Tutorial

Symmetry in Shapes
Theory and Practice

Niloy J. Mitra
University College London

Overview

- Introduction
  - Motivation
  - Classification
- Methods
  - Extrinsic
  - Intrinsic
- Applications

Classification

- Global vs. Partial

(a) complete symmetry group on parts of a shape

(b) partial translational symmetry

(c) partial rotational symmetry

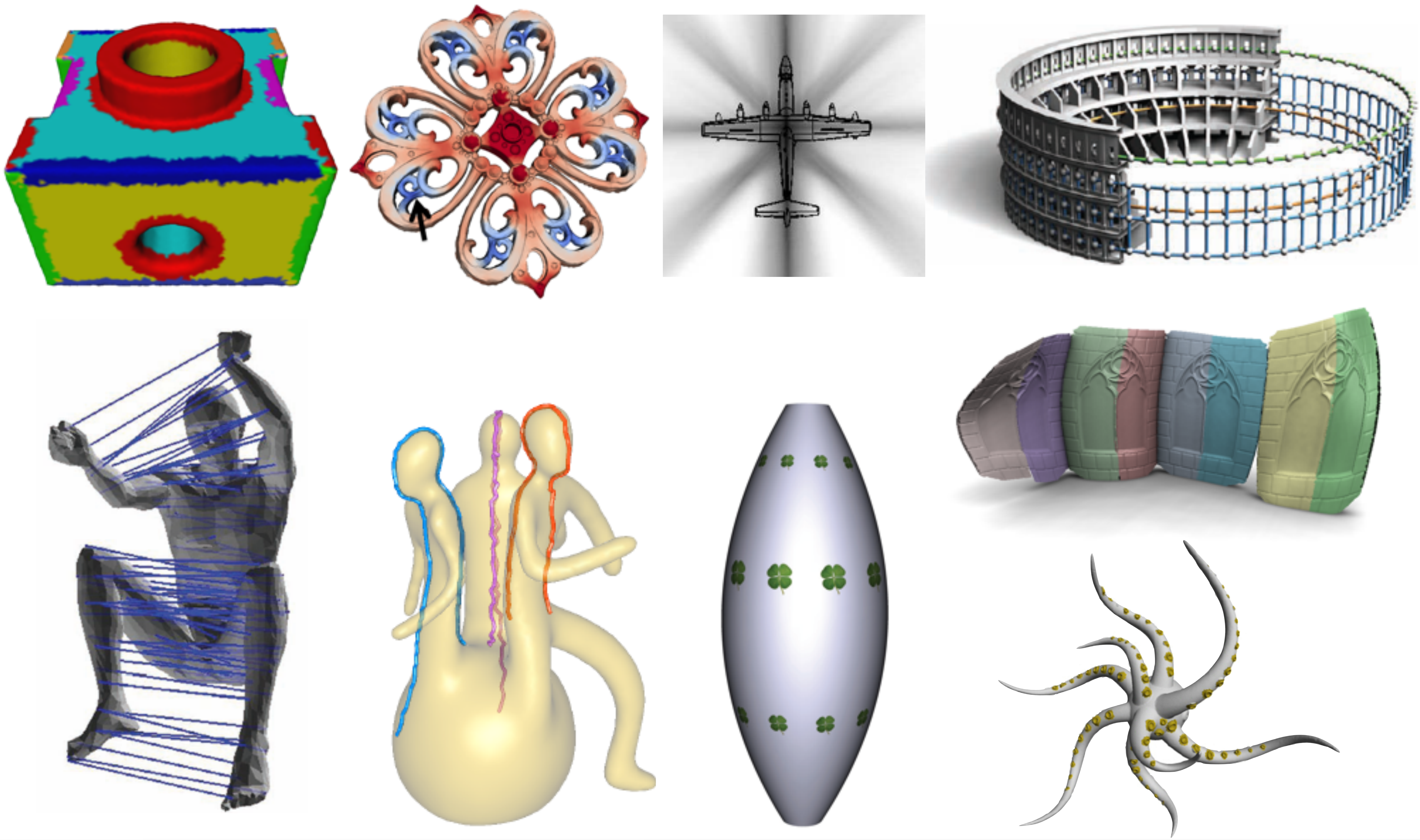
Classification

- Global vs. Partial
- Exact vs. Approximate

Classification

- Global vs. Partial
- Exact vs. Approximate
- Intrinsic vs. Extrinsic

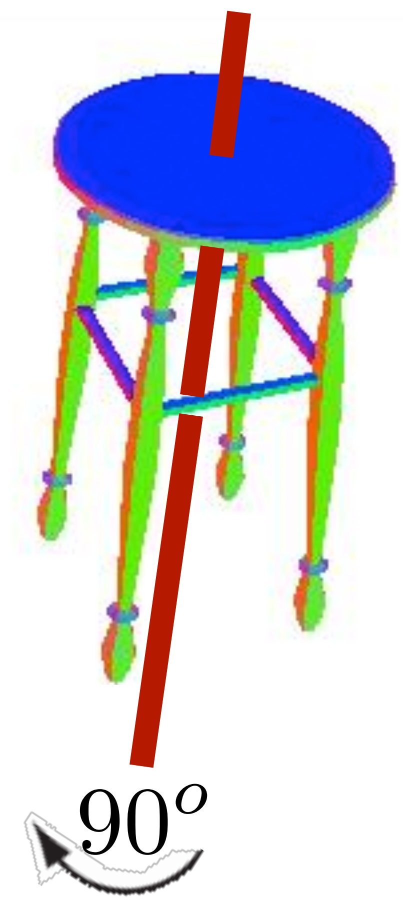
Methods Gallery



Global vs. Partial

Global vs. Partial

reflections
Kazhdan et al. 2003

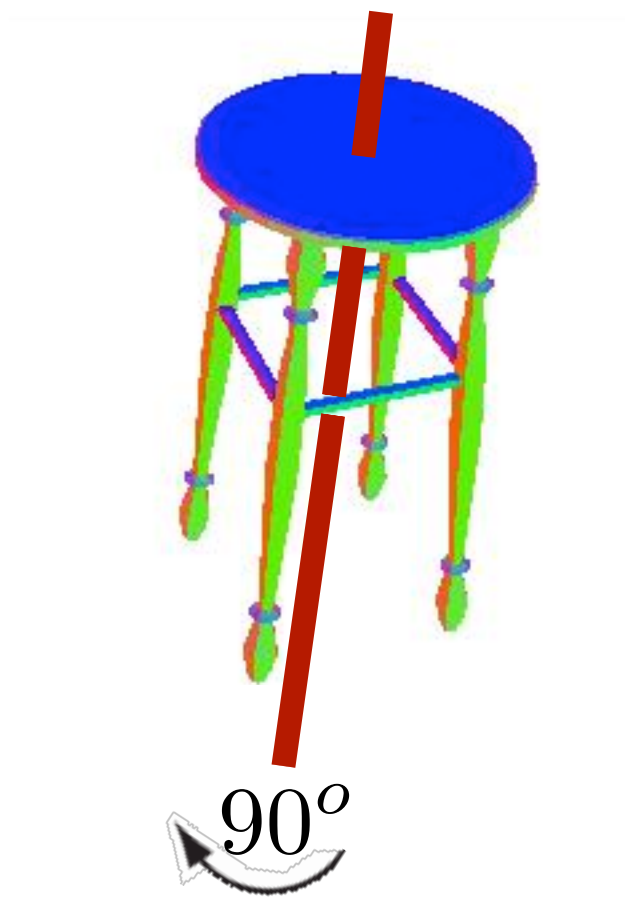


rotations
Kazhdan et al. 2004

Global vs. Partial

reflections

Kazhdan et al. 2003



rotations

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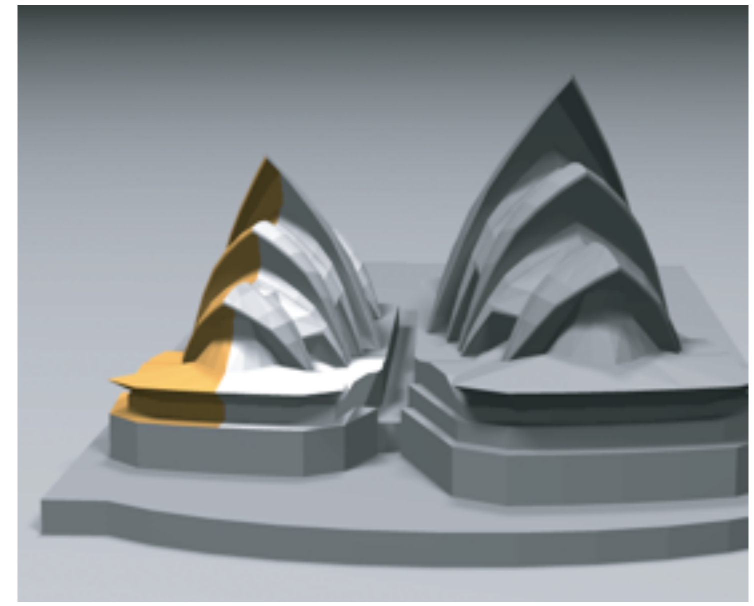
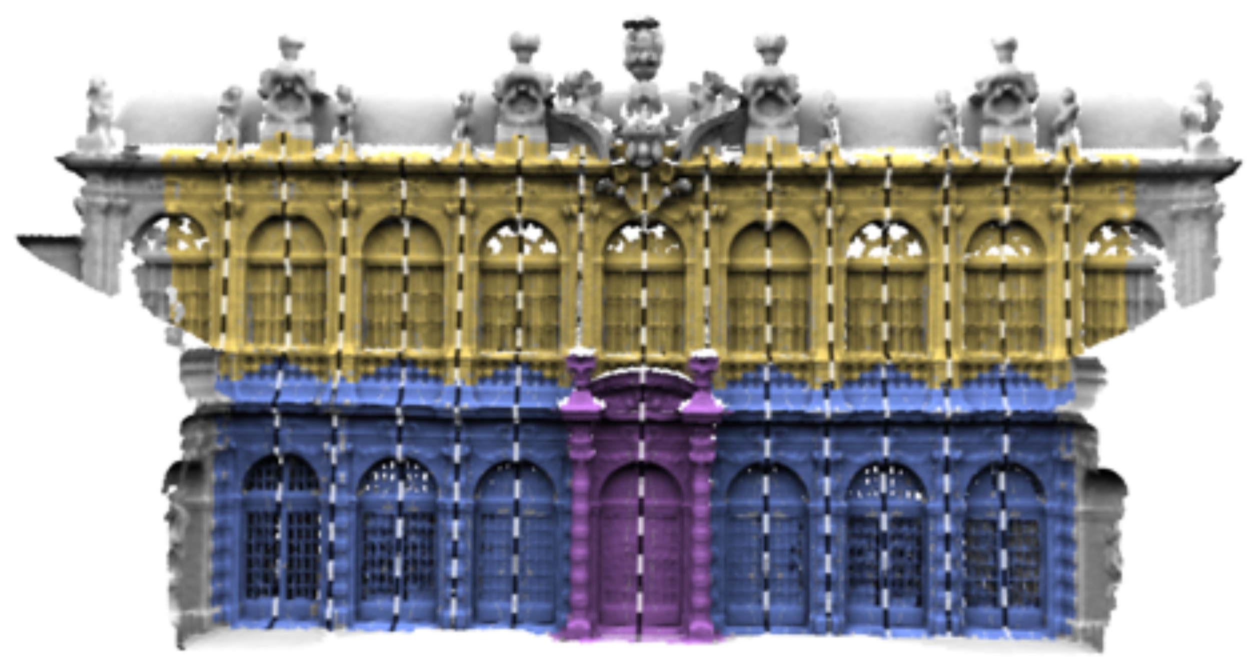


Mitra et al. 2006

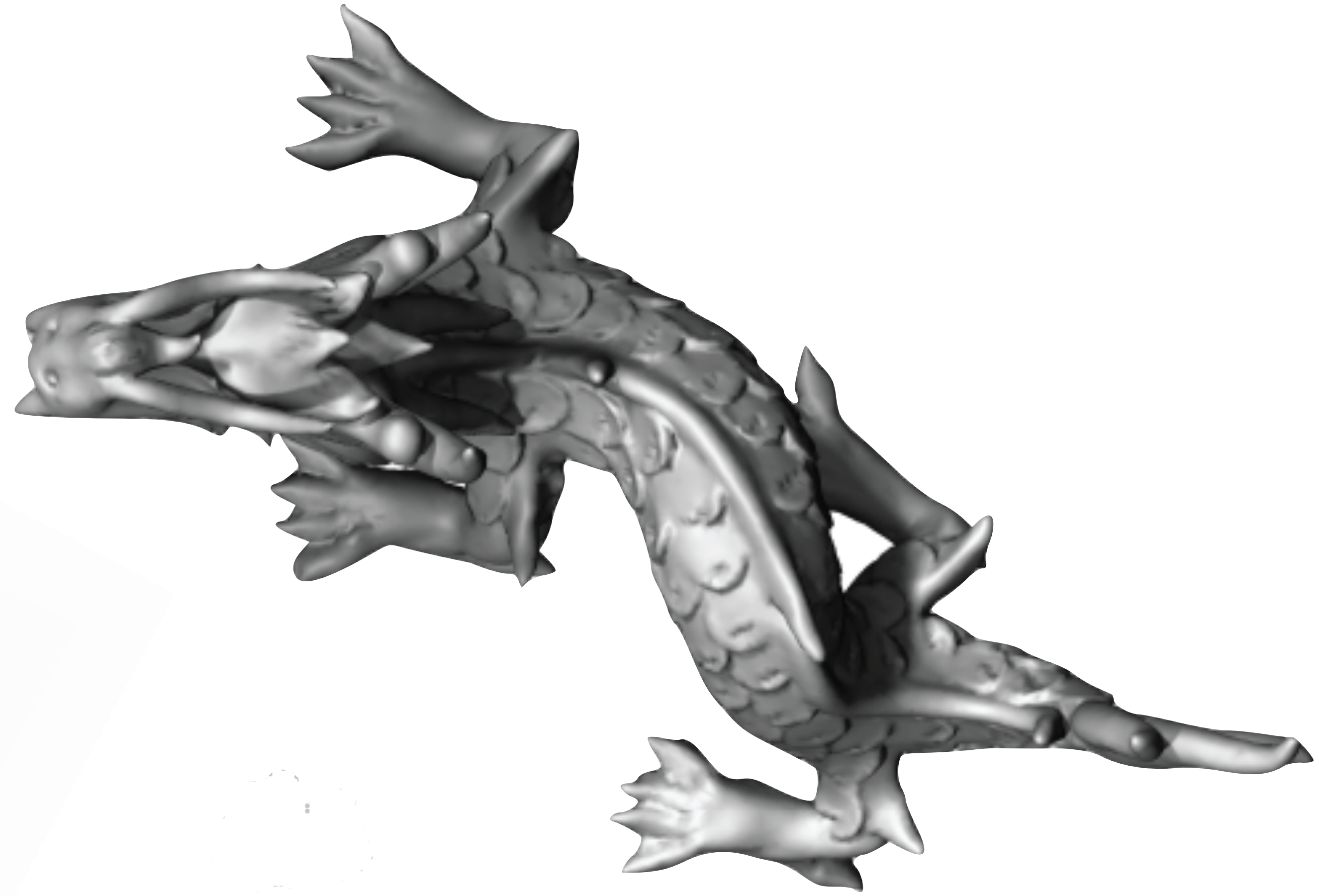


Bokeloh et al. 2009

Extrinsic vs. Intrinsic

Extrinsic symmetries depend on the embedding of the object in space.

Extrinsic vs. Intrinsic

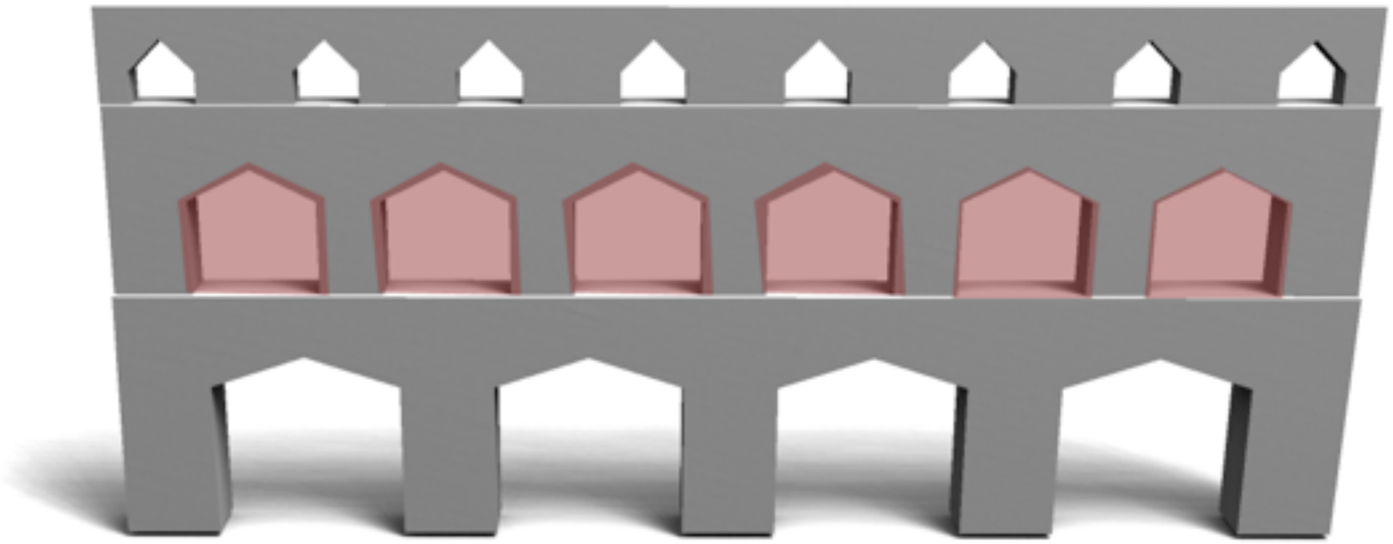


Extrinsic symmetries depend on the embedding of the object in space.

Intrinsic symmetries are defined with respect to an intrinsic metric of the surface.

