Computing Correspondences in Geometric Datasets

4.2 Symmetry & Symmetrization
Symmetry

• Invariance under a class of transformations

  – global vs. partial
  – exact vs. approximative
Symmetry

- Example: Reflection in 2D
Symmetry

• Example: Reflection in 2D
Symmetry

- Example: Reflection in 2D
Symmetry

• Example: Reflection in 2D
Symmetry

• Example: Reflection in 2D
Symmetry

• Example: Reflection in 2D
Symmetry

• Local analysis: Symmetry as a pair relation
Symmetry

- Local analysis: Symmetry as a pair relation
Symmetry

• Local analysis: Symmetry as a pair relation
Symmetry

- Local analysis: Symmetry as a pair relation

![Diagram showing spatial domain and transformation space with a point in each.](image)
Symmetry

- Local analysis: Symmetry as a pair relation

\[ y \]
\[ \phi \]

local evidence for symmetry plane

spatial domain

transformation space

\[ x \]
\[ d \]
Symmetry

• Accumulation of local evidence

\[ y \]
\[ x \]

spatial domain

\[ \phi \]
\[ d \]

transformation space

local evidence for symmetry plane
Symmetry

• Accumulation of local evidence

  – clustering to extract symmetry transformation
  – verification to extract symmetric patches
Symmetry

- Accumulation of local evidence

\[ E(n, \mu, \Delta\sigma) < \left(1 - \sqrt{-2\log \alpha/np}\right) np/2^d \]
Sydney Opera House
Sydney Opera House
Articulated Shapes

- Random samples on two poses
  - Correspondences between points are not known
Articulated Shapes

• Correspondence candidates
Articulated Shapes

segmentation

transform plot
Symmetrization

• Goal: Symmetrize 3D geometry

• Applications
  – reverse engineering
  – recognition, retrieval
  – compression
  – symmetric meshing, etc.

• Approach
  – Minimally deform the model by optimizing the distribution in transformation space
Optimal Displacements

- Find minimal displacements that make two points symmetric with respect to a given transformation
Optimal Displacements

- Find minimal displacements that make two points symmetric with respect to a given transformation

\[
\begin{align*}
&\text{minimize} \\
&\|d_i\|^2 + \|d_j\|^2
\end{align*}
\]
Optimal Displacements

- Find optimal transformation and minimal displacements for a set of corresponding points
Optimal Displacements

• Find optimal transformation and minimal displacements for a set of corresponding points
Optimal Displacements

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Optimal Displacements

• Find optimal transformation and minimal displacements for a set of corresponding points

– closed form solution exists!
Optimization

• Embedded deformation
Optimization

• Embedded deformation
Optimization

- Embedded deformation
Optimization

• Embedded deformation
Optimization

• Embedded deformation
Symmetrization

- 2D Example
Symmetrization

• 2D Example

original

transformation space
Symmetrization

• 2D Example

original

transformation space
Symmetrization

• 2D Example
Symmetrization

• 2D Example

transformation space
Symmetrization

• 2D Example

transformation space
Symmetrization

- 2D Example
Symmetrization

• 2D Example
Dragon
Dragon
Dragon
Symmetrizing the Dragon

Cluster Contraction
Shape Matching

Cluster Merging
References
