Markerless Correspondence
Symmetry Detection and Applications
Correspondence Detection

Given two surfaces, find a set of corresponding points.
**Goal:** Find correspondences likely to participate in an isometry (=geodesic distance preserving)

**Method:** Use the Möbius group as low DOF model for non-rigid alignment.

**Rationale:**
- 6 DOF of the Möbius group
- contains perfect isometries

for devising randomized geometric algorithm.
Mobius Transformation

- All the global 1-1 and onto conformal map on the sphere.

\[ f(z) = \frac{az + b}{cz + d}, \quad ad - bc \neq 0 \]

\[ a, b, c, d \in \mathbb{C} \]

- 6 DOF: prescribing three points uniquely defines a Möbius transformation.

\[ f(z_i) = y_i, \quad i = 1, 2, 3 \quad \Rightarrow \quad (a, b, c, d) \]
Algorithm for Perfect Isometries

\[ \Psi \circ g \circ \Phi^{-1} \in \text{Conformal} \quad \Rightarrow \quad \Psi \circ g \circ \Phi^{-1}(z) = \frac{az + b}{cz + d} \]

search the Möbius group (6 DOF) for your correspondence
Algorithm for Perfect Isometries
Algorithm for Perfect Isometries

A

B

These “triangle laces” are the discrete uniformization to be explained…

3 Correct Correspondences

Symmetry: Mobius Voting
Algorithm for Perfect Isometries

Polynomial time \( O(N^3) \) triplets for discovering isometries!
Even the same shape in different pose is hardly exactly isometric so single global Möbius is not enough...

Furthermore, we want to compare different (non-isometric) surfaces...

How do we extend to “near isometries” – with Voting, locality
Voting for Imperfect Isometries

Symmetry: Mobius Voting
Symmetry: Möbius Voting

Voting for Imperfect Isometries

Key: Uniformization is local
Voting for Imperfect Isometries
Algorithm Overview

sample points  Uniformization  Voting  Extracting Correspondences

Symmetry: Mobius Voting
Algorithm Stages

Sampling points

Uniformization

Scoring Votes
Algorithm Stages

Sampling points

Uniformization

Scoring Votes
Sampling points

Sample by:
1) Extrema of Gauss curvature (isometry invariant)
2) Uniform samples

Each point represents a surface patch of “equal importance”

\[ \{ z_1, z_2, \ldots, z_N \} \quad \text{and} \quad \{ w_1, w_2, \ldots, w_N \} \]
Algorithm Stages

Sampling points

Uniformization

Scoring Votes
Map the surface to space where Möbius is easy to apply and the metric represented by density.

Every genus-0 surface can be mapped globally to a sphere conformally (angle preserving).
Uniformization

Natural definition of discrete conformal: \textit{piecewise similarity}

\[ T = s \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \]

Too many constraints: generally \textbf{not possible}.

\textbf{Possible}: using Pinkall & Polthier [93] conjugate discrete harmonics.

Symmetry: Mobius Voting
Uniformization

\( \{ w_1, w_2, \ldots, w_N \} \)

Symmetry: Mobius Voting
Algorithm Stages

Sampling points

Uniformization

Scoring Votes
Scoring votes

Symmetry: Mobius Voting
The vote value is the transportation “effort”:

\[
E(c) = \int_{\mathbb{C}} d(z, c(z))d\lambda \approx \sum_k d(z_k, c(z_k))\text{area}(\Omega_k) \quad d(z, w) = \frac{|z - w|}{1 + \bar{z}w}
\]
Scoring Votes

Symmetry: Mobius Voting
Results

Symmetry: Mobius Voting
Cross Correspondence

Symmetry: Mobius Voting
Mobius Voting for Surface Correspondence, Yaron Lipman, Thomas Funkhouser, SIGGRAPH 2009.
Applications
Symmetry Detection and Applications
Pipe Tree
Random (Castle) Variations
Random (Playground) Variations
Bus Stop Variations
Relations in Man-made Objects

i) orthogonal/parallel relations; equal angle

ii) placement relation, e.g., coplanar, coaxial

iii) equal length/radii relations
Parallel/Orthogonal Relations

\[ C_o = \{ c_1, c_2, \ldots \} \]

\[ C_o^* \subset C_o \]
Equal Angle Relations
Wheel Dataset

input model

input scan

RANSAC primitives

misaligned primitive axes

aligned primitives

exactly aligned primitive axes

Symmetry: Applications
References

A Connection between Partial Symmetry and Inverse Procedural Modeling,
Martin Bokeloh, Michael Wand, Hans-Peter Seidel,
SIGGRAPH 2010.

GlobFit: Consistently Fitting Primitives by Discovering Global Relations,
Yangyan Li, Xiaokun Wu, Yiorgos Chrysanthou, Andrei Sharf,
Daniel Cohen-Or, Niloy J. Mitra,
SIGGRAPH 2011 (conditional accept).