Extrinsic Symmetries

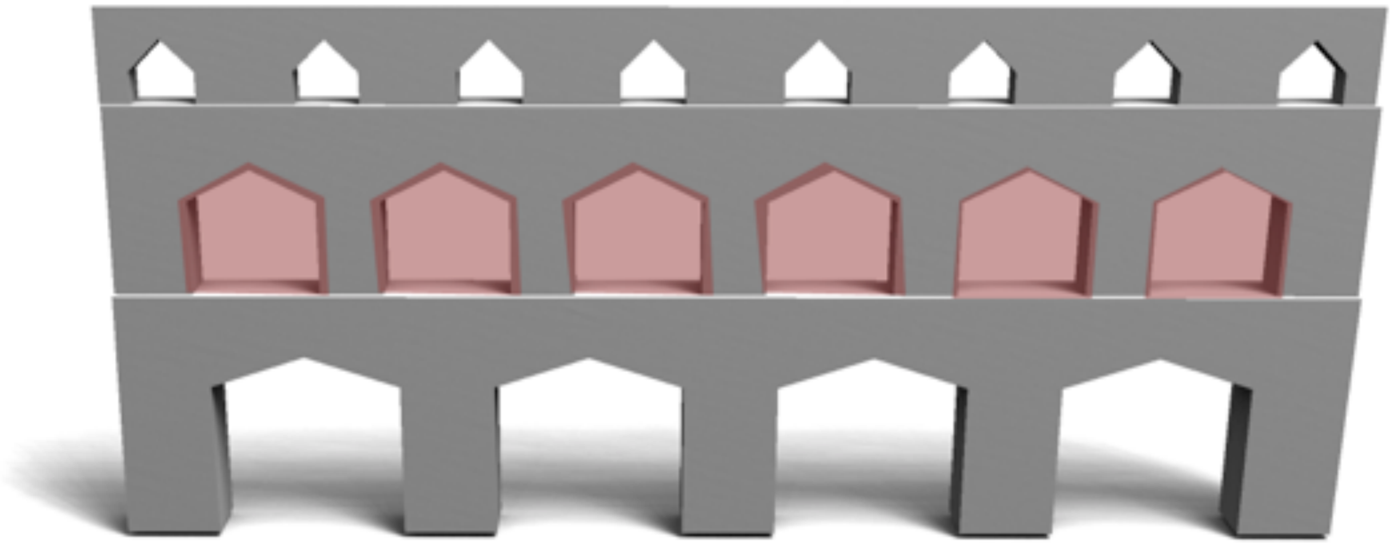
Extrinsic Symmetries



translations

Bokeloh et al. 2011

Extrinsic Symmetries



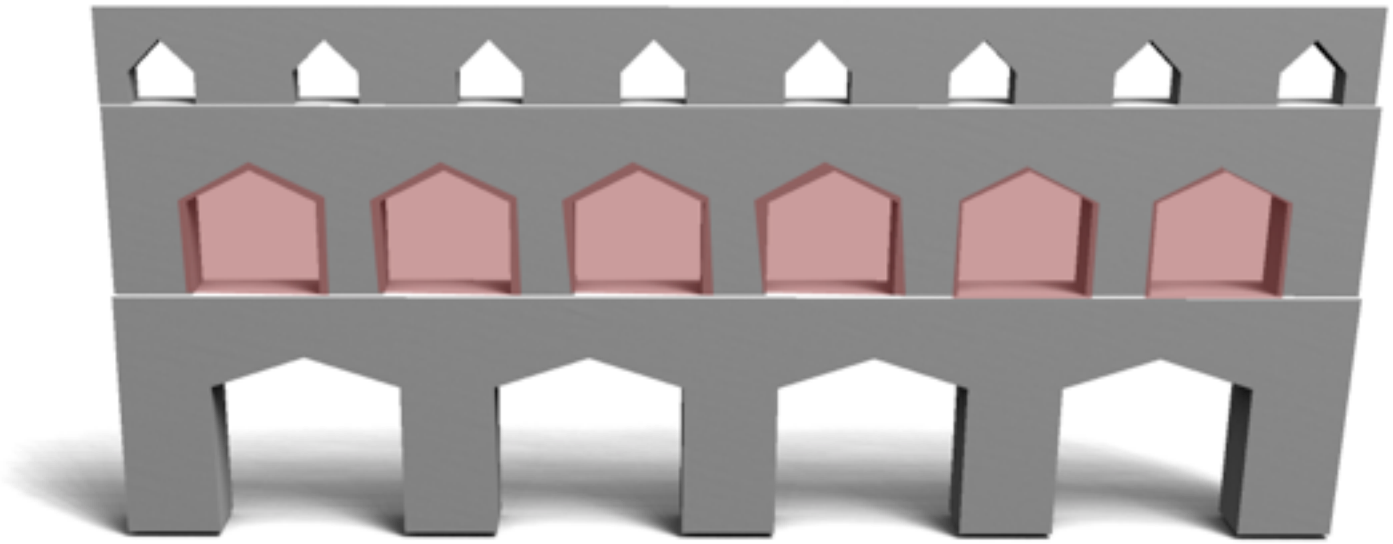
translations

Bokeloh et al. 2011

rotations reflections

Martinet et al. 2006

Extrinsic Symmetries



translations

Bokeloh et al. 2011

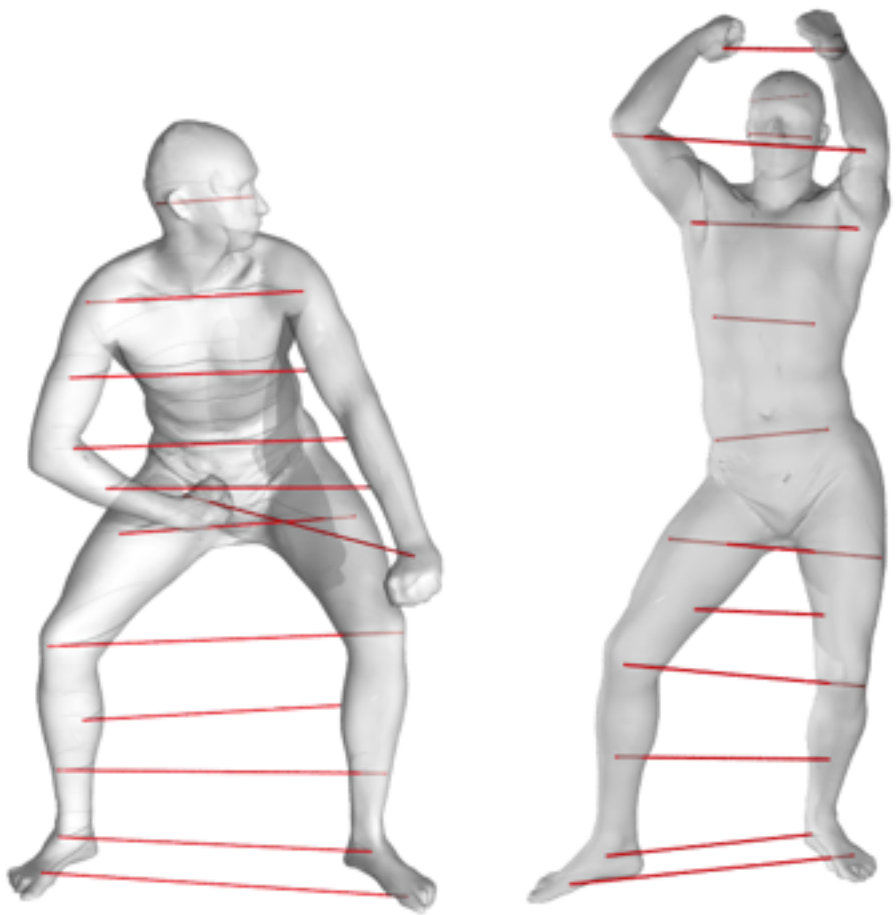
rotations reflections

Martinet et al. 2006

similarity transforms

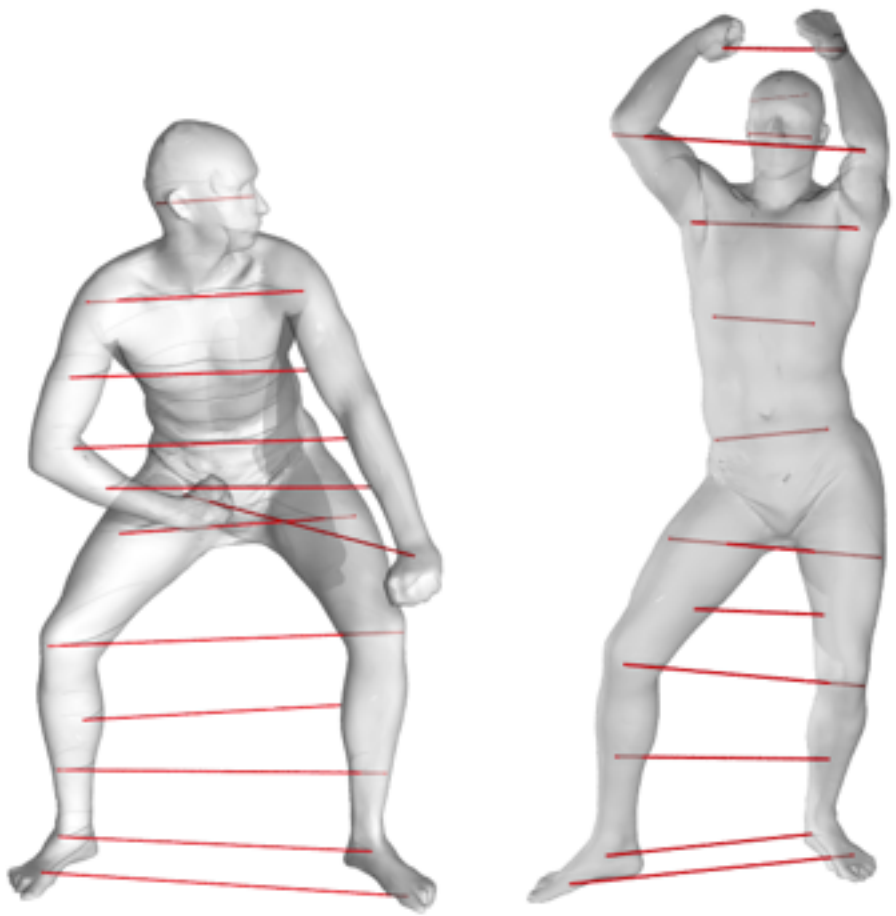
Gal et al. 2006

Intrinsic Symmetries



point-to-point
correspondences
Ovsjanikov et al. 2008

Intrinsic Symmetries



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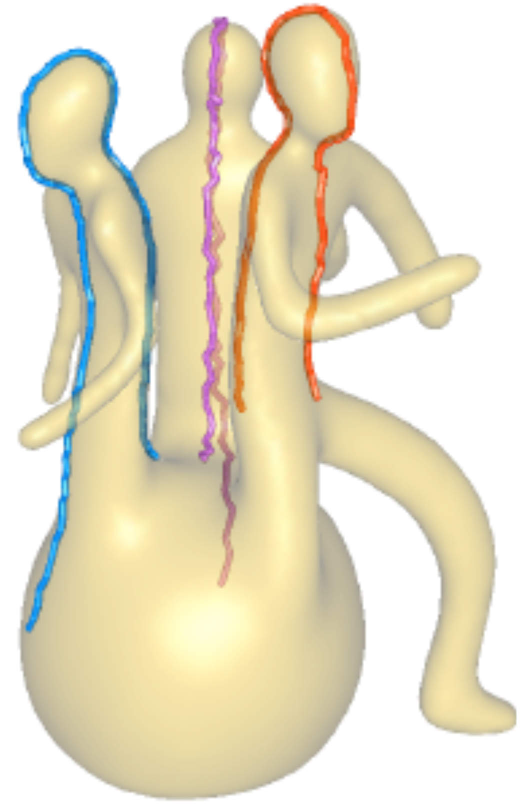
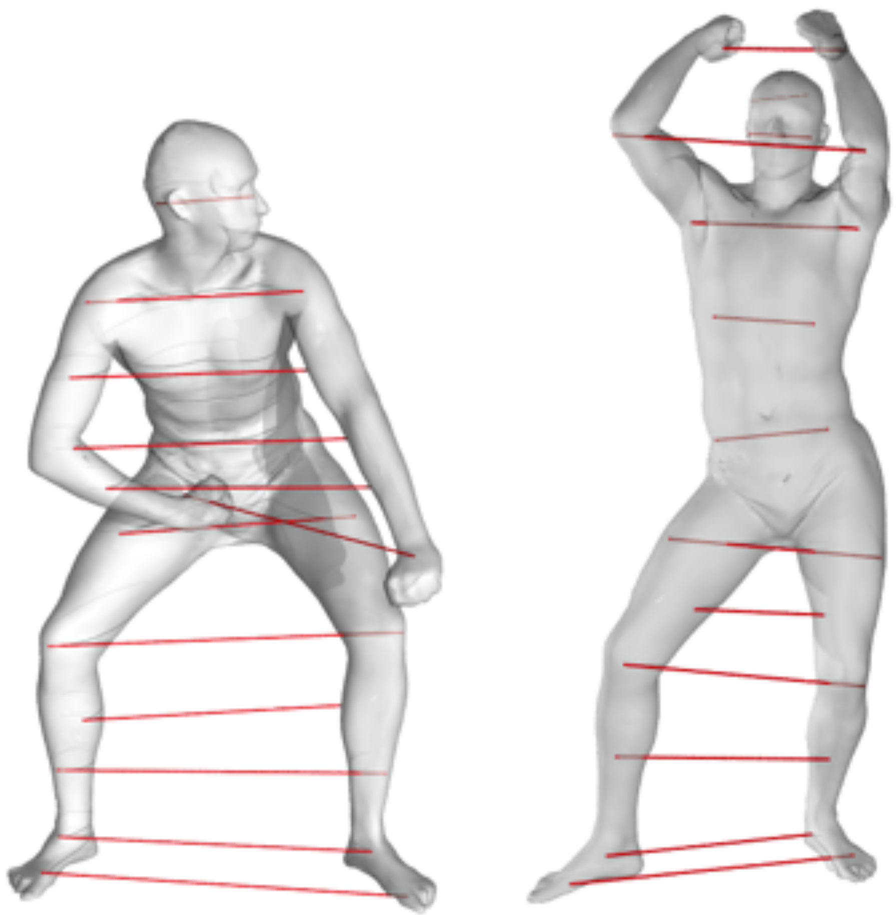


partial intrinsic
reflectional symmetries
Xu et al. 2009

Intrinsic Symmetries



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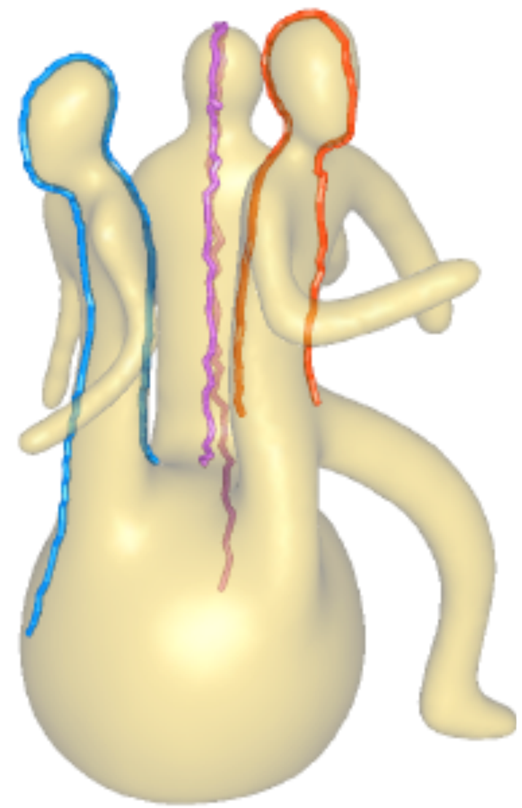

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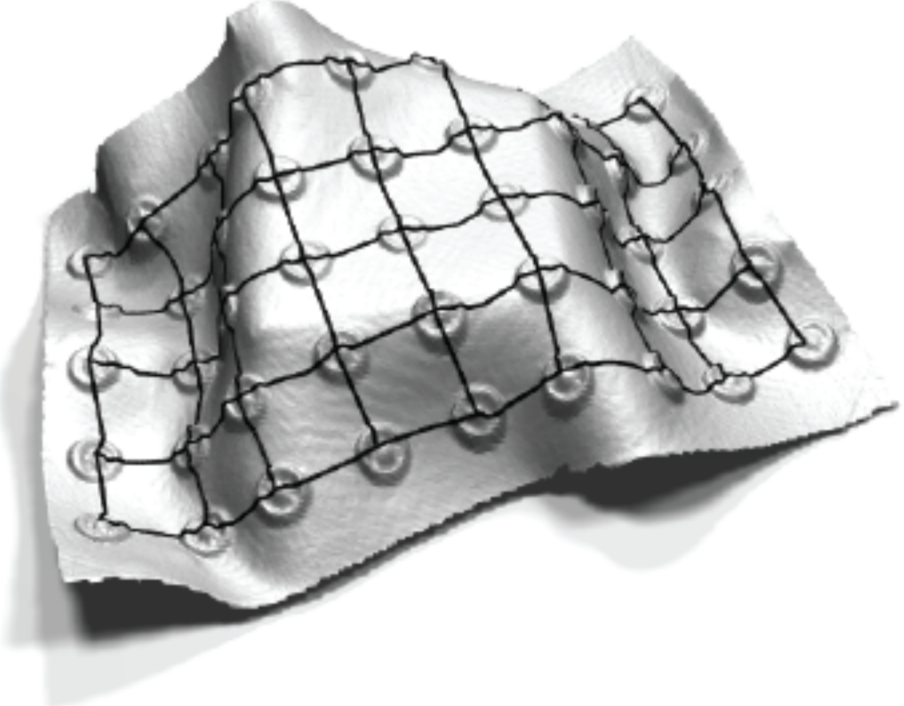
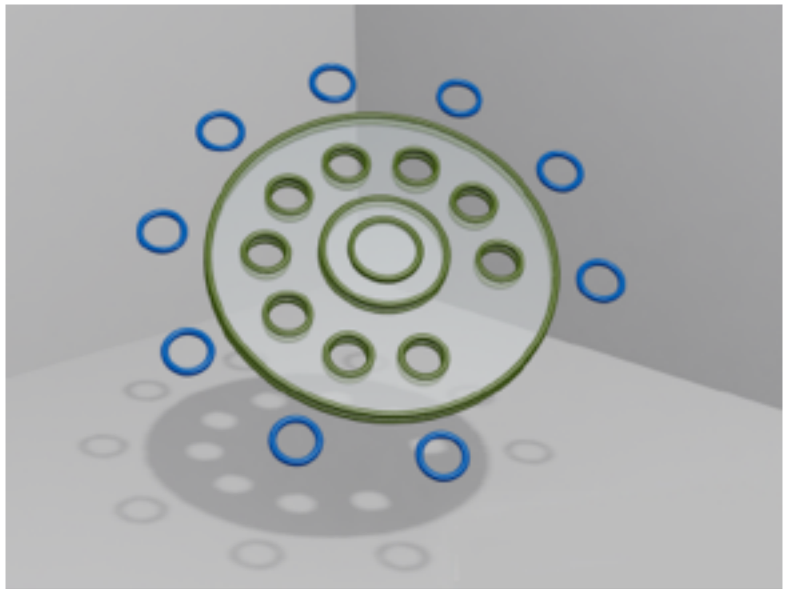


intrinsic regularities
Mitra et al. 2009

Discrete vs. Continuous

Discrete vs. Continuous



Gal et al. 2009

Berner et al. 2009

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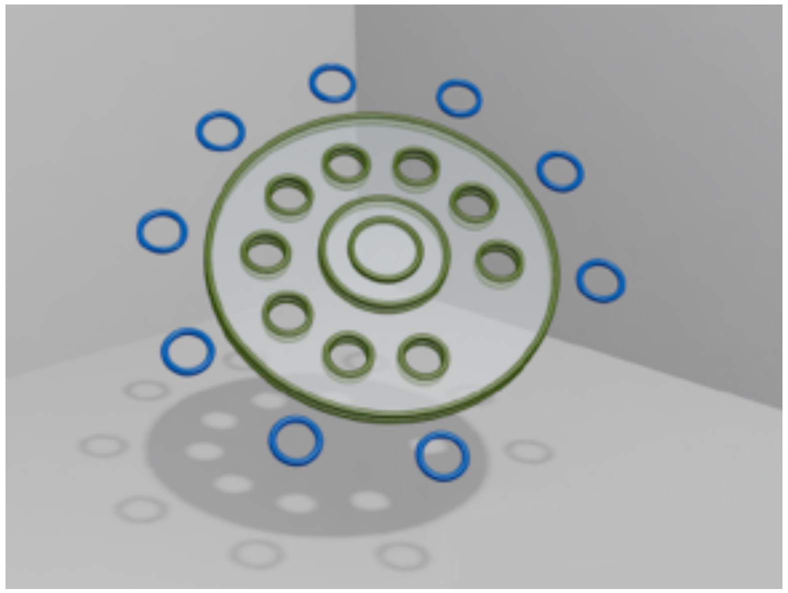

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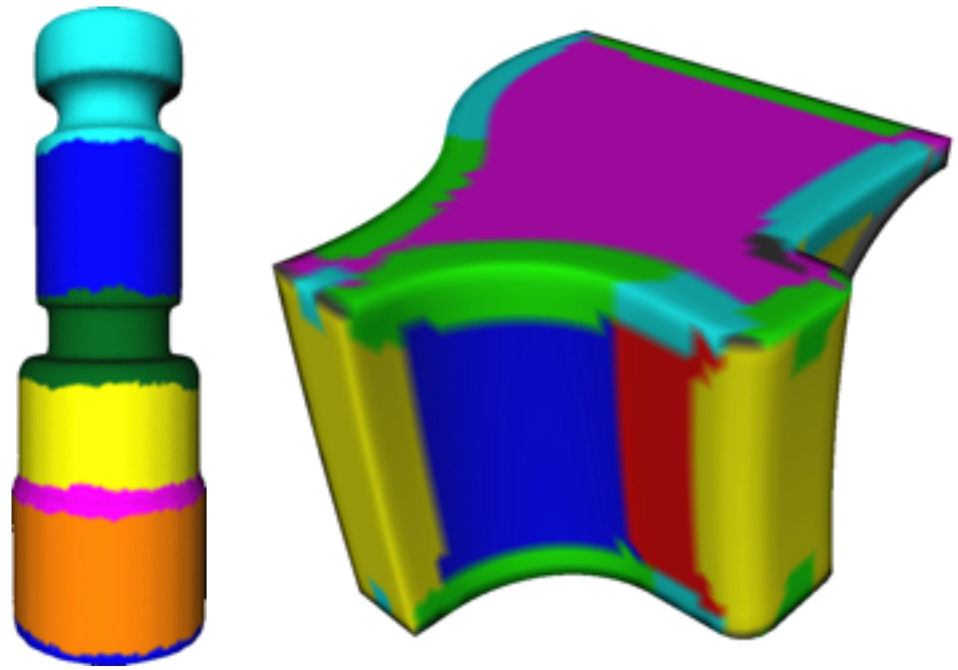
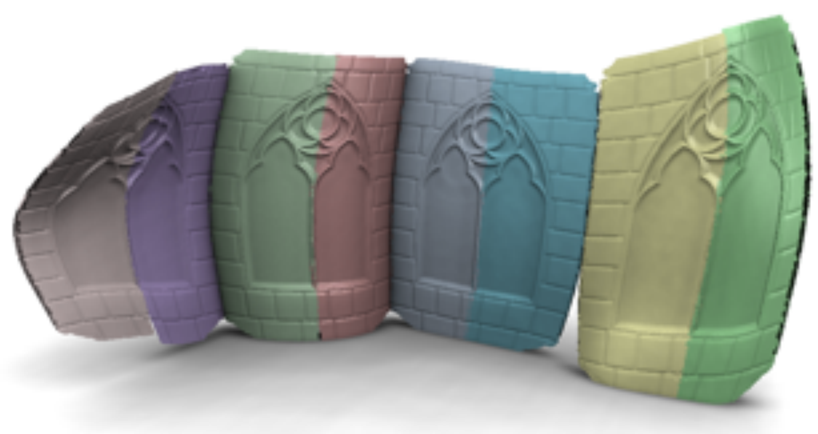


