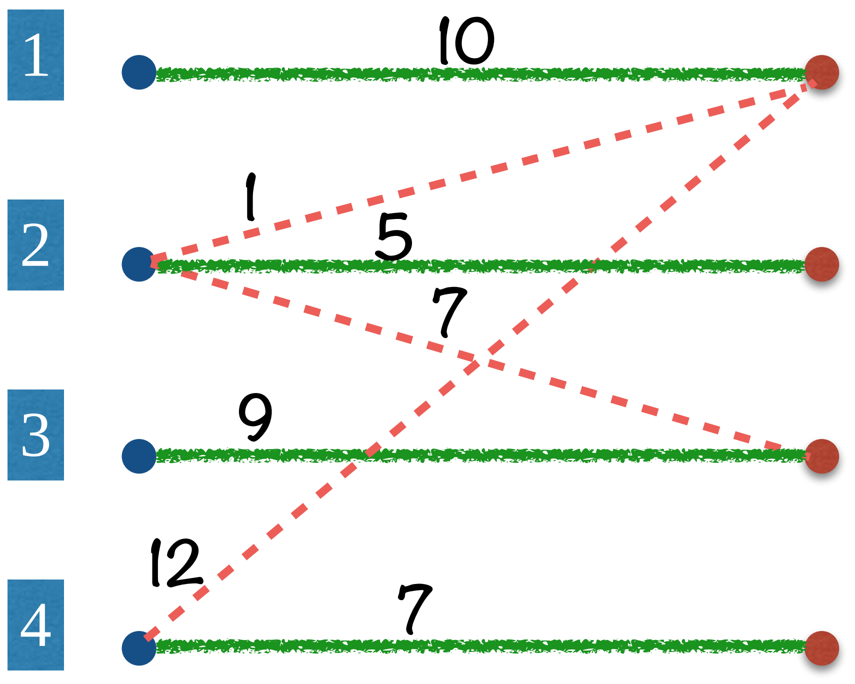

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Our results:

- Any max-weight matching **has** a corresponding *action sequence* of *n* agents that induces it.
- 2-approximation polynomial-time algorithm.
  Can we do better?

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Maximum Satisfiability (weighted version):

$v_i(S) =$ Maximum weight of ***new*** clauses satisfied by variable x_i
after the variables in ordered set S have been set as T or F.

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Our results:

- An optimal assignment for MAX-SAT may ***not*** be produced from *any* action sequence of *n* variables!
- *Given an instance of MAX-SAT, does there exist an action sequence for all 1's assignment?* NP-complete

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The Big Picture

- Introduce a *query model for understanding serial dictatorship* in the abstract setting.
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✓ Maximum-weight Matching in bipartite graph
✗ Maximum-weight Matching in non-bipartite graph
✗ Maximum Satisfiability (weighted version)
✗ Longest path with maximum-weight
✓ Maximum-weight Arborescence
Maximum-weight Cut 

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Thank you!

Specific Problems

Longest path with maximum weight:

$v_i(S) =$ Maximum weight that node i can achieve such that the underlying structure is a union of paths

Our results:

- An optimal assignment for Longest-Path may not be produced from any action sequence of n nodes.
- For any instance of Longest-Path, there always exists an action sequence that recovers **1/2** of the optimal value.

Conjecture: The above factor is 2/3.

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