Chapter X: Classification*

1. Basic idea
2. Decision trees
3. Naïve Bayes classifier
4. Support vector machines
5. Ensemble methods

* Zaki & Meira: Ch. 24, 26, 28 & 29; Tan, Steinbach & Kumar: Ch. 4, 5.3–5.6

X.5 Ensemble methods*

1. Basic idea
2. Bagging
3. Boosting
   - 3.1. AdaBoost

* Zaki & Meira: Ch. 29; Tan, Steinbach & Kumar: Ch. 5.6; Bishop: Ch. 14.2–3

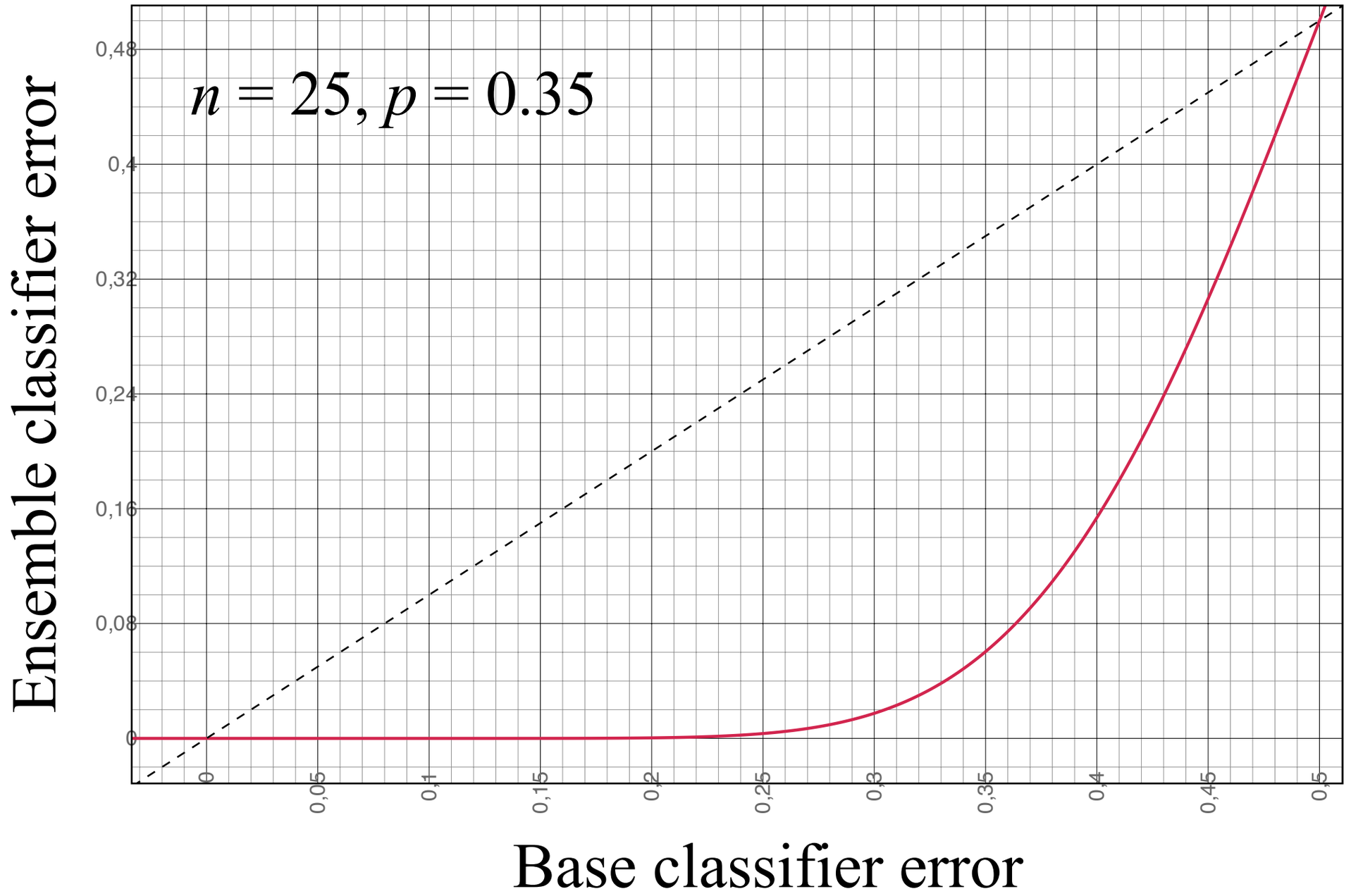
Basic idea

- Suppose we have multiple classifiers for the data
  - Each is good in some parts and bad in other parts
- Can we get better results if we combine these methods?
- How can combine them?
  - Simple committee solution: take the majority label
  - If we have confidence to the classifiers, we can weight their solutions and take the weighted majority label

Rationale of ensembles

- 25 binary classifiers (*base classifiers*)
  - Each base classifier has error rate 0.35