Gelfand et al. 2004



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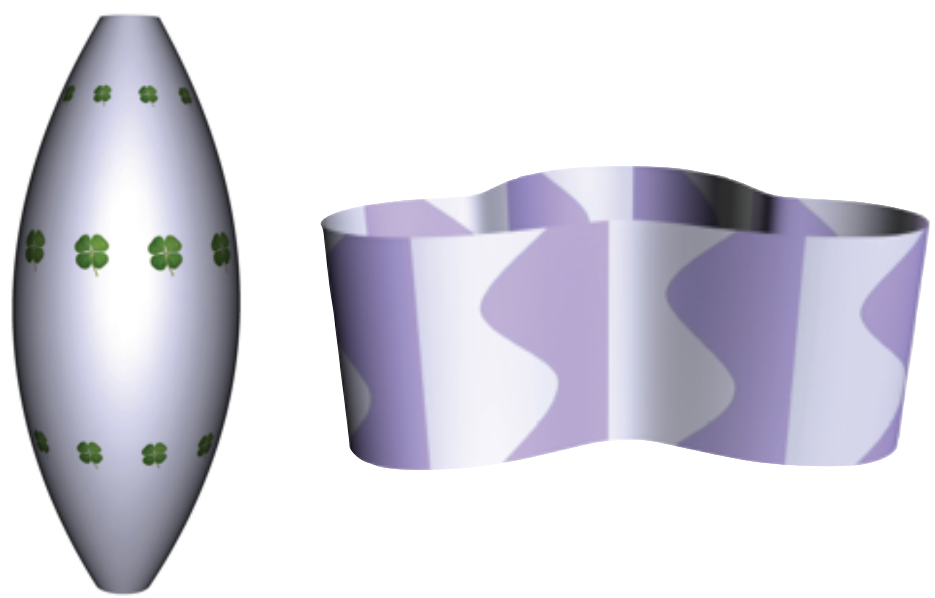

Ben-Chen et al. 2010

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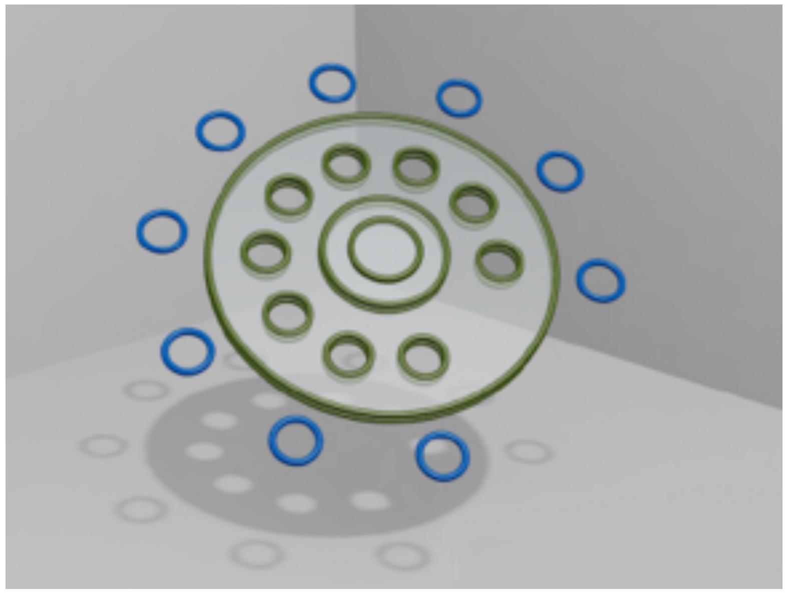
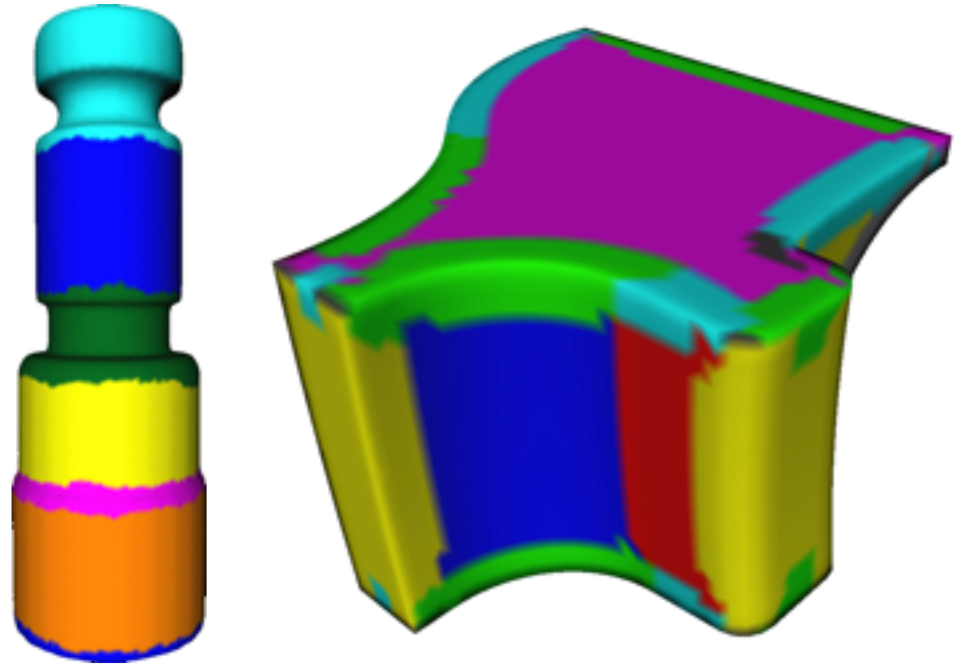
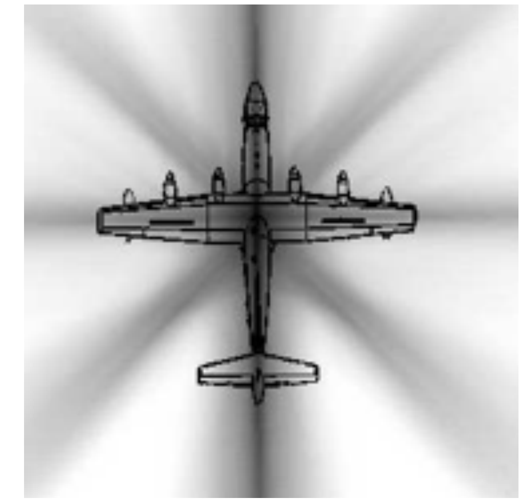


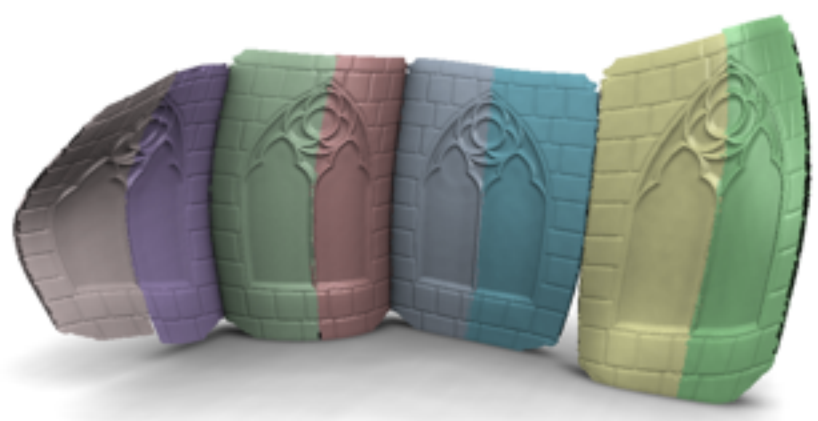
Gal et al. 2009



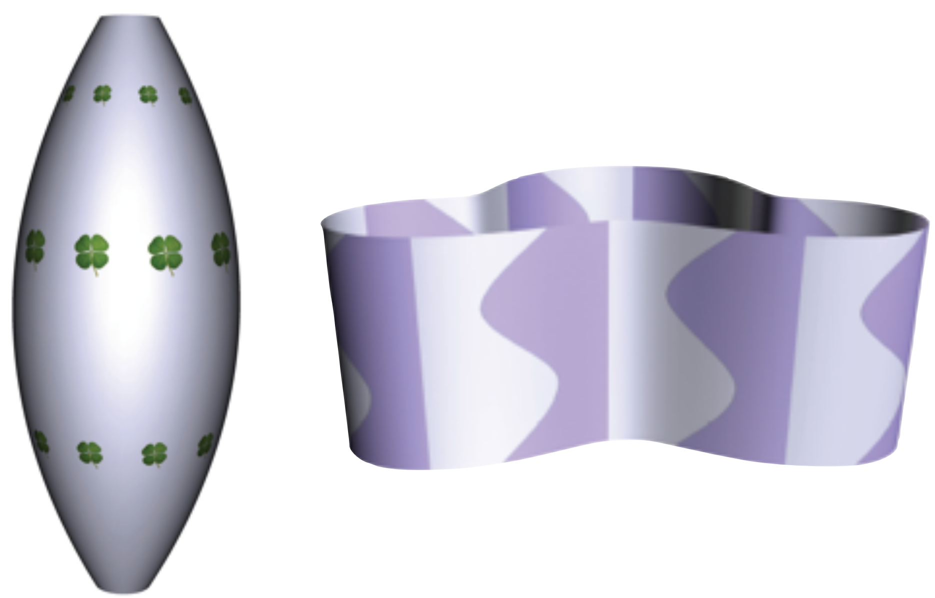
Gelfand et al. 2004



Podalak et al. 2006



Berner et al. 2009

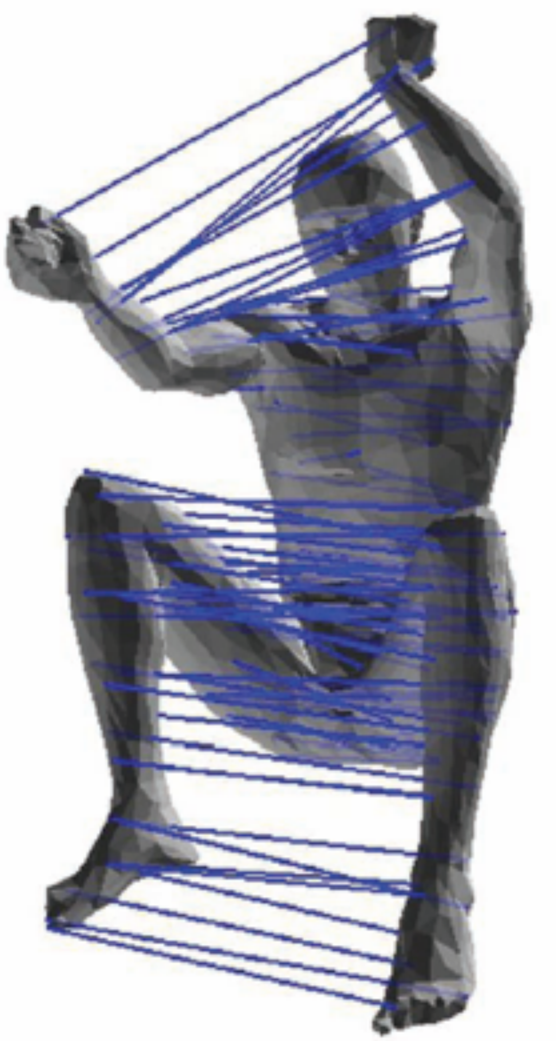


Ben-Chen et al. 2010

Lipman et al. 2010

Output Structure

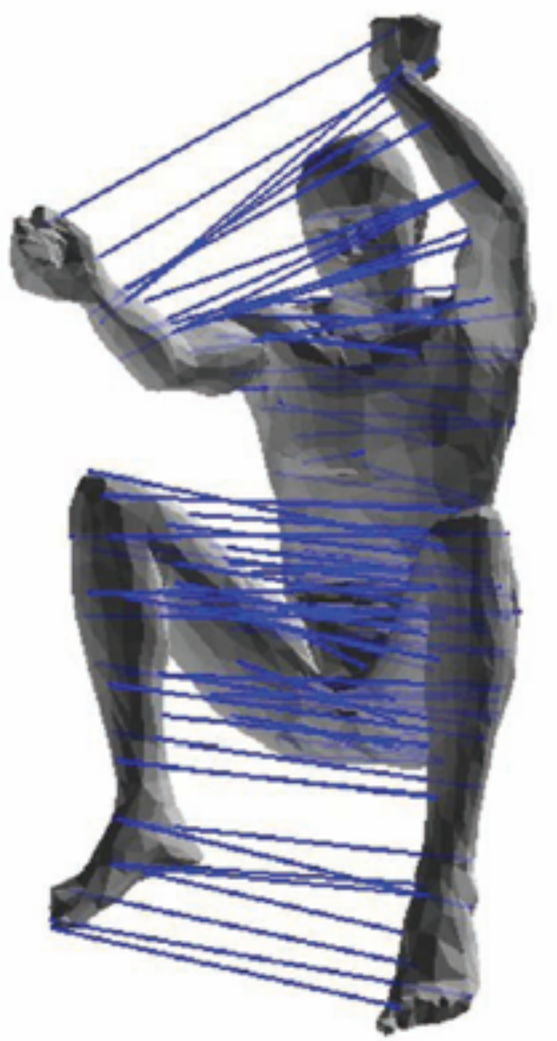
Output Structure



pairwise

Kim et al. 2010

Output Structure



pairwise

Kim et al. 2010

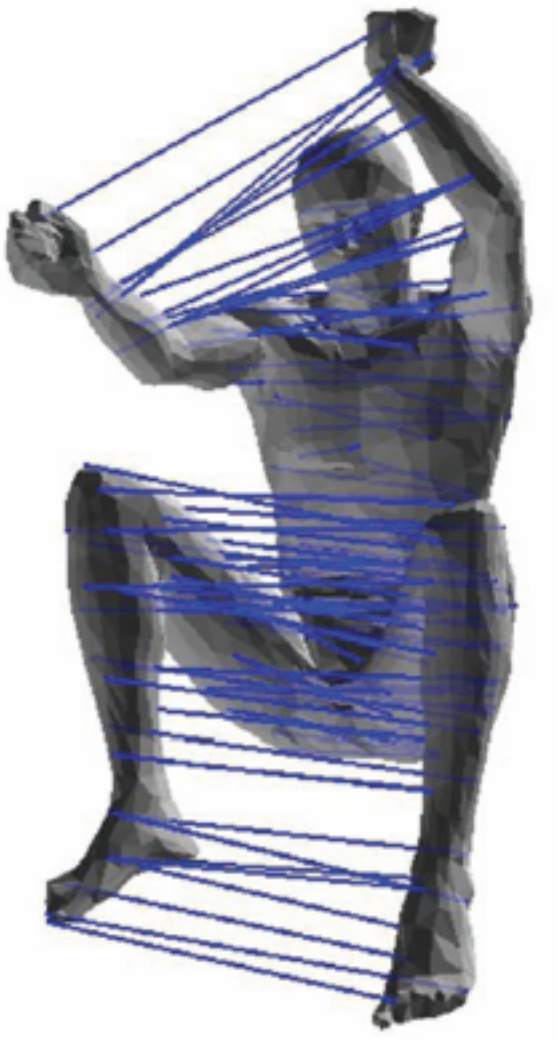
segmentation

Mitra et al. 2006

Output Structure

segmentation

Mitra et al. 2006



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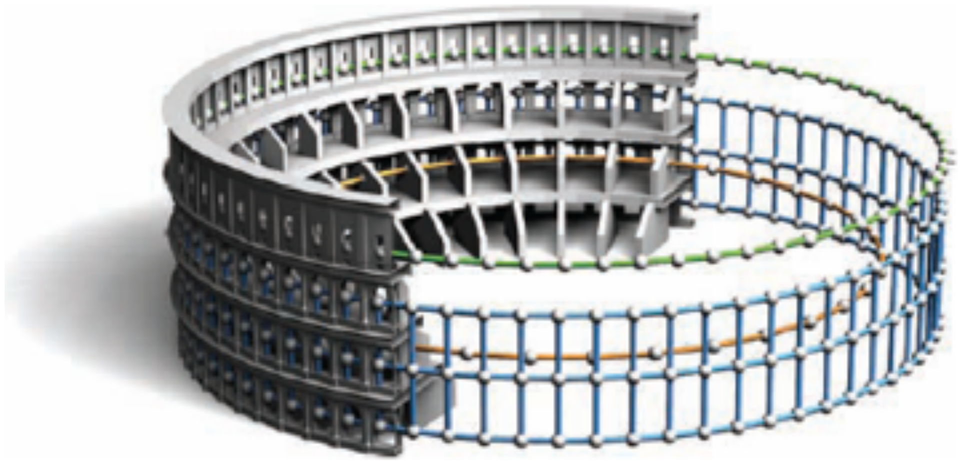


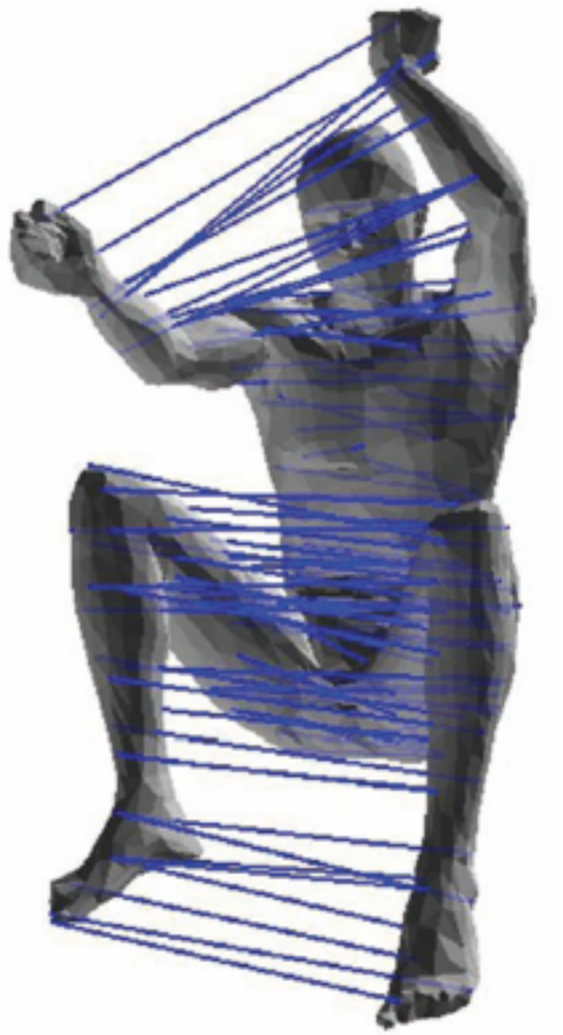
symmetry groups

Pauly et al. 2008

Output Structure

segmentation

Mitra et al. 2006



pairwise

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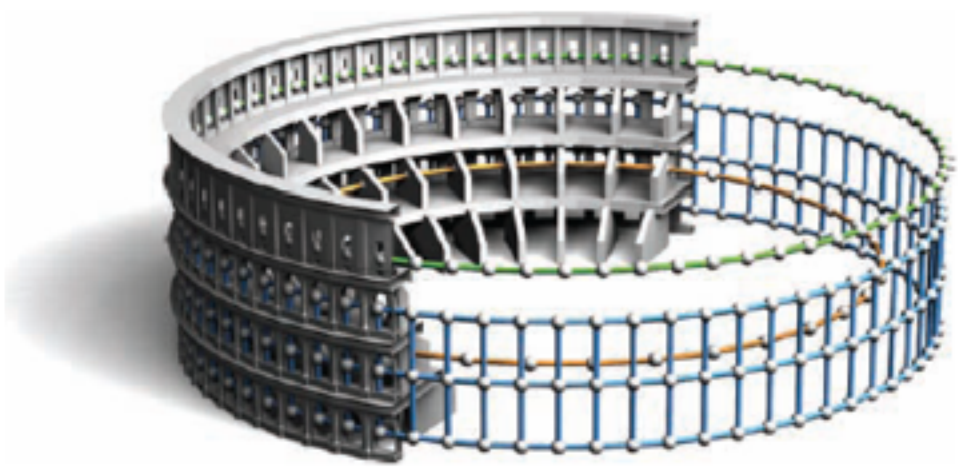

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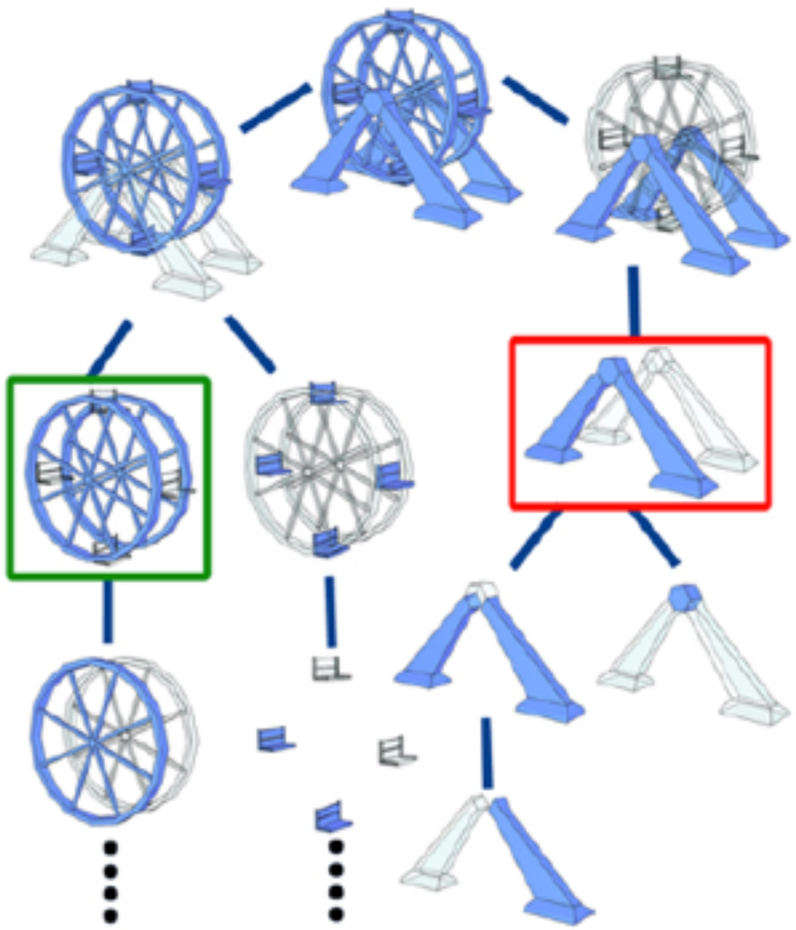
Pauly et al. 2008

