Global, Isometric, Pairwise: Isometric Matching and Quadratic Assignment
How many meshes?

- **Two**: Pairwise registration
- **More than two**: multi-view registration

Initial registration available?

- **Yes**: Local optimization methods
- **No**: Global methods

Class of transformations?

- **Rotation and translation**: Rigid-body
- **Non-rigid deformations**
Overview and Motivation
Global Isometric Matching

Goal

- We want to compute correspondences between deformable shape
- *Global algorithm*, no initialization
Global Isometric Matching

Approach & Problems
• Consistency criterion: global isometry

Problem
• How to find globally consistent matches?

Model
• Quadratic assignment problem
  ▪ General QA-problem is NP-hard
  ▪ But it turns out: Isometric matching can be solved more efficiently
Feature Based Matching

Quadratic Assignment Model
Feature-Matching

- Detect feature points

- Local matching: potential correspondences

- Global filtering: correct subset
Matching model

- Preserve *descriptors*
- Preserve *geometric relations*
Quadratic Assignment

- $n$ potential correspondences
- Each one can be turned on or off
- Label $x_i$
Quadratic Assignment

- Compatibility score:
  - Singeltons: Descriptor match

\[ x_i = 0 \]

\[ x_j = 1 \]
Quadratic Assignment

- Compatibility score:
  - **Singeltons:**
    - Descriptor match
  - **Doubles:**
    - Compatibility

\[ x_j = 1 \]
Quadratic assignment problem

Quadratic Error Score:

\[ E(x_1, \ldots, x_n) = xs + x^T Dx \]

- Pairwise scores are encoded in Matrix \( D \)
- Linear scores are encoded in Vector \( s \)
- Task: find optimal binary vector \( x \)

Trade Off:

- Maximize number of matches
Randomized Matching
Deformable Matching

- Two shapes: original, deformed
- How to establish correspondences?
- Looking for global optimum
  - Arbitrary pose

Assumption

- Approximately isometric deformation
RANSAC/FWS Algorithm

**Idea**
- Starting correspondence
- Add more that are consistent
  - Preserve intrinsic distances
- Importance sampling algorithm

**Advantages**
- Efficient (small initial set)
- General (arbitrary criteria)
Step 1: Start with one correspondence

- Importance sampling: prefer good descriptor matches
Step 2: „Posterior“ using geodesics

- Importance sampling:
  sample according to descriptor match $\times$ distance score
Step 2: „Posterior“ using geodesics

- Importance sampling:
  sample according to descriptor match × distance score
Step 3:

- Same as step 2, continue sampling...
Step 3:

- Same as step 2, continue sampling...
Typically: 100 random trials required
Step 1: Start with one correspondence
- Target side importance sampling: prefer good descriptor matches
- Source side importance sampling: descriptor entropy
Step 2: „Posterior“ using geodesics

- Target side importance sampling: sample according to descriptor match × distance score
- Source side importance sampling: minimize posterior marginals entropy
Step 3:
- Same as step 2, continue sampling...
Step 3:
- Same as step 2, continue sampling...
Entropy-based planning

- More efficient
- Typically 1-15 trials
Landmark Coordinates

- Distance to already established points give a charting of the manifold
Robust Landmark Distances

- Gaussian noise
- Likelihood
- Deviation
Variance Provides Additional Cues

Numerical Instability:

Missing Geometry:
Results: Topological Noise

Spectral Quadratic Assignment
[Leordeanu et al. 05]

Ransac Algorithm
[Tevs et al. 09]
Global Animation Reconstruction

input data  reconstruction  correspondences

[Data set: Hao Li, ETH Zürich]