  - Majority vote to select the class label
- If the base classifiers are identical, the ensemble will have error rate 0.35
  - If the base classifiers are independent, the ensemble will have error rate $\Pr[X \geq 13]$ with $X \sim \text{Binom}(25, 0.35)$
  - $\Pr[X \geqslant 13] = \sum_{i=13}^{25} \binom{25}{i} 0.35^i (1 - 0.35)^{25-i} = 0.06$
- Two conditions:
  - Base classifiers must be (reasonably) independent
  - Base classifiers must do better than purely random

Ensemble error example



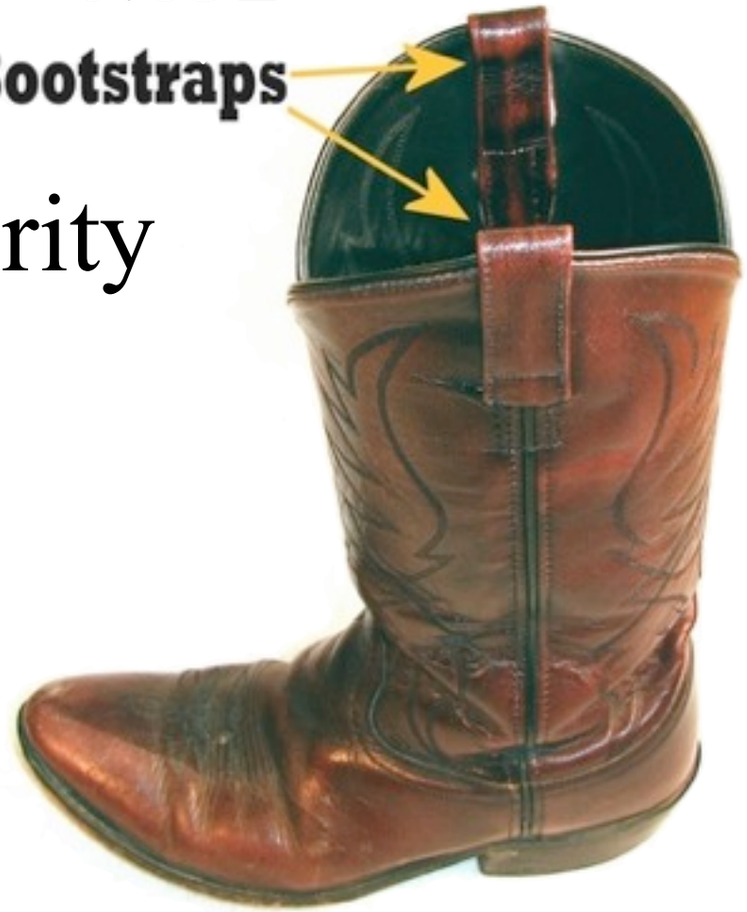
How to make independent classifers

- Manipulate the training set
  - Bagging
  - Boosting
- Manipulate the input features
  - Random forest
- Manipulate the class labels
  - Different splits from multi-class to two-class
- Manipulate the learning algorithm
  - Add randomness

Bagging (a.k.a. bootstrapping)