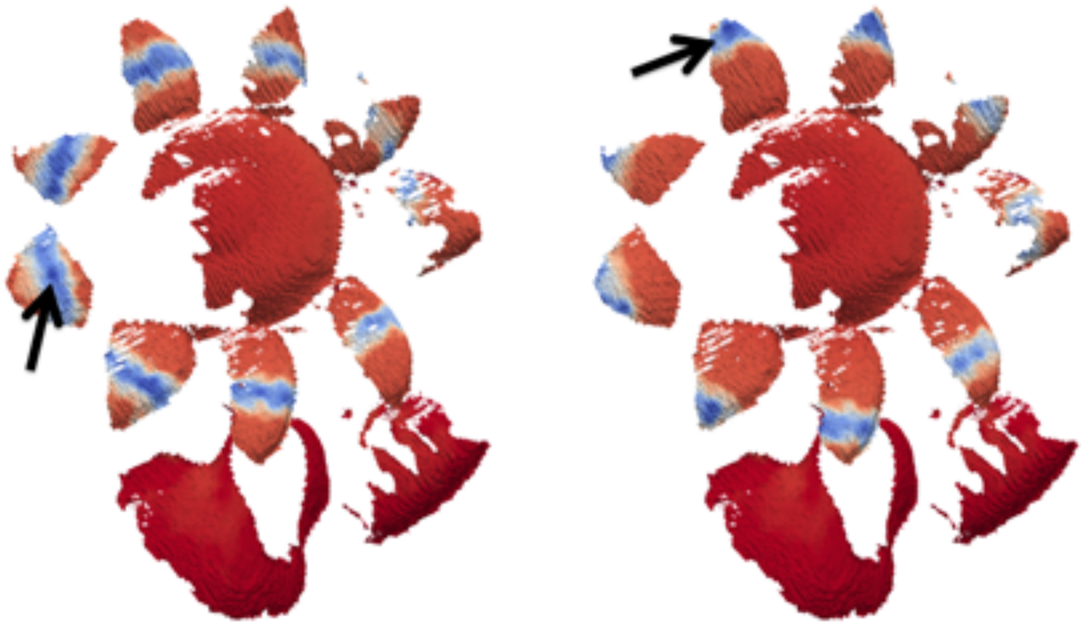


hierarchy

Wang et al. 2011

Output Structure

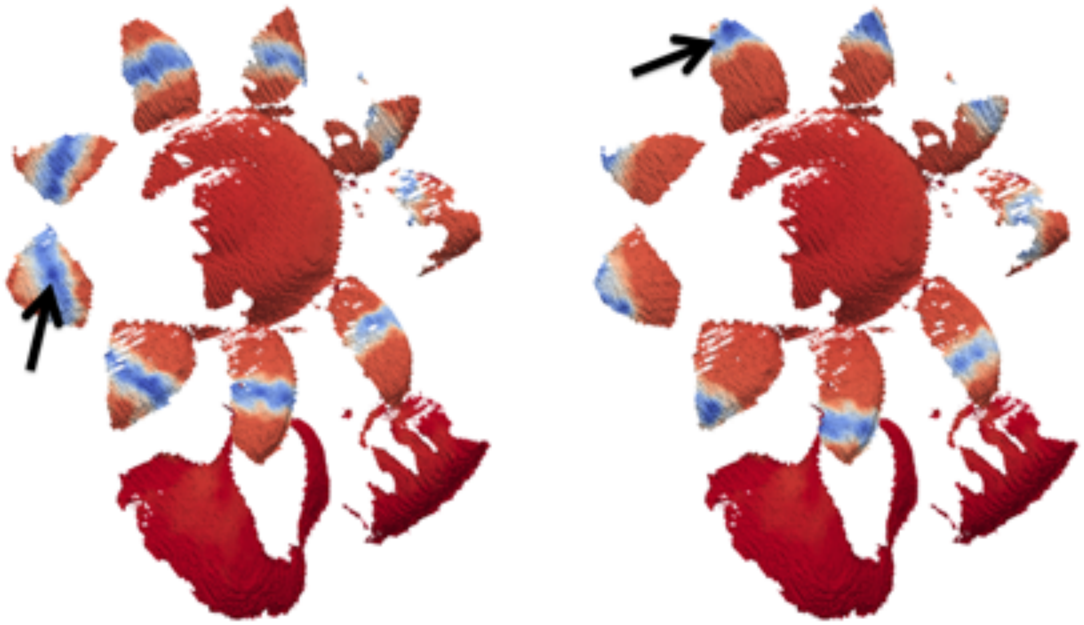
Output Structure



symmetry orbits

Lipman et al. 2010

Output Structure



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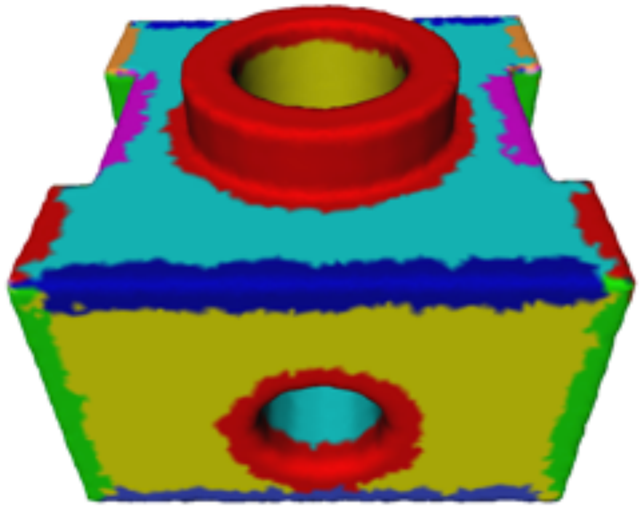

slippable regions

Gelfand et al. 2004

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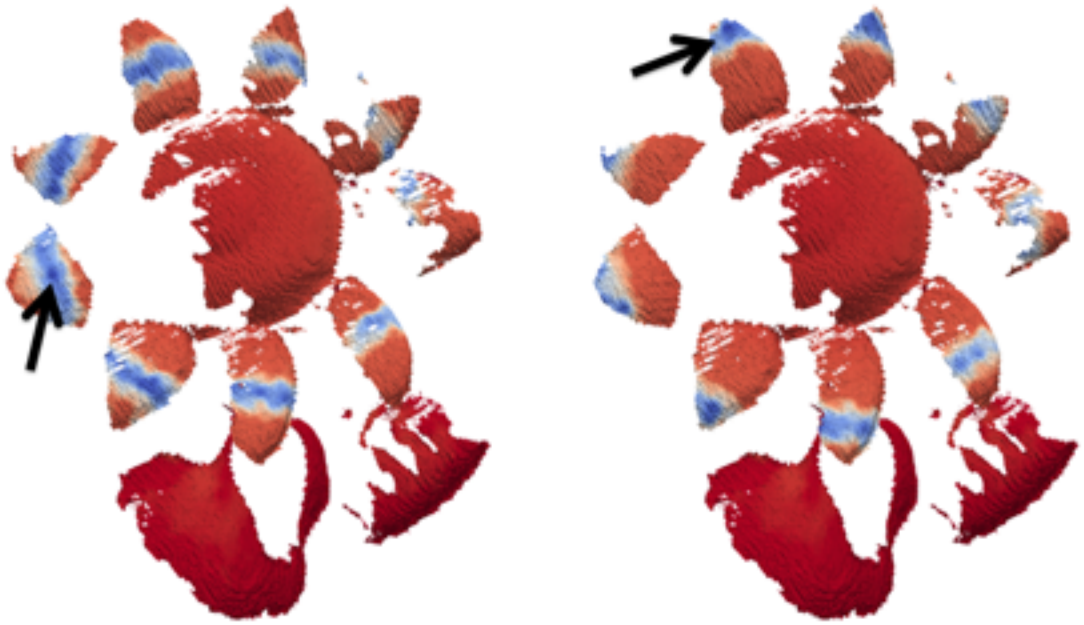

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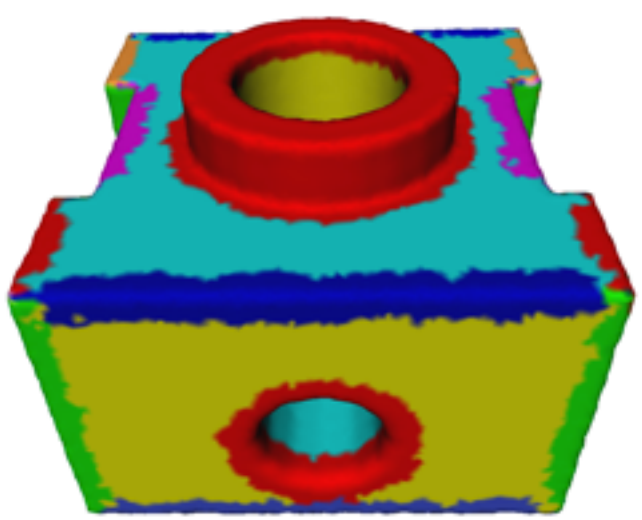

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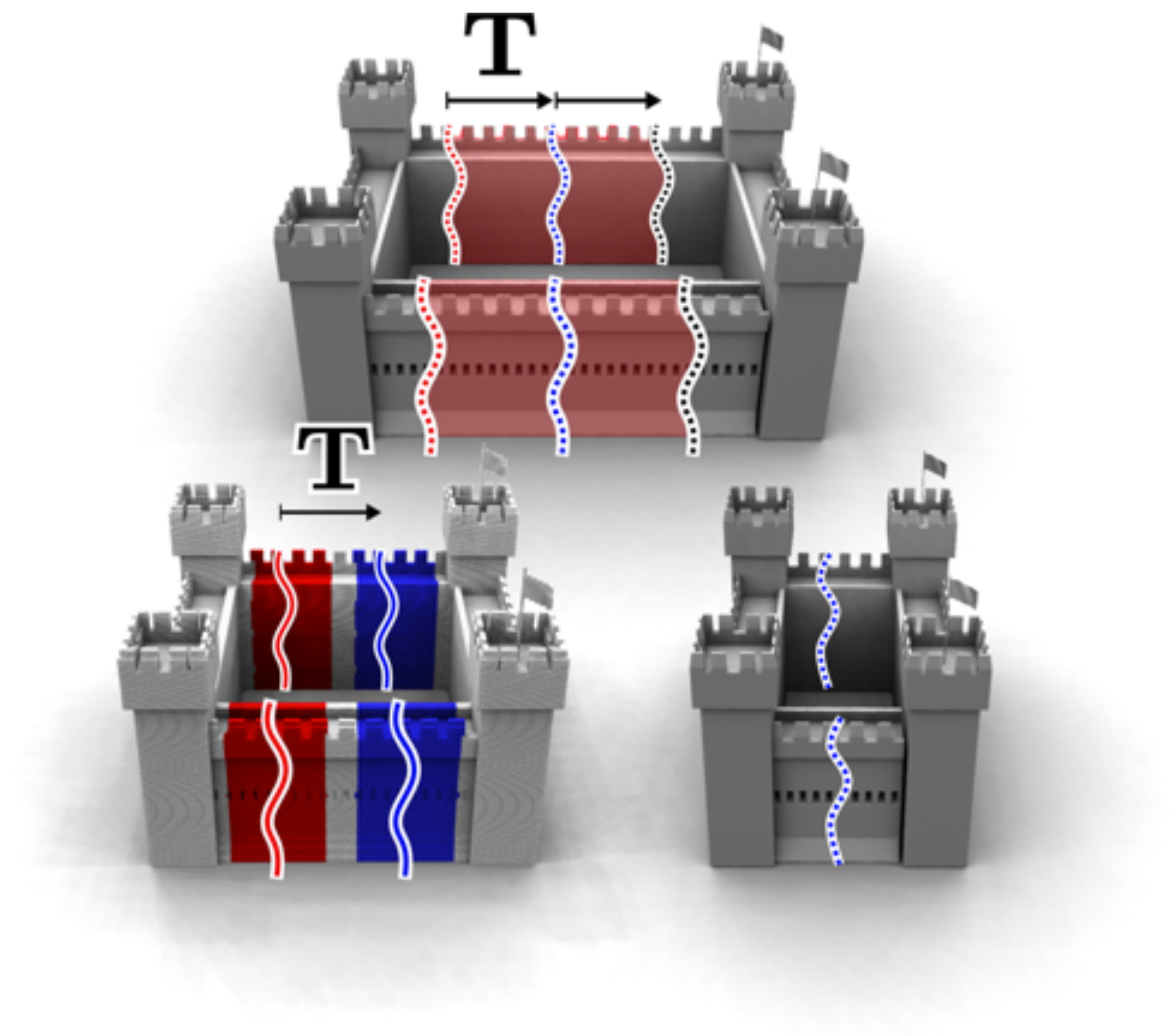


