1. Describe a $O(2^{\omega n/3})$-time algorithm for the Max Cut problem.

**Max Cut**

**Input:** Graph $G = (V, E)$, a number $k \in \mathbb{N}$

**Question:** Is there a subset $S \subseteq V$ such that there are at least $k$ edges between $S$ and $V \setminus S$?

2. In the weighted $k$-path problem, we are given a directed graph $G = (V, E)$ with a weight function $w : E \to \{0, \ldots, W\}$ and the goal is to find a $k$-path $P$ of smallest weight (i.e. $\sum_{u,v \in E(P)} w(u,v)$). Describe a $O(2^k \cdot W \cdot p(k, n))$-time algorithm for this problem, for some polynomial $p$.

   **Hint:** Introduce a new variable.

3. Describe a $O^*(2^{3k})$-time algorithm for the Triangle Packing problem.

**Triangle Packing**

**Input:** Graph $G = (V, E)$, a number $k \in \mathbb{N}$

**Question:** Does $G$ contain $k$ disjoint triangles?

4. In the lecture we have seen a $O^*(2^{3/4k}) = O^*(1.682^k)$-time algorithm for undirected $k$-path. This can be tuned to $O^*(1.66^k)$. Can you find a possible way of doing it? (Warning: although the idea is simple, calculating the 1.66 constant can be hard, especially without the aid of a computer.)

   **Hint:** Reduce the number of labels, but not for free.

5. Consider the following problem:

**Colorful Graph Motif**

**Input:** Graph $G = (V, E)$, a coloring $c : V \to C$, a set of colors $M$

**Question:** Is there a subset $S \subseteq V$ such that $G[S]$ is connected, and $c(S) = M$?

Denote $k = |M|$. Describe a $O^*(c^k)$-time algorithm for a constant $c$, ideally a $O^*(2^k)$-time algorithm.

You can also consider a (slightly different, perhaps slightly simpler) problem, where the vertices in the subset $S$ must form a path.