- Sample the data *uniformly with replacements*
  - Each sample D_i has the same size as the original data D
  - Each data point $\boldsymbol{x} \in D$ has
    $\Pr[\boldsymbol{x} \in D_i] = 1 - (1 - 1/|D|)^{|D|} \rightarrow 1 - 1/e \approx 0.632$
    when $|D| \rightarrow \infty$
- Final classifier usually uses the majority voting

Image: http://www.lemen.com



Bagging example

x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y	+1	+1	+1	–1	–1	–1	–1	+1	+1	+1

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$x \leq 0.35$

Single one-split decision tree's accuracy $\leq 70\%$

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10 bagging samples:

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x	0.1	0.1	0.1	0.1	0.3	0.3	0.8	0.8	0.9	0.9
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x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Σ	2	2	2	–6	–6	–6	–6	2	2	2

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2-level decision tree!

x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Σ	2	2	2	–6	–6	–6	–6	2	2	2

Boosting

- In bagging, each item gets sampled u.a.r. independent of how hard they're to classify
  - Easy items don't need ensembles
  - Shouldn't we concentrate on hard cases?
- Boosting adds weights to the training items
  - More often misclassified items get bigger weights
  - Weights can be used to learn biased classifiers
    - Pay more for misclassifying heavier items
  - Weights can be used to weight bootstrap sampling
    - Heavier items get selected more often

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This is what we cover

Basic idea

1. Initialize all weights to $1/N$
   - Uniform distribution
2. Perform classification
3. Increase the weights of misclassified items and reduce the weight of correctly classified items

4. Aggregate the predictions

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1. Initialize all weights to $1/N$
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4. Aggregate the predictions

- Methods differ on how the weights are changed and how the aggregation works

AdaBoost

- Let C_i be a base classifier
  - *Error rate* of C_i is $\epsilon_i = N^{-1} \sum_{j=1}^{N} w_j \mathbf{1}(C_i(\mathbf{x}_j) \neq y_j)$
    - $\mathbf{1}(p) = 1$ if p is true and 0 o/w
    - w_j is the weight of training item $(\boldsymbol{x}_j, y_j)$
  - *Importance* of C_i is $\alpha_i = \ln(1/\epsilon_i - 1)/2$
  - *Weight* of $(\boldsymbol{x}_j, y_j)$ for iteration i+1 is

$$w_j^{i+1} = \frac{w_j^{(i)}}{Z_i} \times \begin{cases} e^{-\alpha_i} & \text{if } C_i(\mathbf{x}_j) = y_j \\ e^{\alpha_i} & \text{if } C_i(\mathbf{x}_j) \neq y_j \end{cases}$$

    - Z_i is a normalization constant s.t. w_j's sum to 1
  - If error rate goes above 0.5, all weights are set to $1/N$
- For aggregation, each classifier is weighted by α_i

AdaBoost

Importance w.r.t. error rate

- Let C_i be a base classifier
  - *Error rate* $\ldots \neq y_j)$
    - $\mathbf{1}(p) = 1$ if
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  - *Importance*
  - *Weight* of (