docking sites

Bokeloh et al. 2010

Understanding Geometry

Physical Object

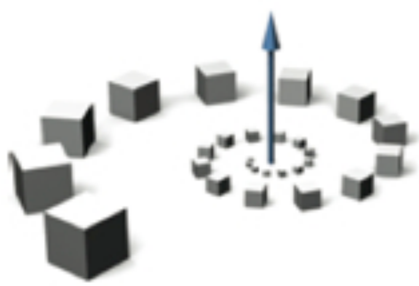
Understanding Geometry

Physical Object

Digital Measurement

Acquisition

Understanding Geometry

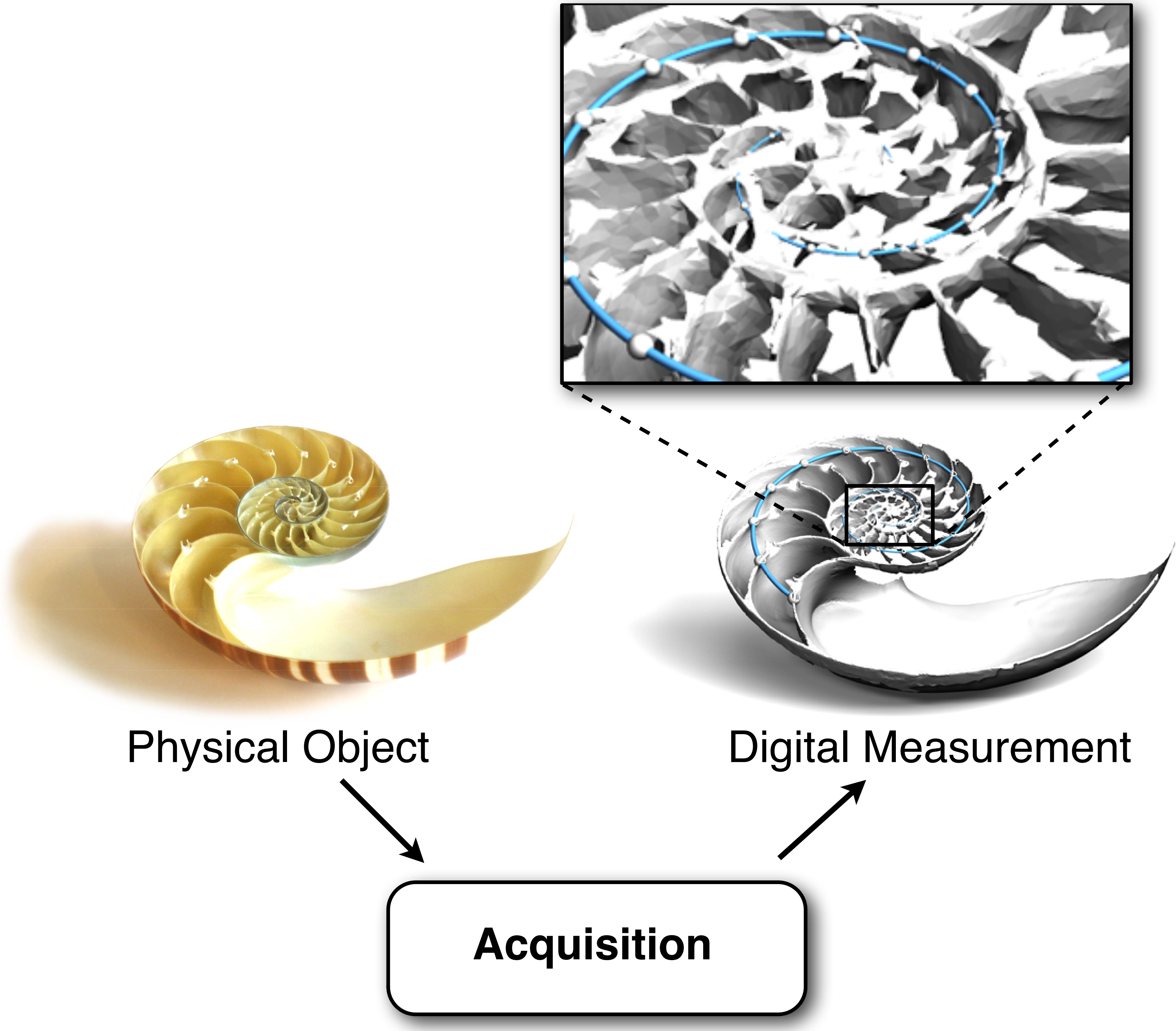


Physical Object

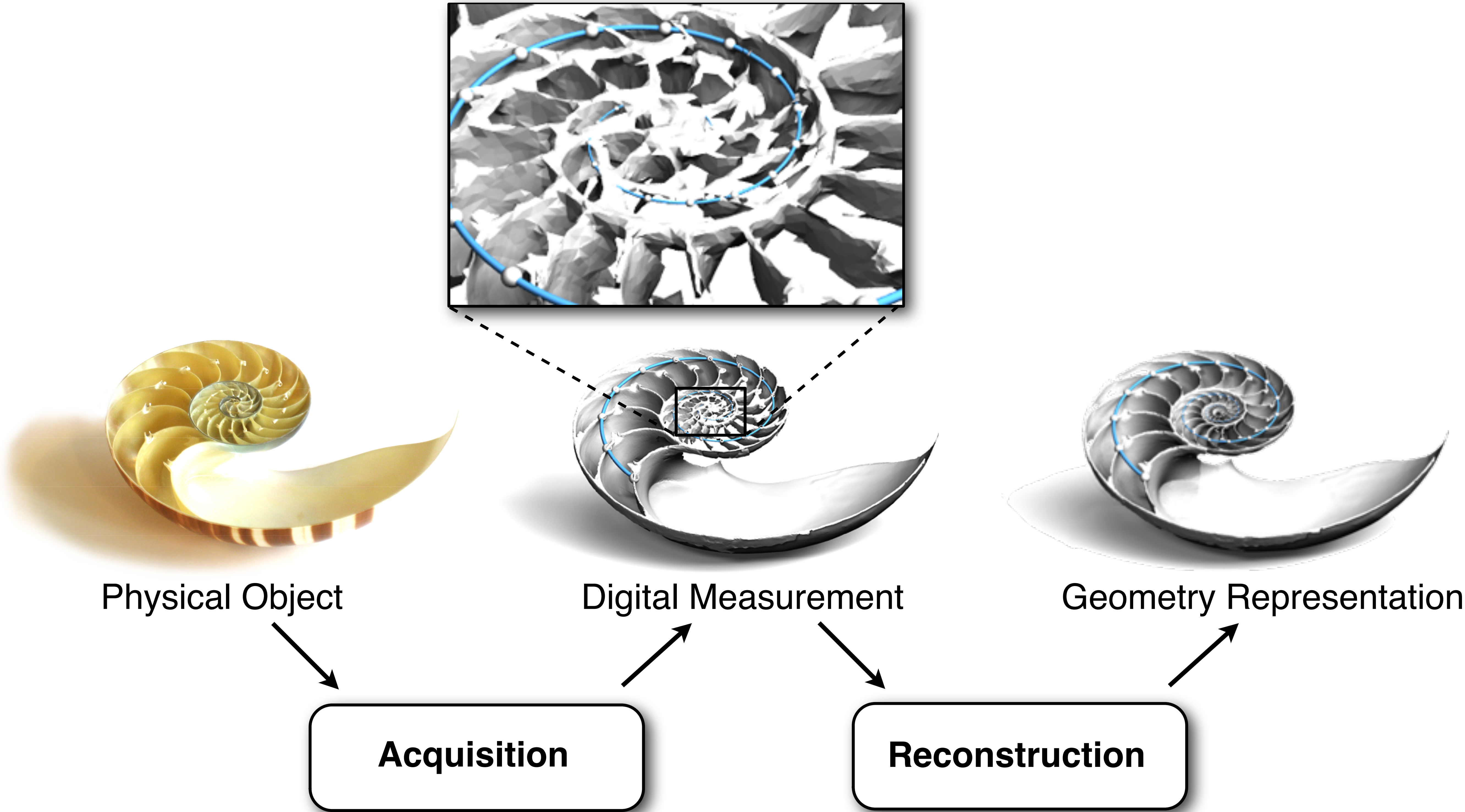
Digital Measurement

Acquisition

Understanding Geometry



Understanding Geometry



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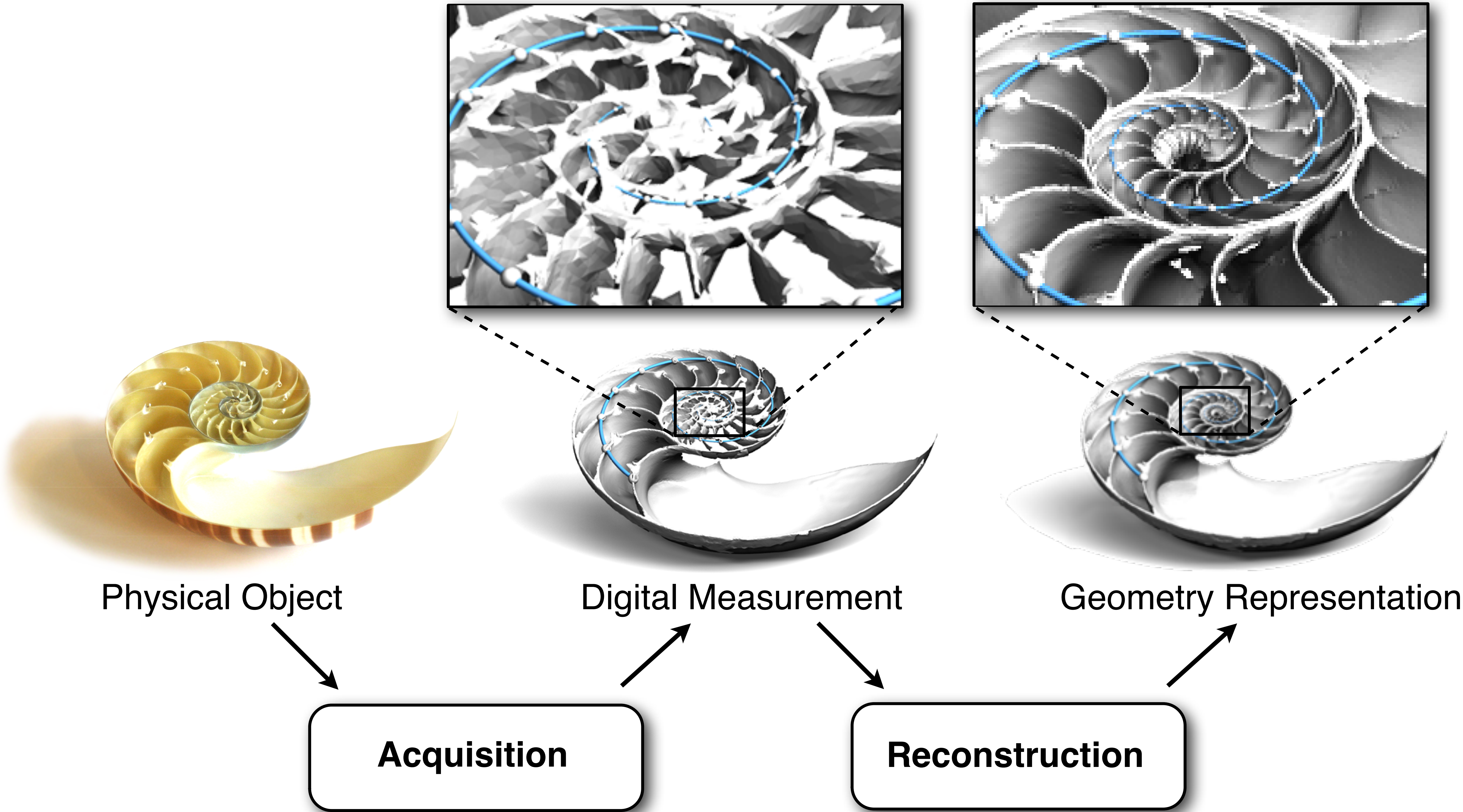
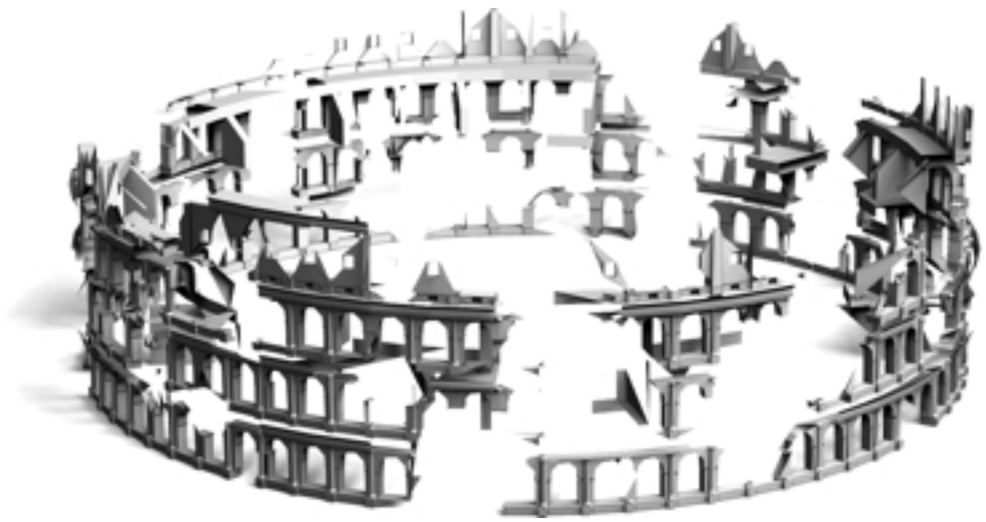


Understanding Geometry

Symmetry encodes Redundancy



Understanding Geometry

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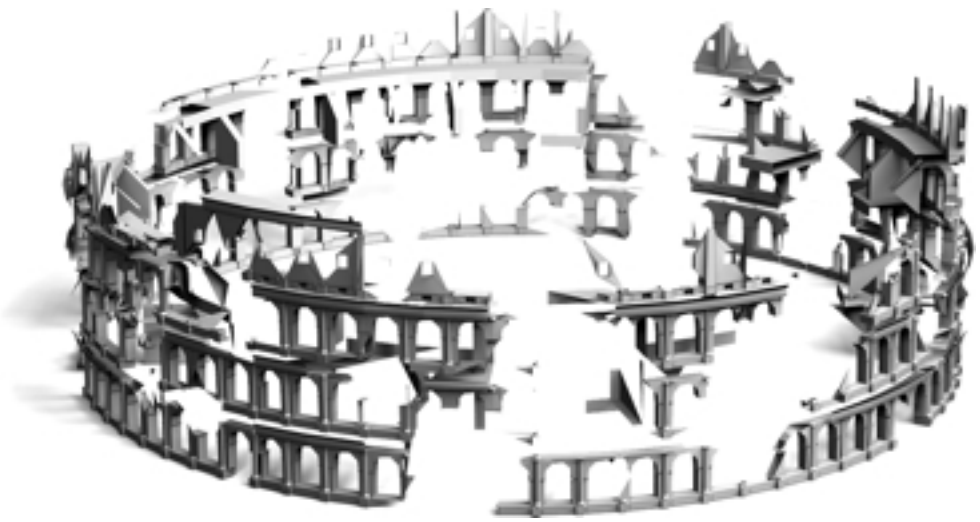
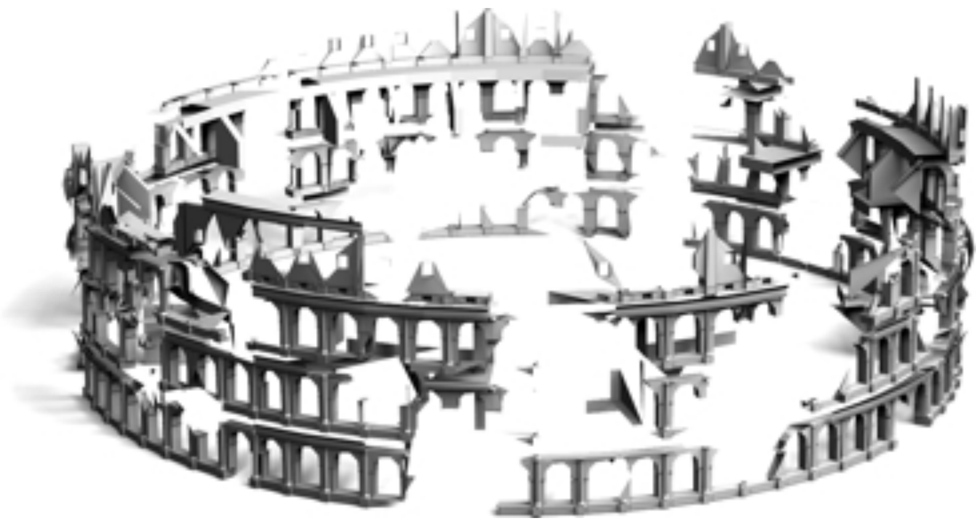
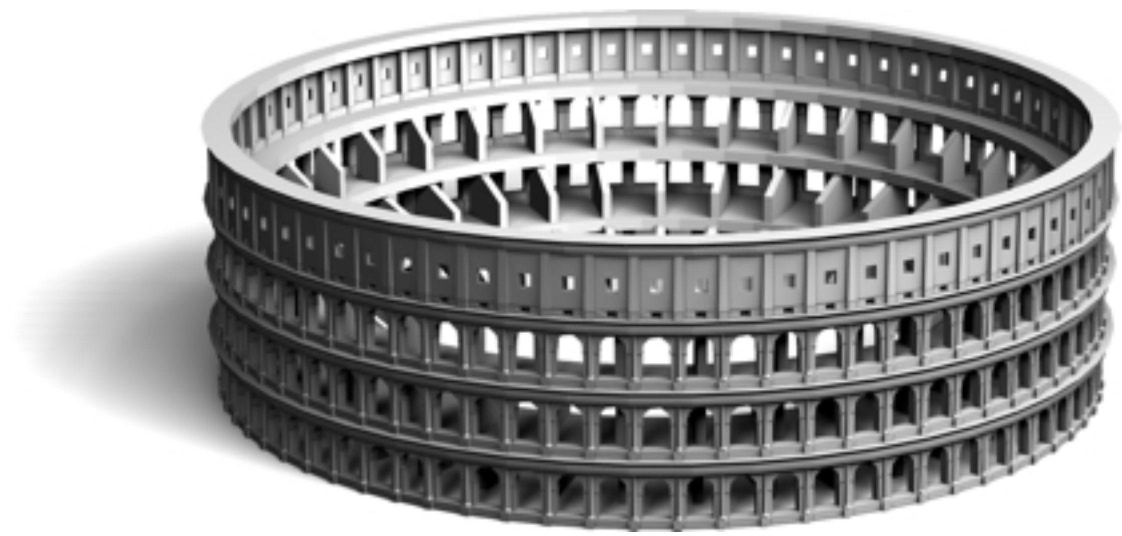


Symmetry Analysis

Understanding Geometry

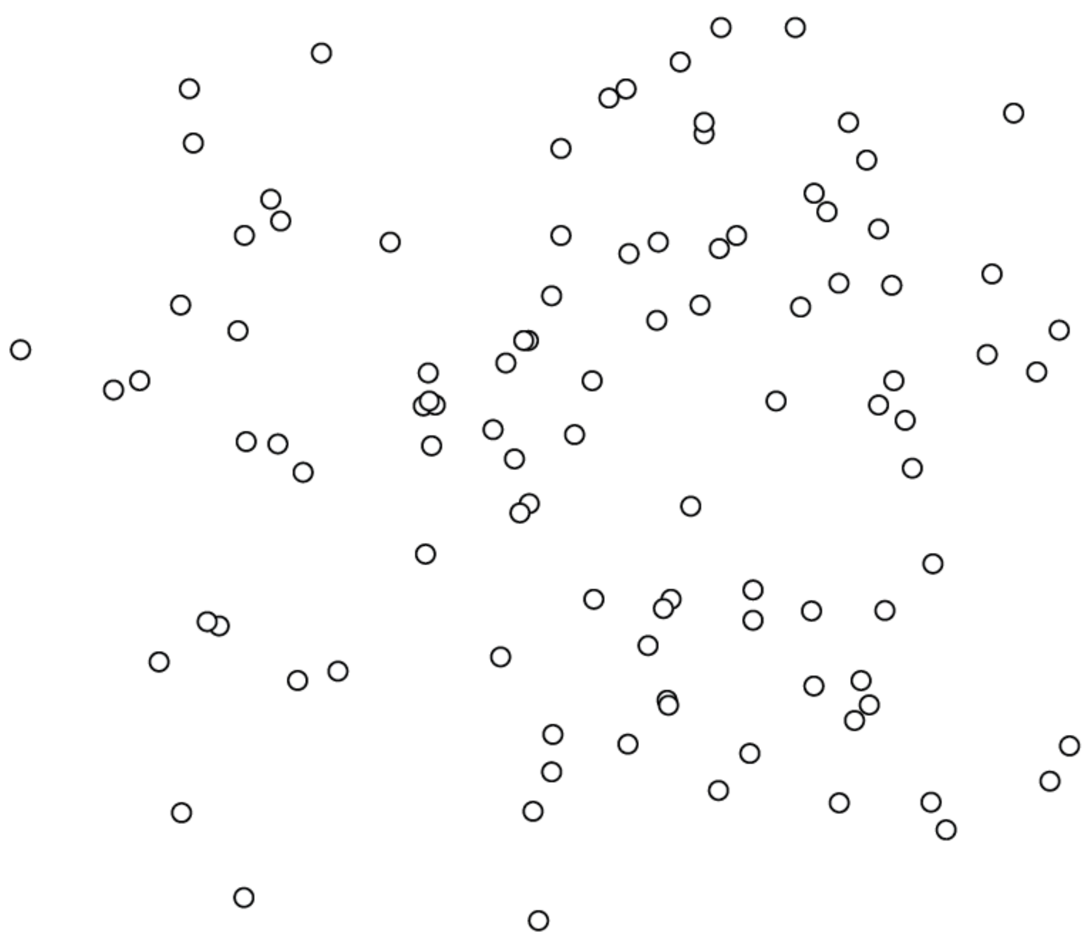
Symmetry encodes Redundancy



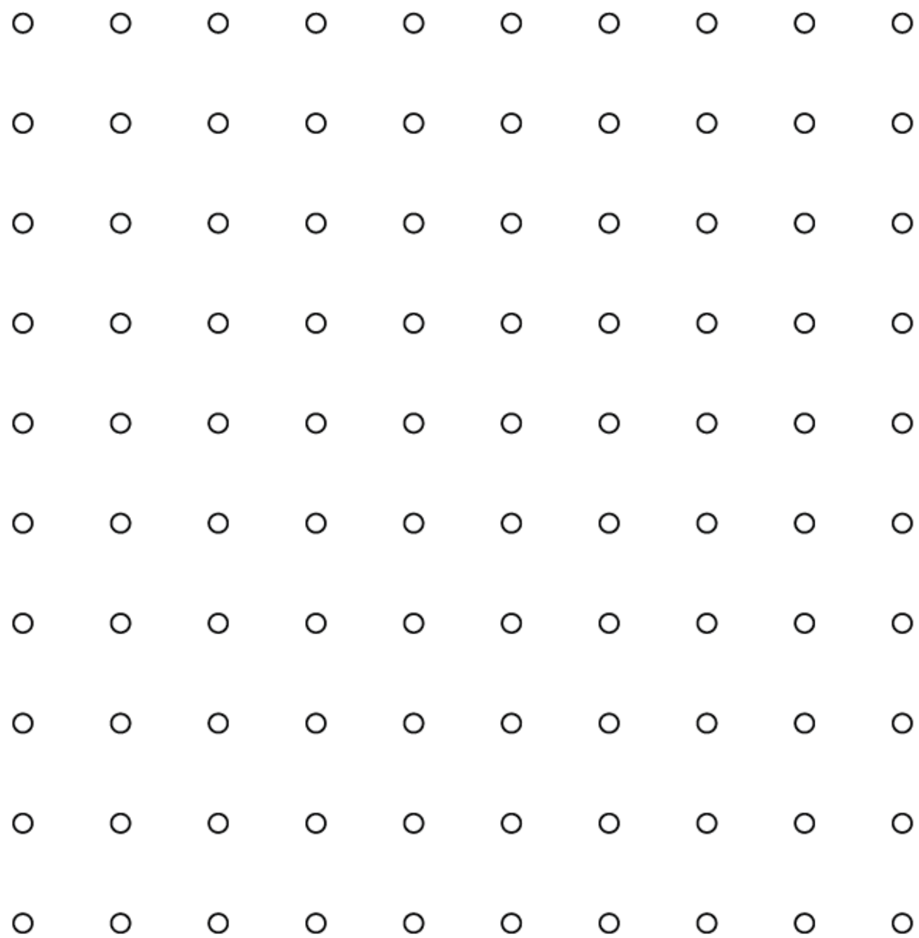


Symmetry Analysis → *Reconstruction*

Symmetry and Information

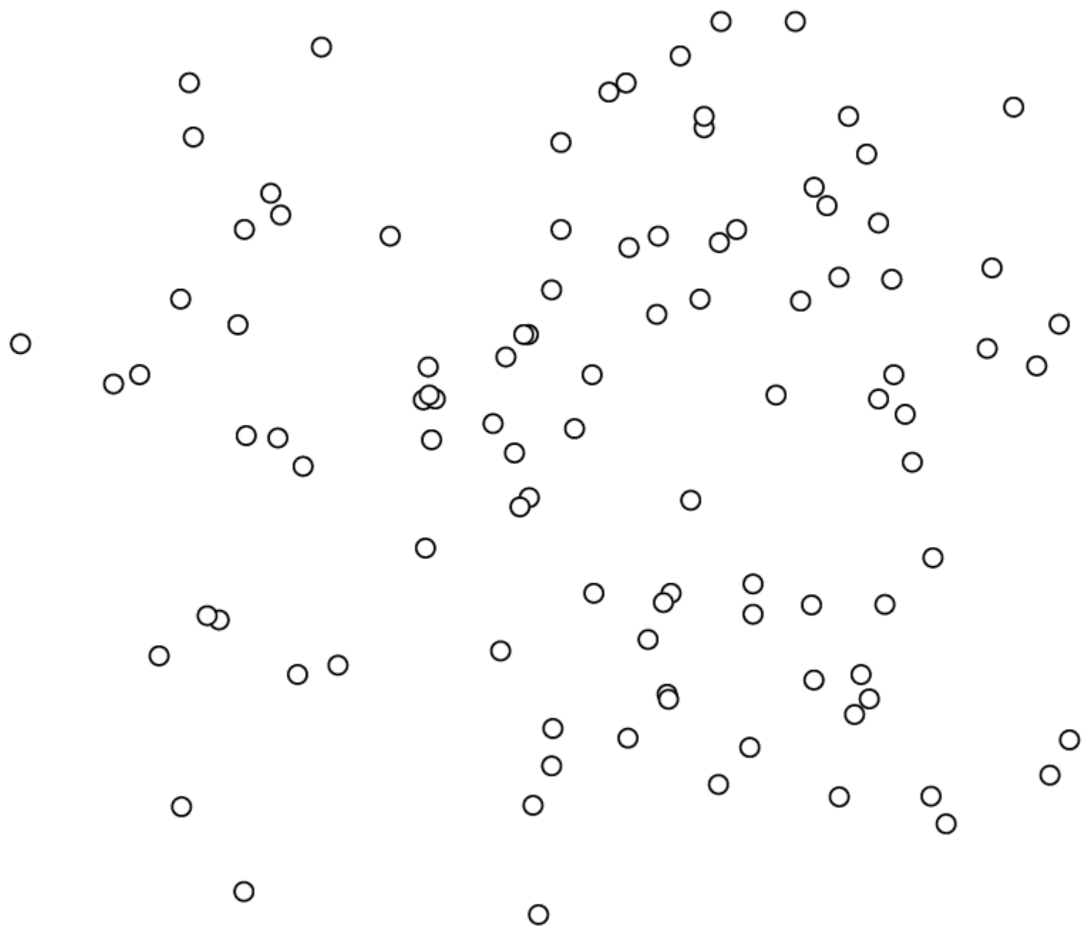


"100 Random Points"



"A 10x10 Regular Grid of Points"

