Solve the following problems.

**Problem 1: Subquadratic algorithms for 3-sum**

Let \( a_i, b_i \) and \( c_i \) for \( 1 \leq i \leq n \) be three lists of integers in the range \([0, d]\). Find an algorithm that runs in time \( O((n + d) \log(nd)) \) to determine whether there exist \( i, j \) and \( k \) such that \( a_i + b_j = c_k \). You can assume that the basic arithmetic operations on the integers in the range \([0, d]\) can be performed in unit time.

**Hint:** Use Fast Fourier Transform.

**Problem 2: Variations of the 3-sum problem**

Consider the following variation of the standard 3-SUM problem.

**Tripartite 3-sum:** Let \( a_i, b_i \) and \( c_i \) for \( 1 \leq i \leq n \) be three lists of integers in the range \([0, d]\). Determine whether there exist \( i, j \) and \( k \) such that \( a_i + b_j = c_k \).

Prove that (3-SUM, \( n^2 \)) and (TRIPARTITE 3-SUM, \( n^2 \)) are fine-grained reducible to each other.

Standard 3-SUM is problem is defined below.

**3-sum:** Given a sequence of integers \( x_1, x_2, \ldots, x_n \) where \( x_i \in [0, 1, \ldots, d] \), do there exist \( i, j \) and \( k \) such that \( x_i + x_j = x_k \)?

**Problem 3: Collinear points**

Show that 3-SUM can be reduced to the following problem so that a polynomial improvement in solving the problem would result in an algorithm for 3-sum with polynomial improvement over \( n^2 \).

Given a set of points in the plane with integer coordinates, is there a line that contains at least 3 points?

**Problem 4: Orthogonal Vectors problem in low dimension**

Show that the ORTHOGONAL VECTORS problem for \( n \) vectors of dimension \( c \log n \) can be solved in time \( n^{c+1} \).
Problem 5: Minimum Hamming Distance problem

Assuming SETH, show that for any $\varepsilon > 0$ there is a $c$ such that solving the minimum distance problem on $d = c \log n$ dimensions requires time $\Omega(n^{2-\varepsilon})$.

Minimum Hamming Distance problem: For any $d$ and $l$, decide if two input sets $U, V \subseteq \{0, 1\}^d$ with $|U| = |V| = n$ have a pair $u \in U$ and $v \in V$ such that $\|u - v\|_2^2 < l$. 