$$w_j^{i+1} = \frac{}{}$$

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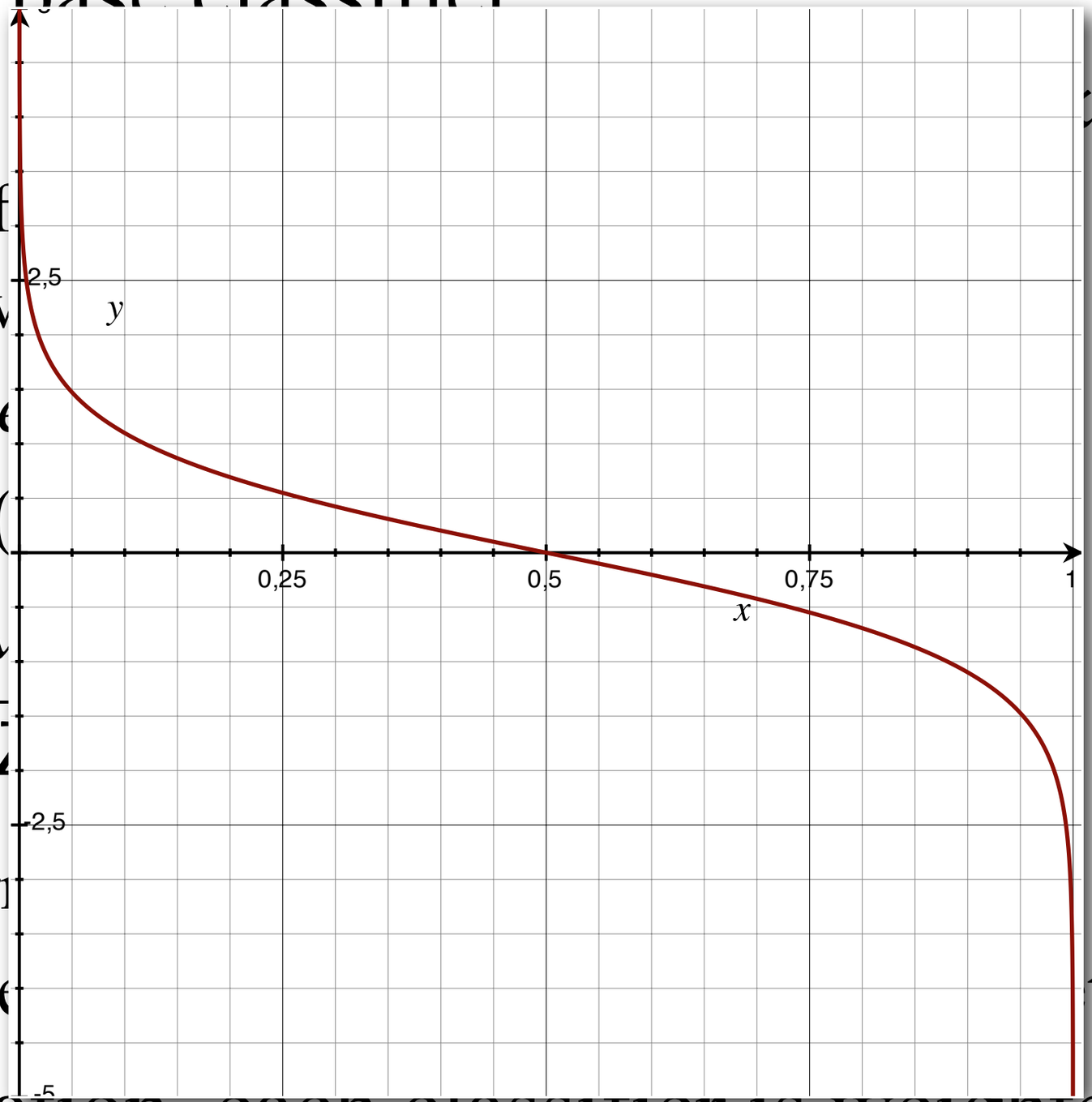

error rate

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Error rate

- Let ε_i be the error rate of classifier i in boosting
  - Let $\varepsilon_i < 0.5$ for all i and write $\varepsilon_i = 0.5 - \gamma_i$
- The total error rate of the ensemble can now be bounded by

$$\epsilon_E \leqslant \prod_i \sqrt{\epsilon_i(1-\epsilon_i)} = \prod_i \sqrt{1/4 - \gamma_i^2}$$

$$= \exp\left(-O\left(\sum_i \gamma_i^2\right)\right)$$

  - The error decreases exponentially if $\gamma_i > \gamma^* > 0$ for all i's
    $\Rightarrow$ Fast convergence

Summary of ensemble methods

- Combining many classifiers usually helps
- Base classifiers must be reasonably independent
- Bagging is simple, but doesn't improve bad classifiers much
- Boosting helps in particular with almost-random classifiers
  - Boosting is prone to overfitting
- How many classifiers can be combined depends on how much time we can spend