Symmetry and Information



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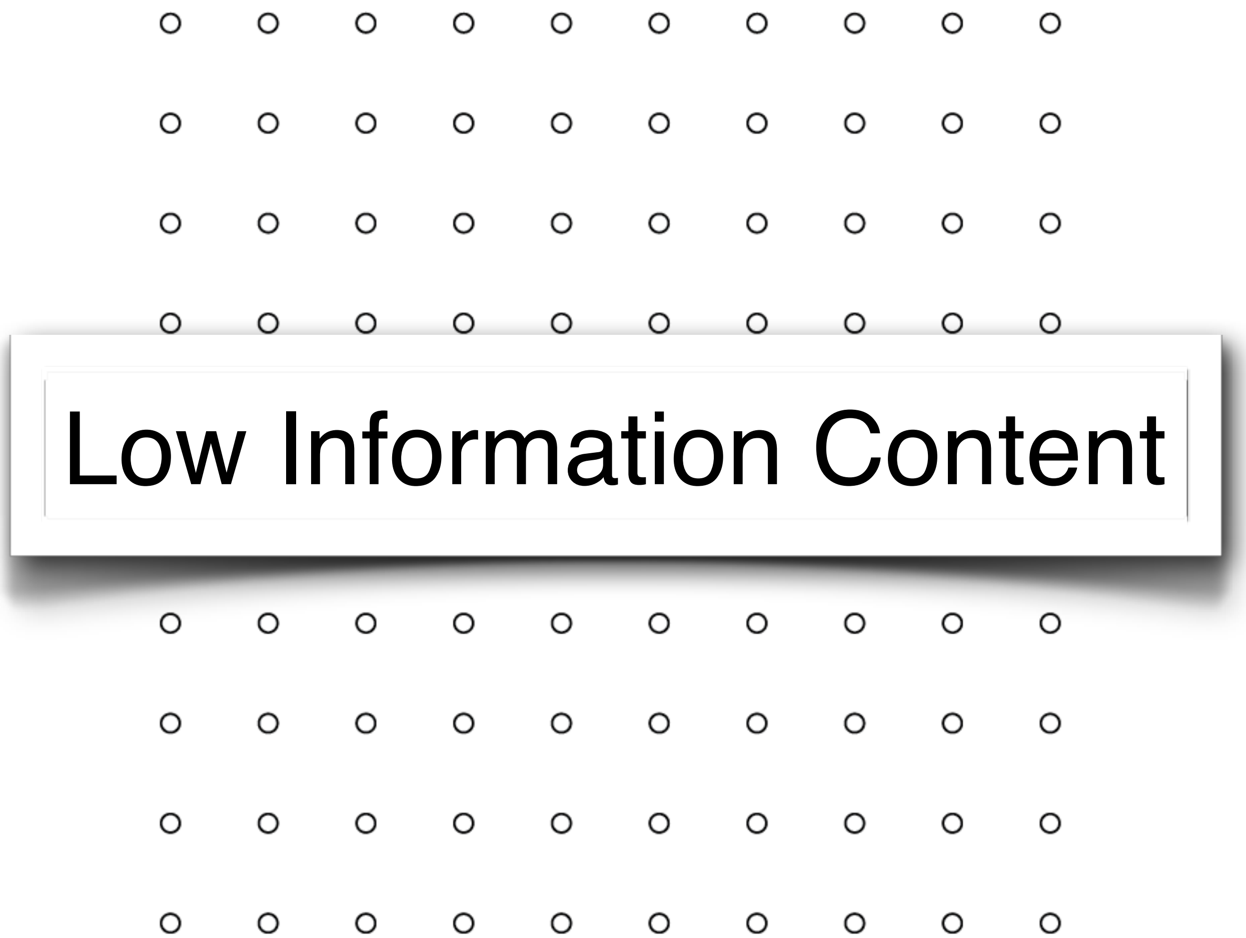

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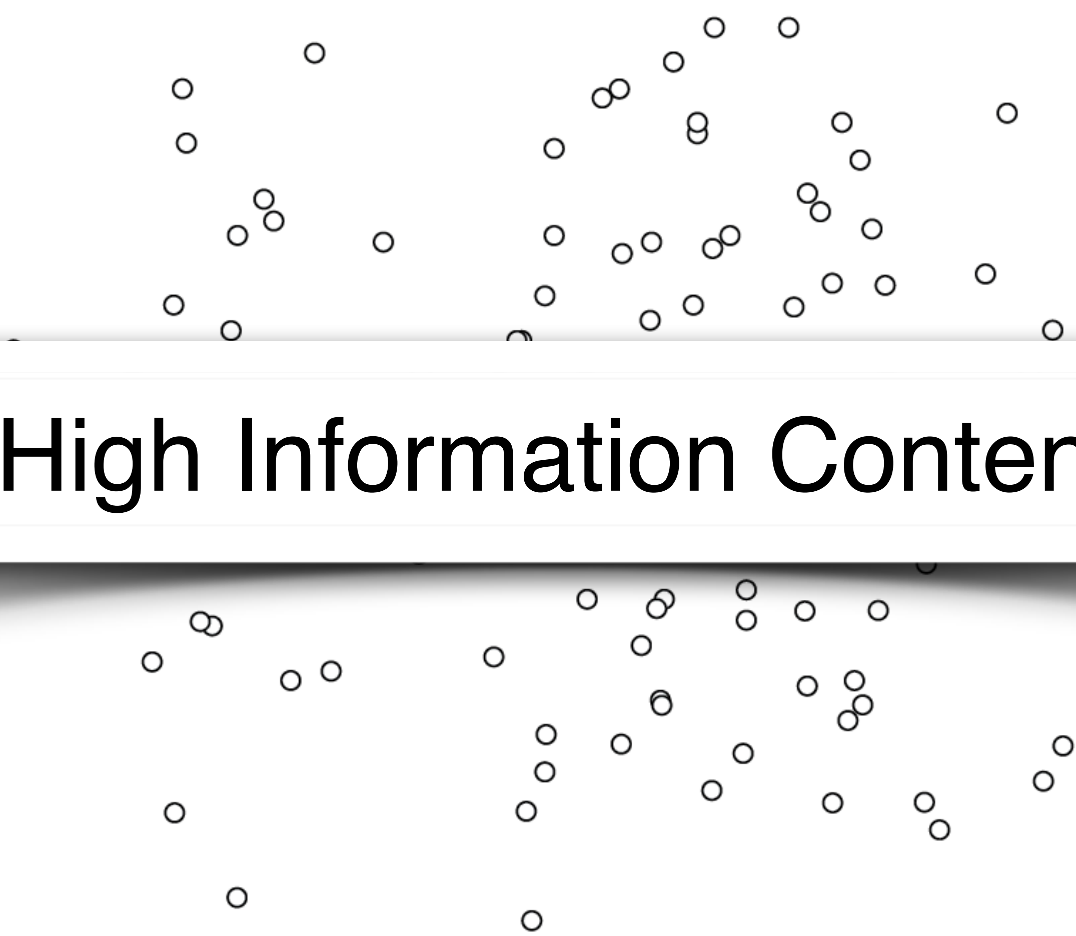

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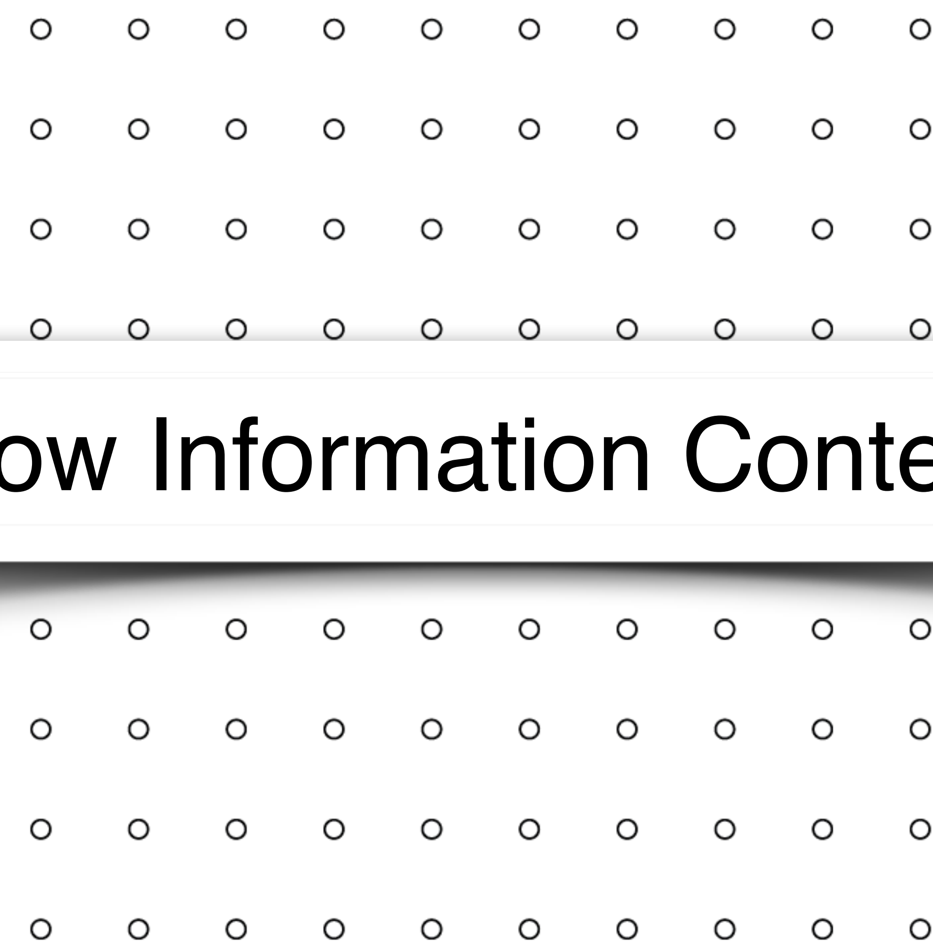
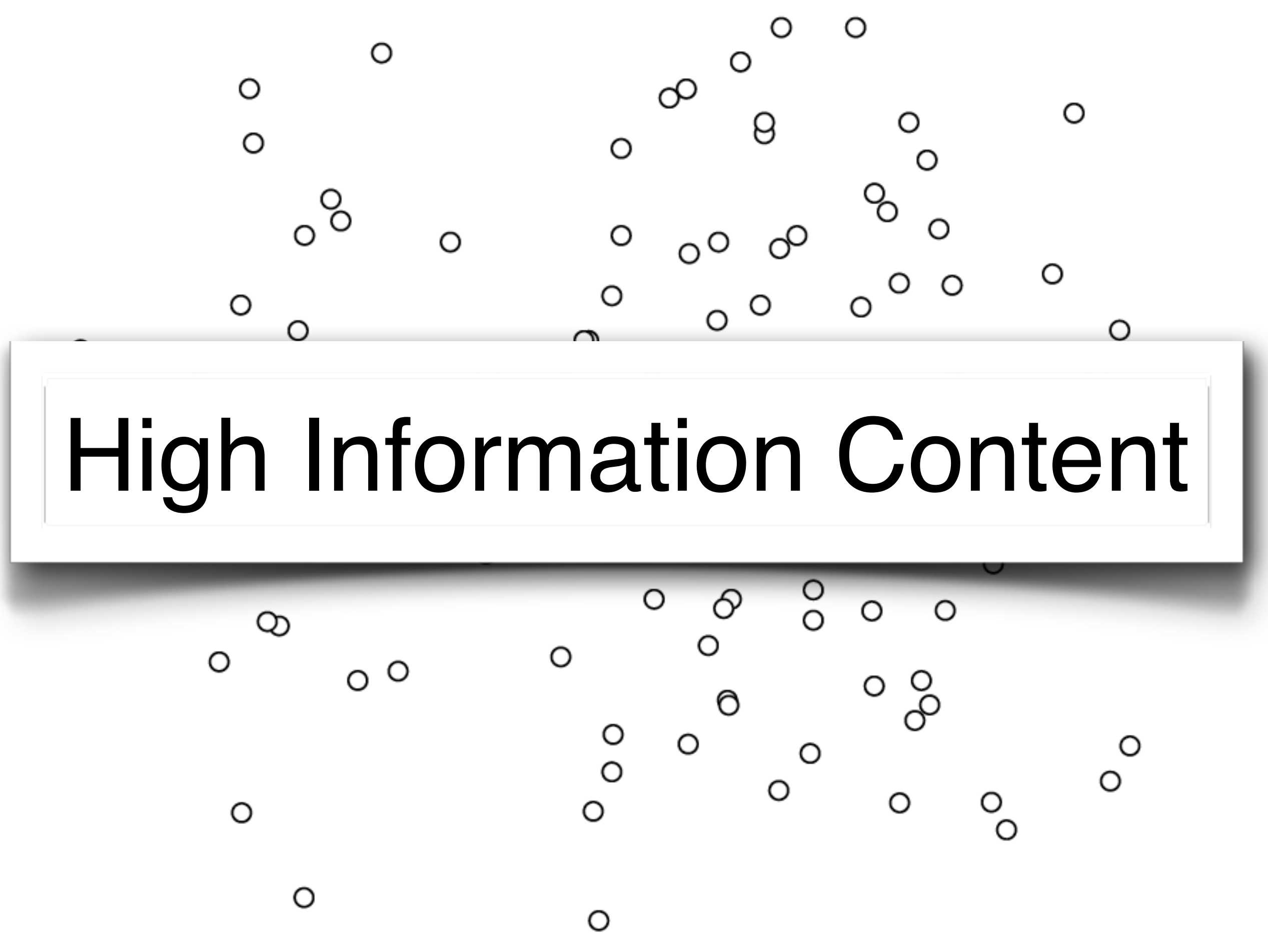


"A 10x10 Regular Grid of Points"

Symmetry and Information

Symmetry is **absence** of information



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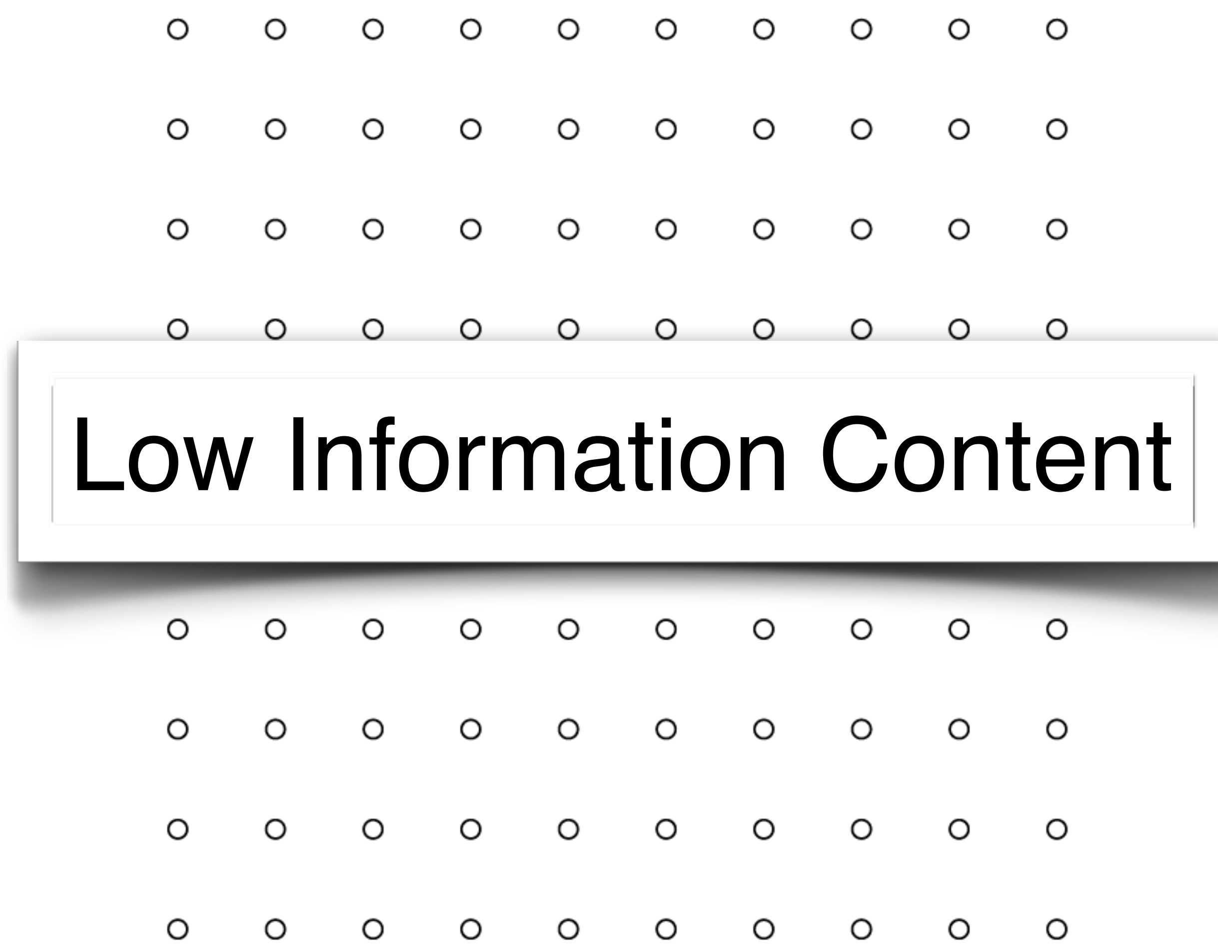
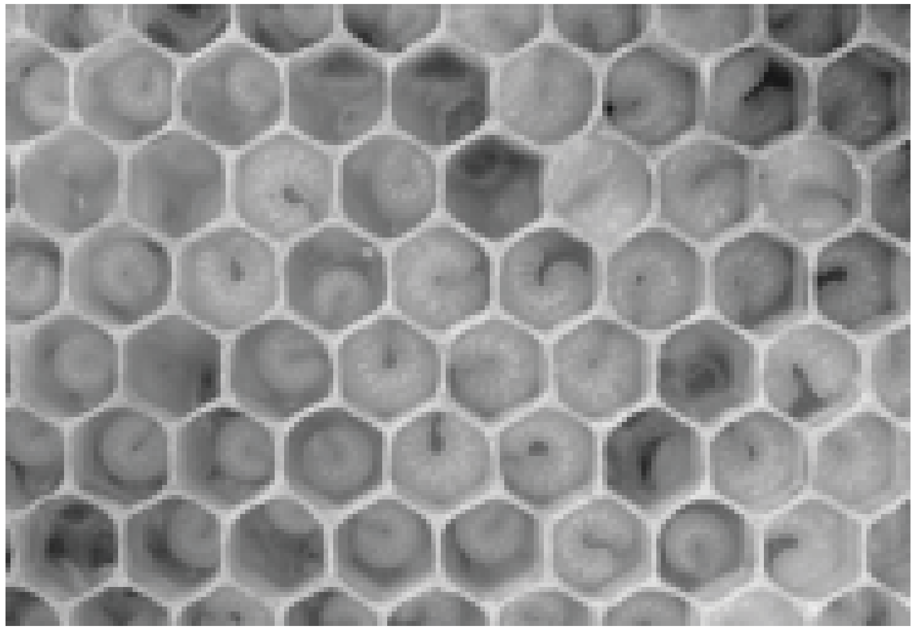


"A 10x10 Regular Grid of Points"

Regular Structure



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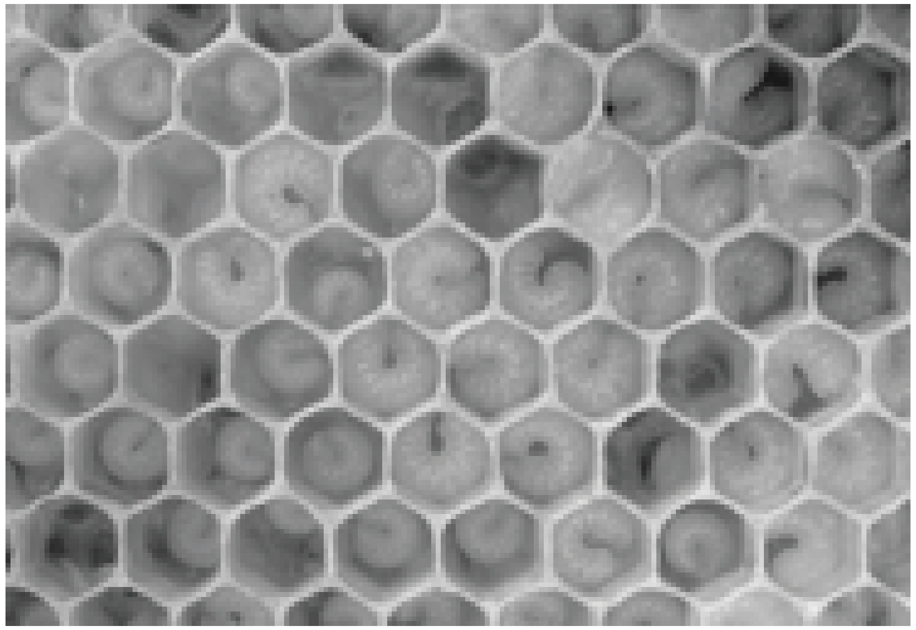

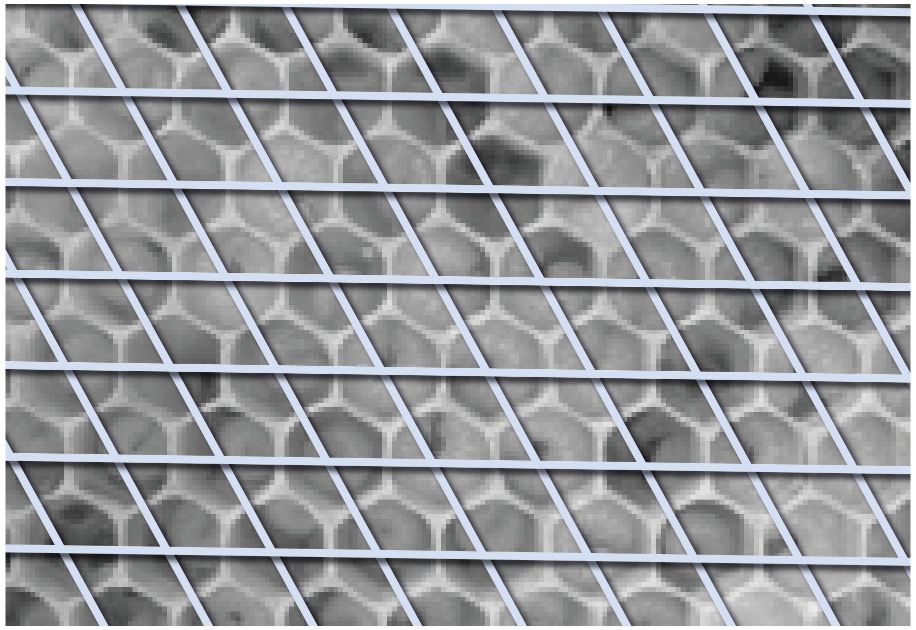

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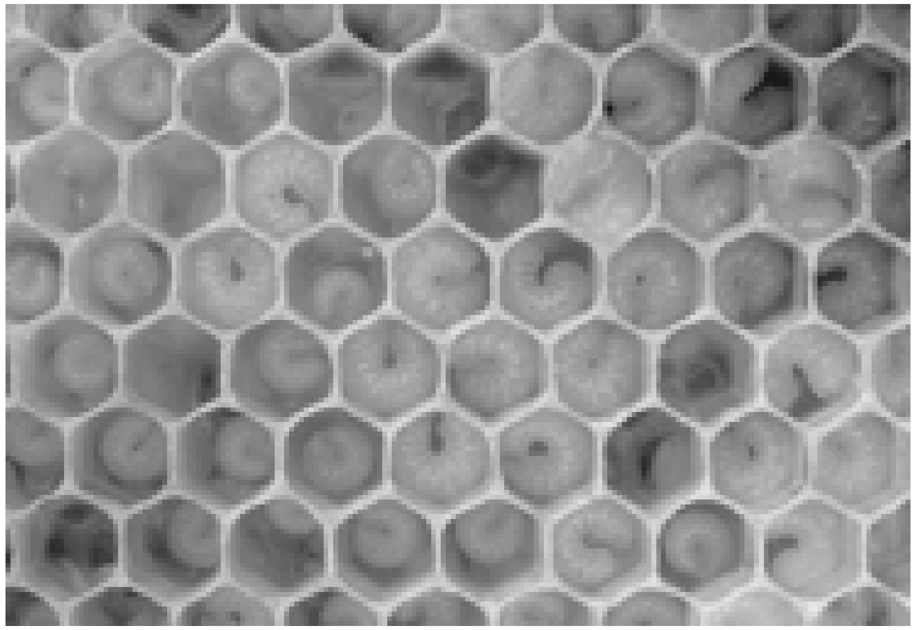

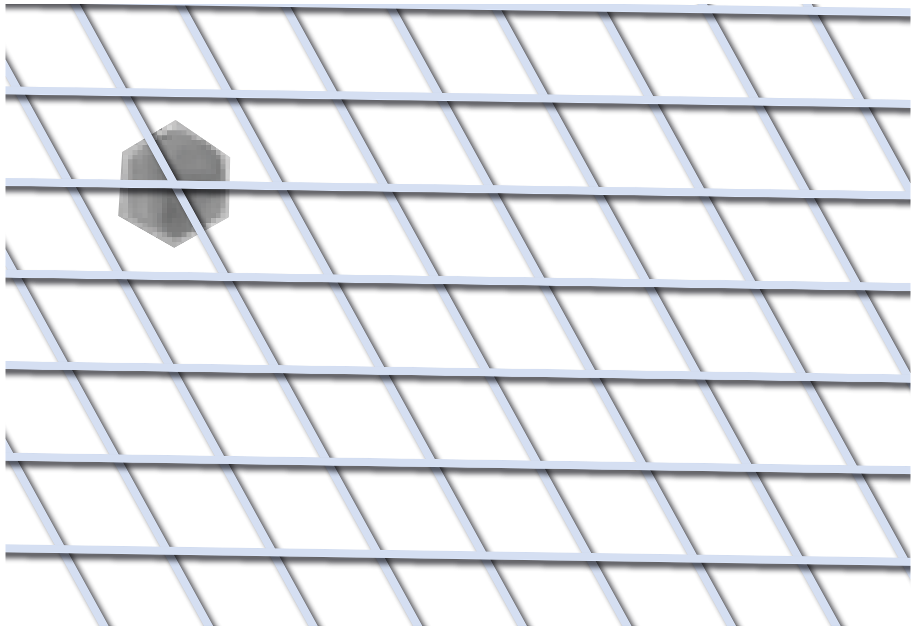

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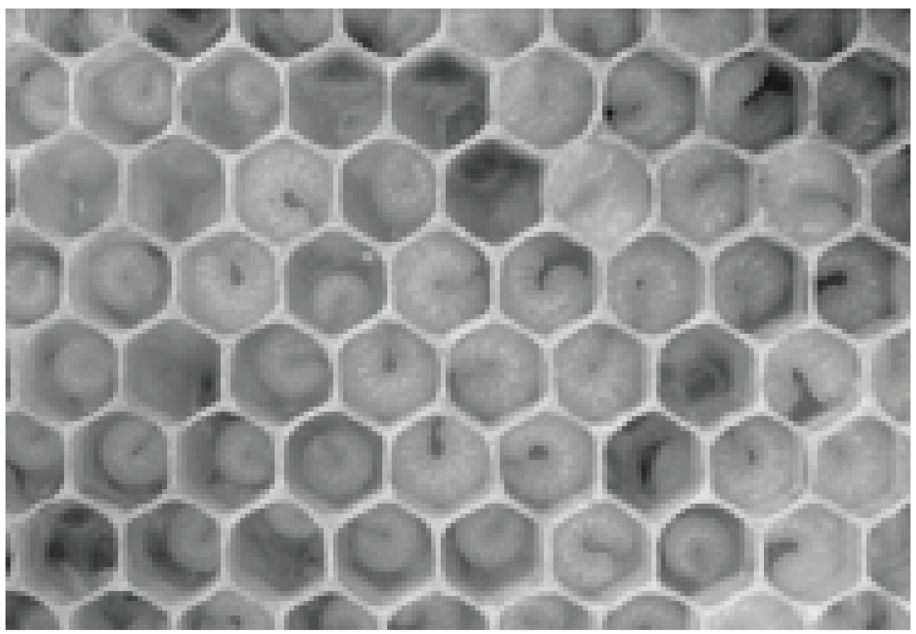

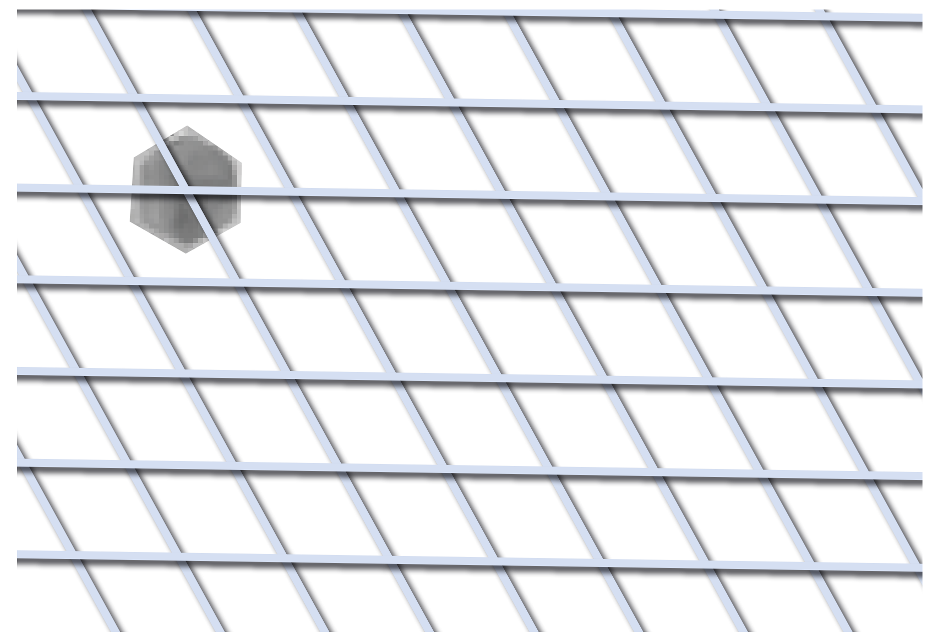

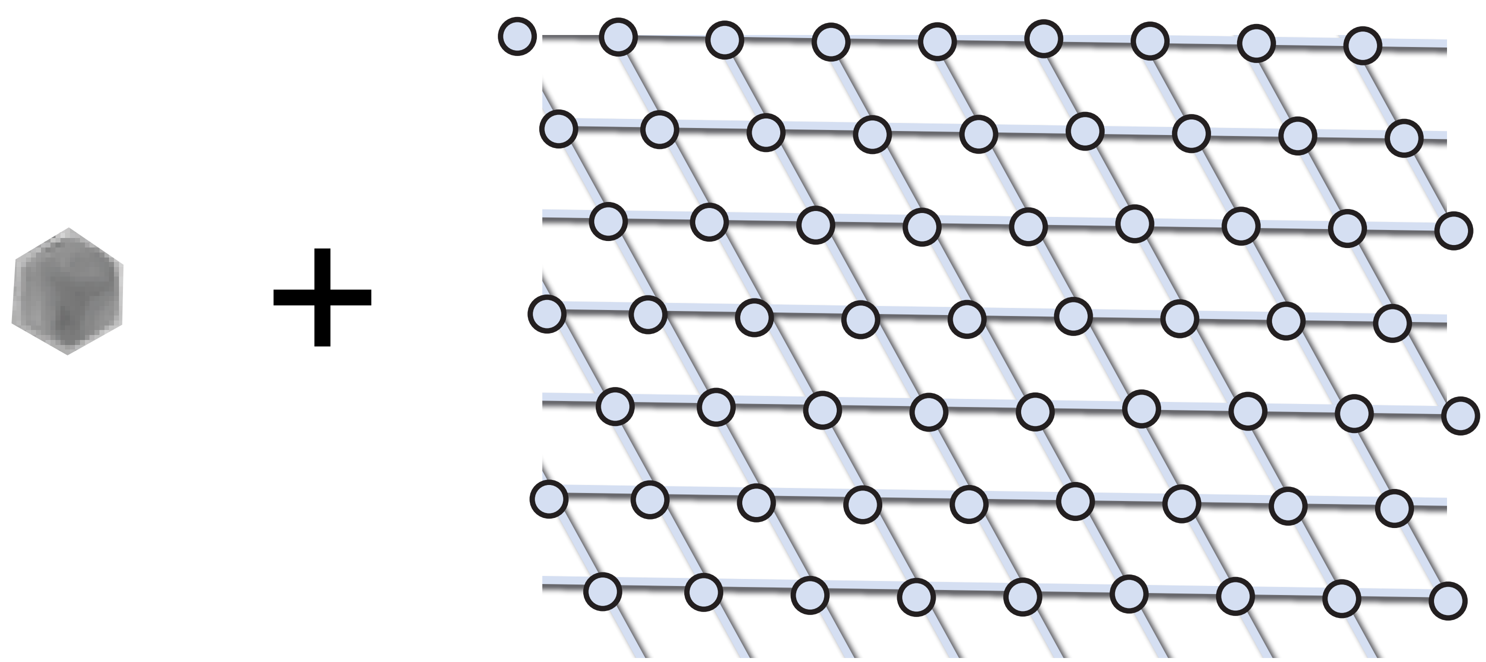

Problem Characteristics

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Difficult

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- Which parts are symmetric → objects are not pre-segmented

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- Space of transforms: rotation, translation, scaling, etc.

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Easy

Problem Characteristics

Difficult

- Which parts are symmetric $\longrightarrow$ objects are not pre-segmented
- Space of transforms: rotation, translation, scaling, etc.
- Brute force search is not feasible

Easy

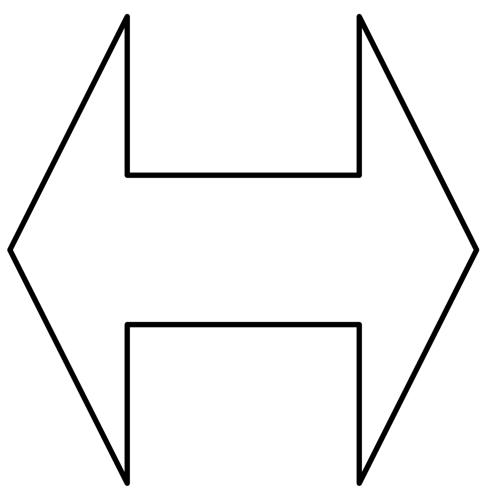
- Proposed symmetries $\longrightarrow$ easy to validate

Relation to Shape Matching

- General setup
- Global registration
- Local registration (refinement)

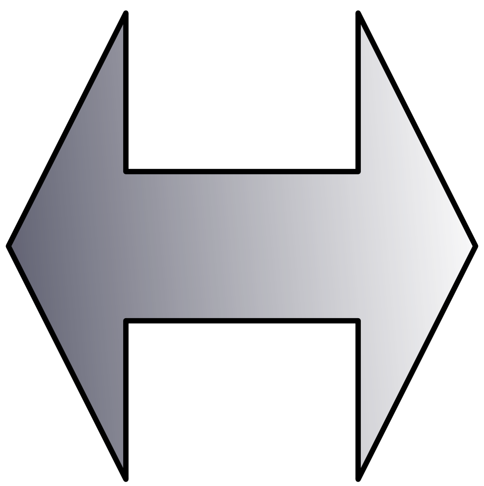
Symmetry Detection

$$M$$



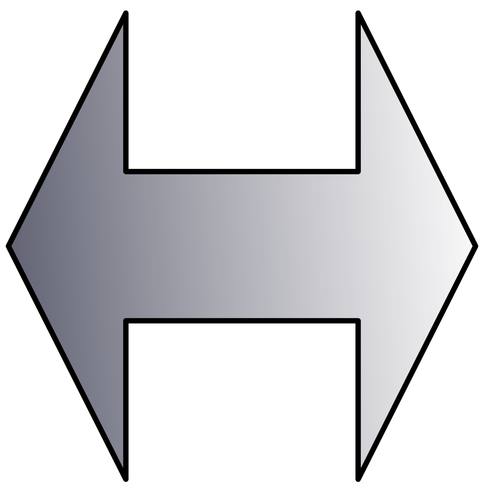
Geometric Matching

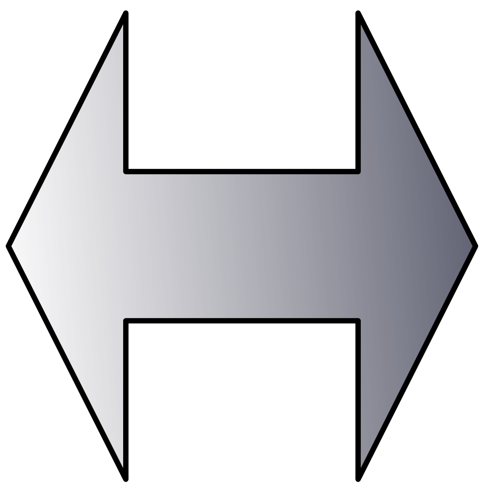
$$M_1$$



Geometric Matching

M_1 M_2





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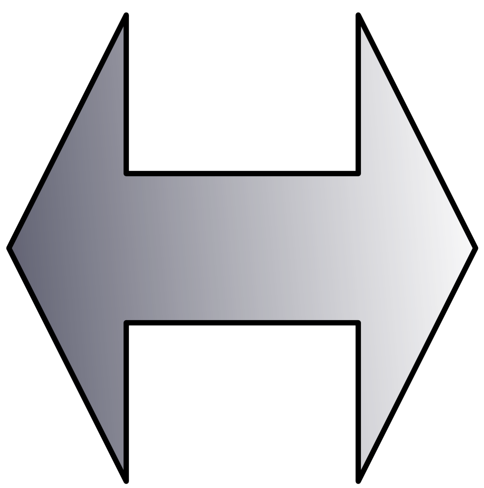
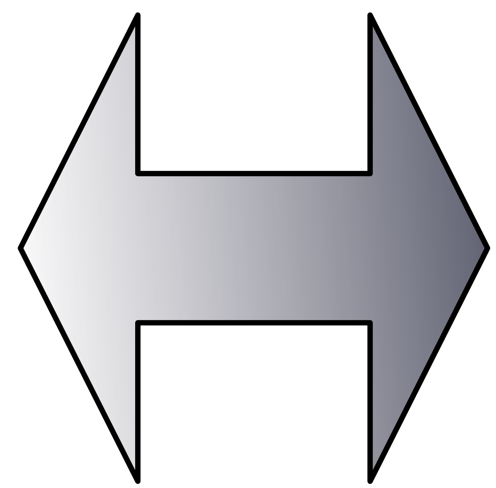


M_2



$$M_1 \approx T(M_2)$$

Geometric Matching

M_1 M_2

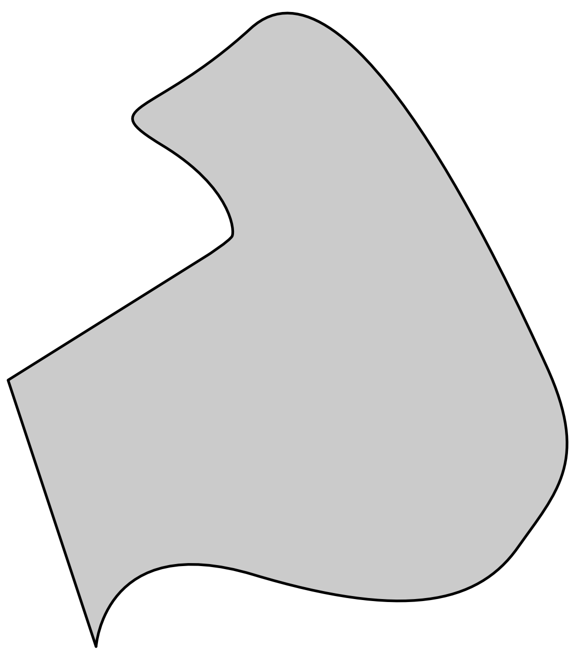
$$M_1 \approx T(M_2)$$

symmetry detection $M \approx T(M)$

Geometric Matching

Geometric Matching

M_1



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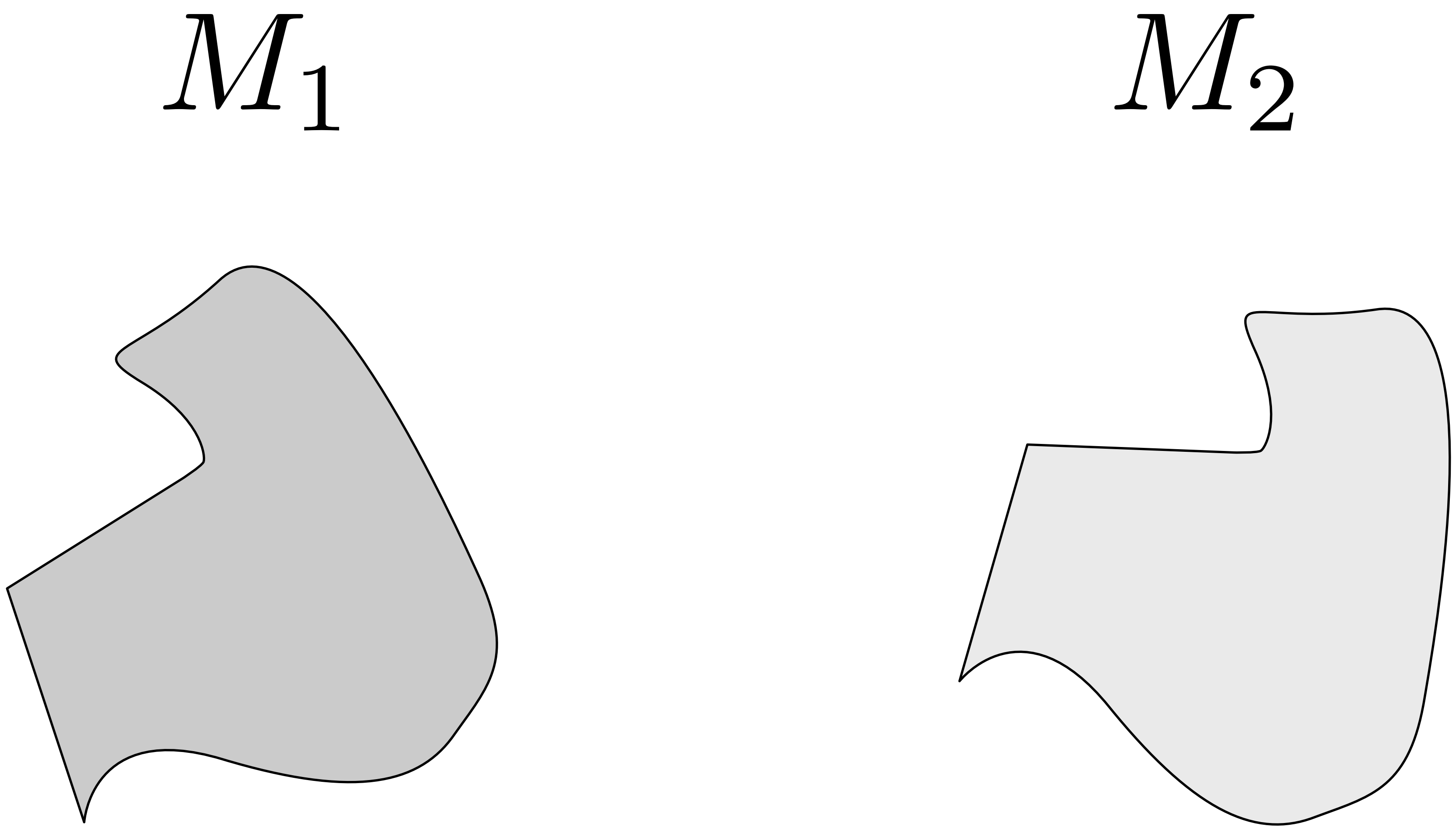

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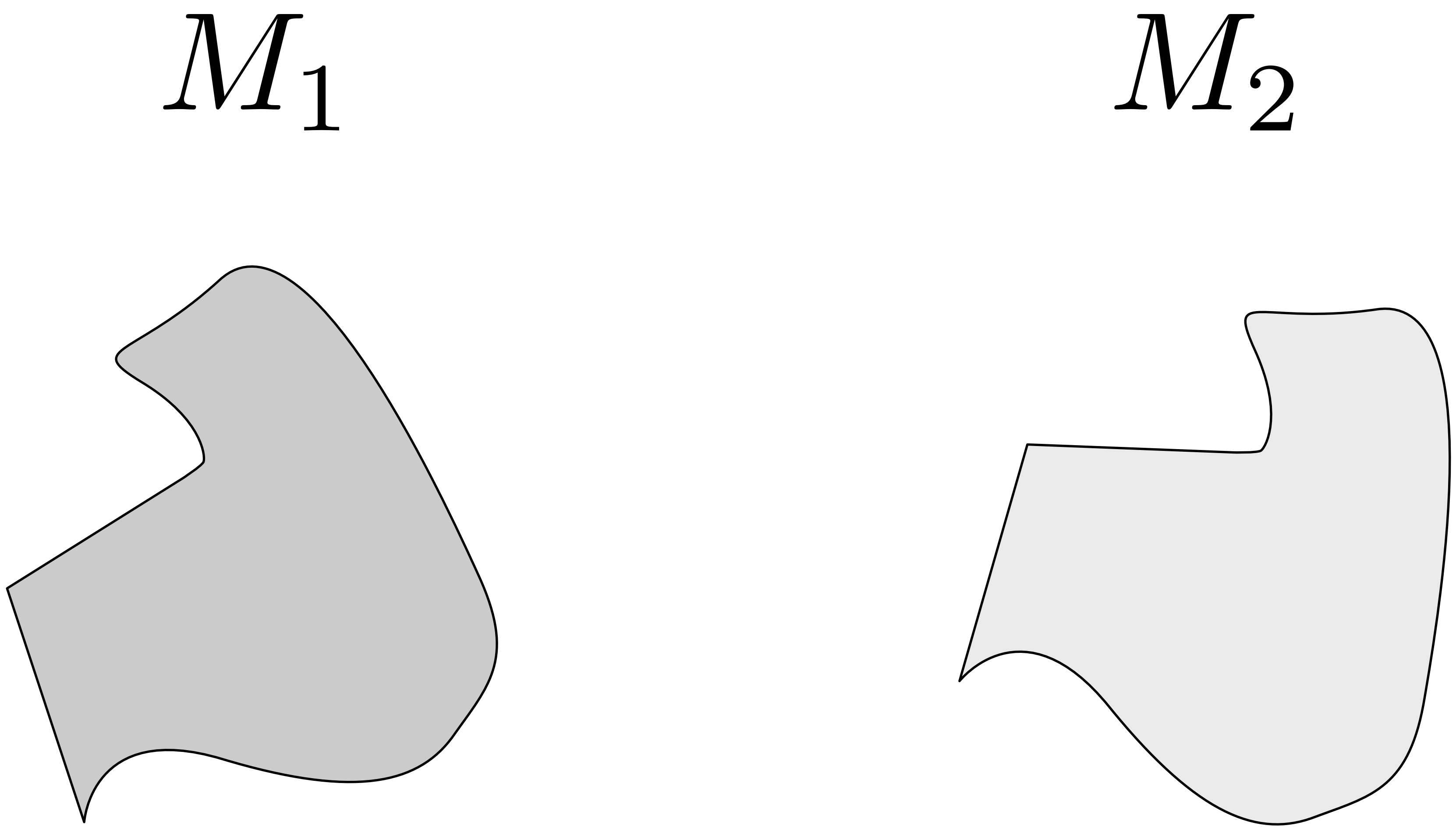

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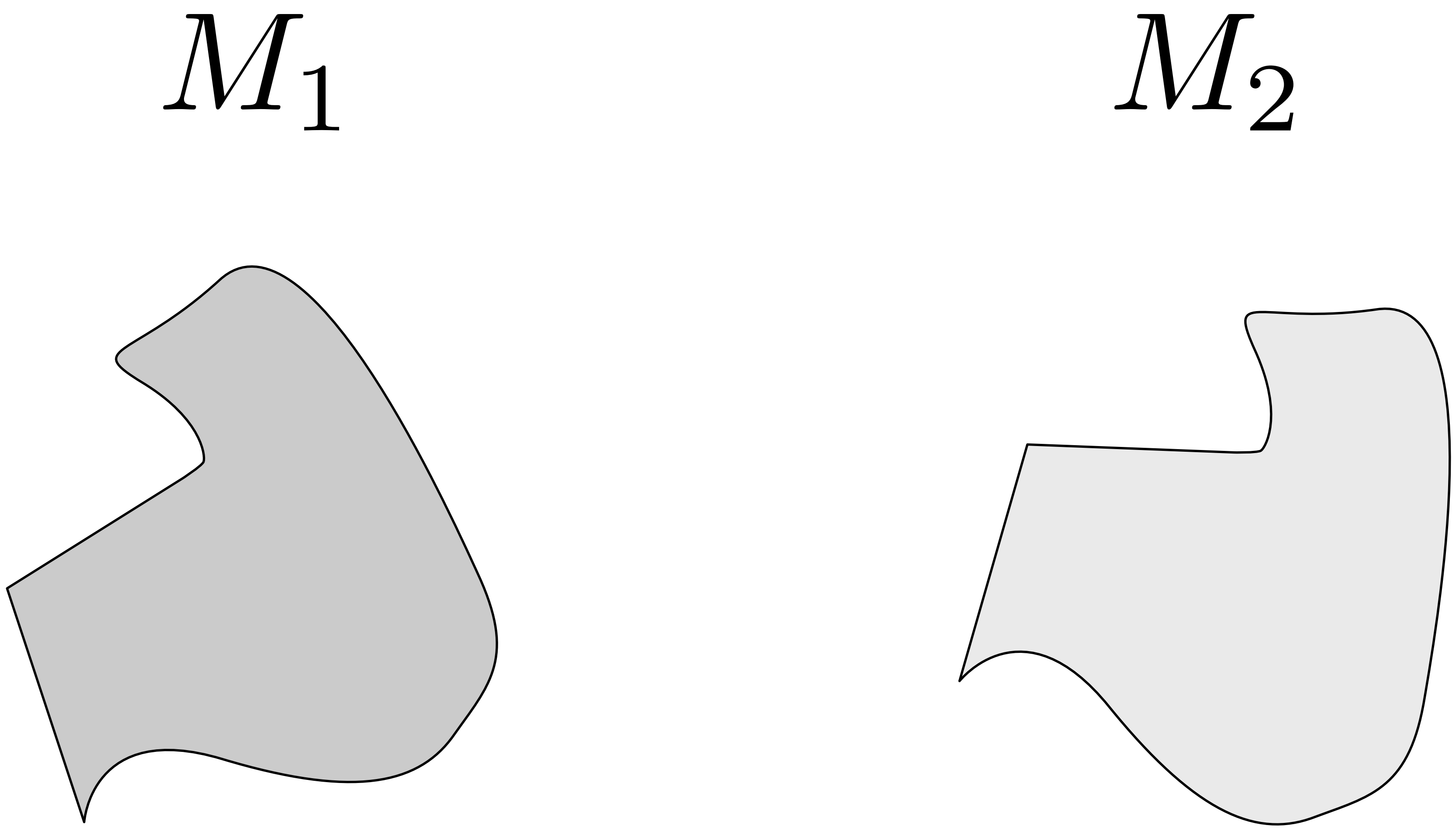
Matching with Translation



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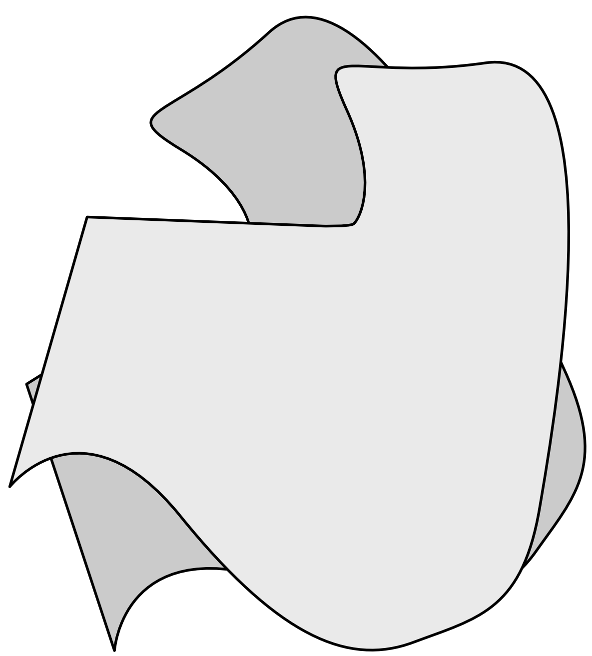
$$T : \text{translation}$$

Matching with Translation

M_1 M_2



$$M_1 \approx T(M_2)$$
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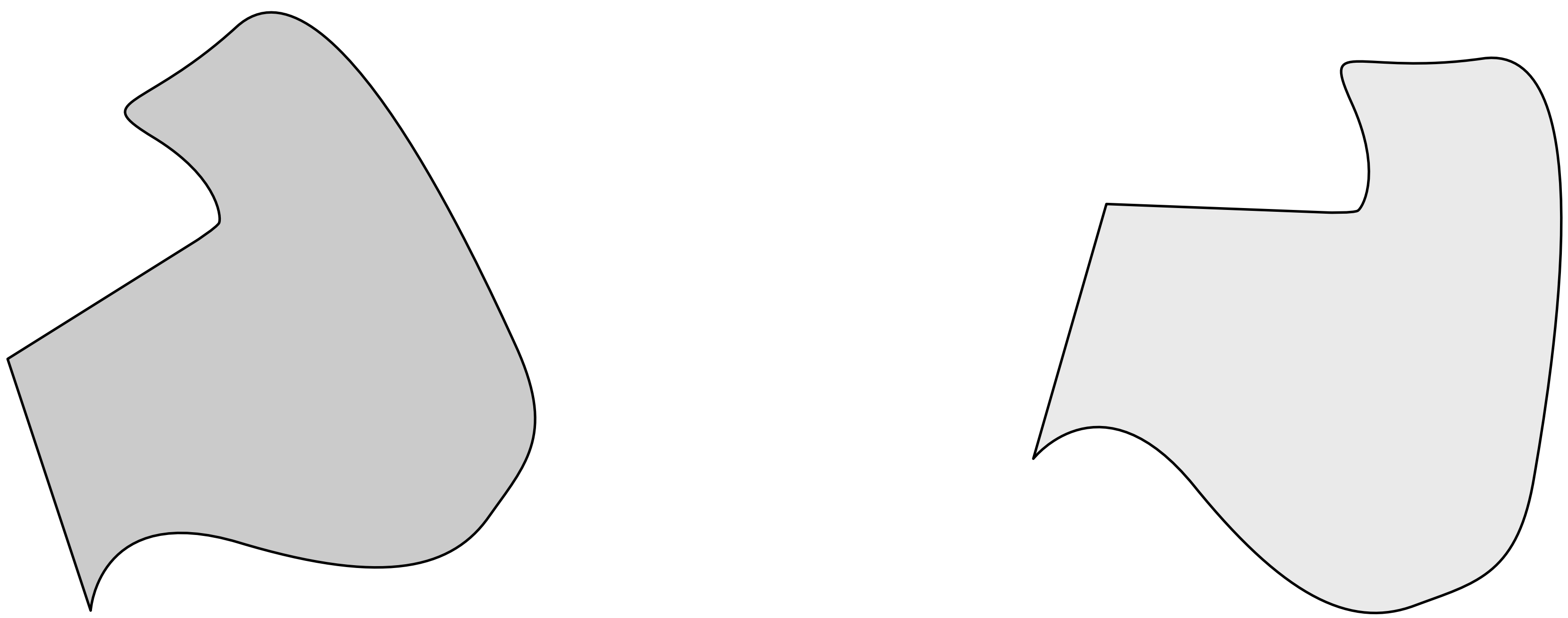
Matching with Rigid Transforms

M_1 M_2



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$$T : \text{translation} + \text{rotation}$$

Matching with Rigid Transforms

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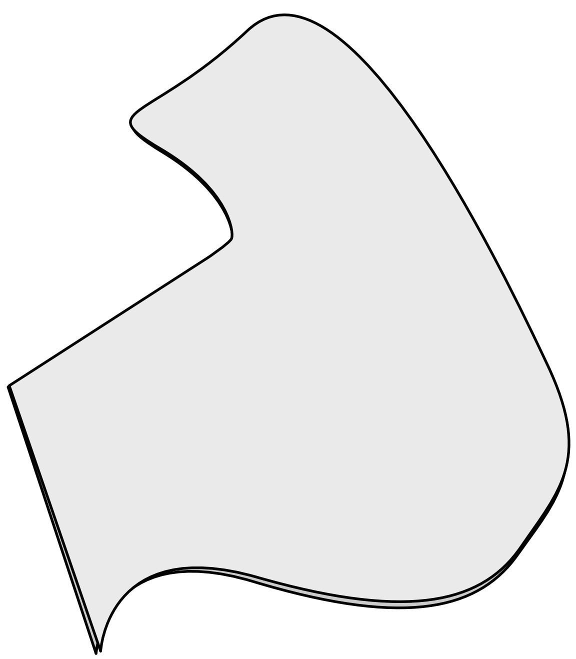

$$M_1 \approx T(M_2)$$

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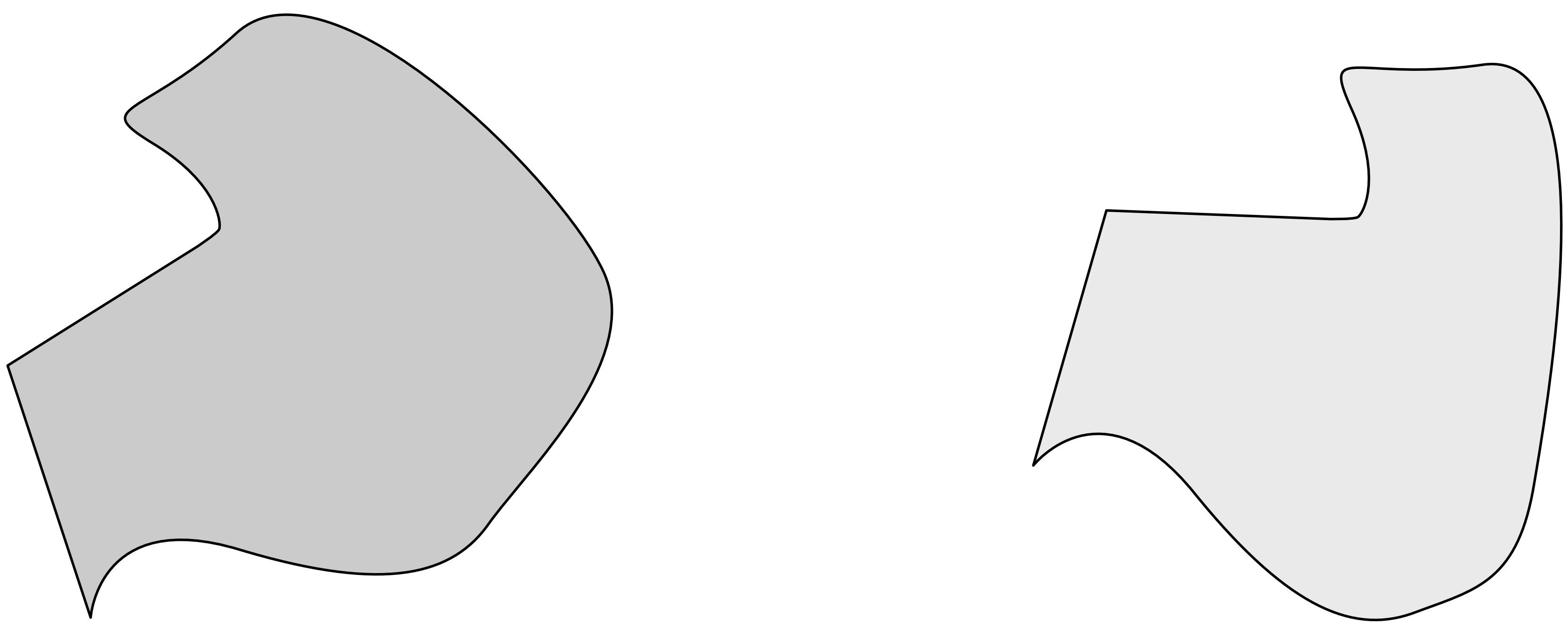
Partial Matching

$$M_1 \qquad\qquad M_2$$



$$M_1 \approx T(M_2)$$

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Partial Matching

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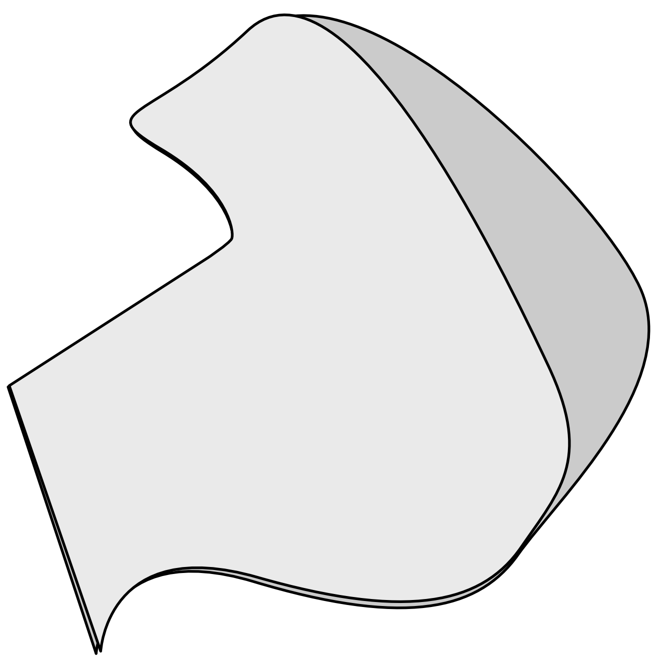

$$M_1 \approx T(M_2)$$

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Local vs. Global Matching

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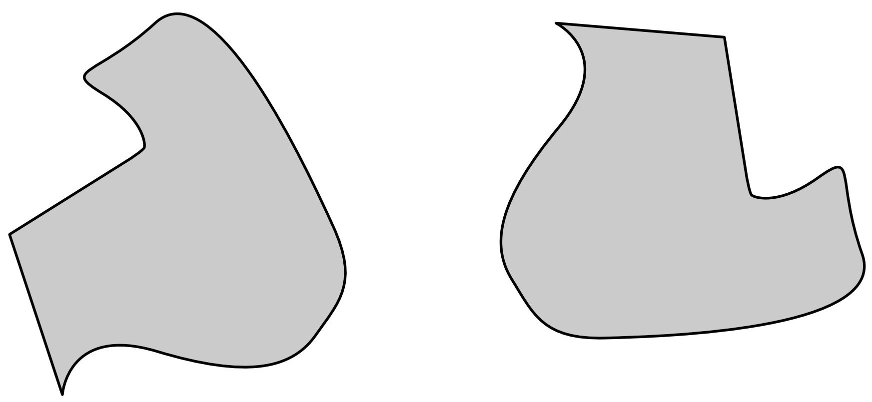
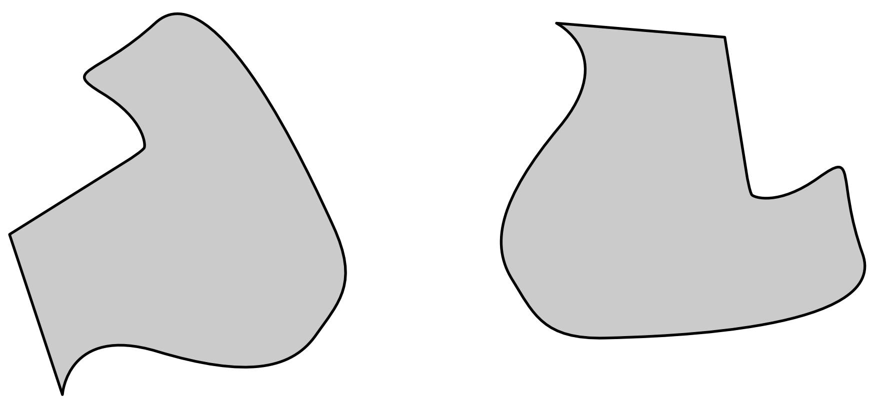


global registration

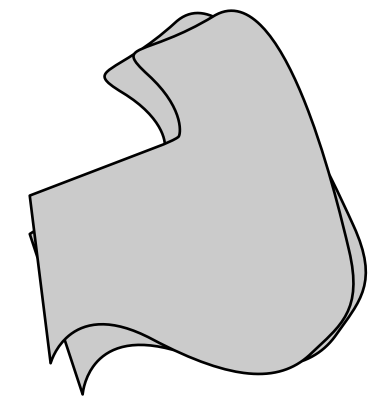
any rigid transform

Local vs. Global Matching



global registration

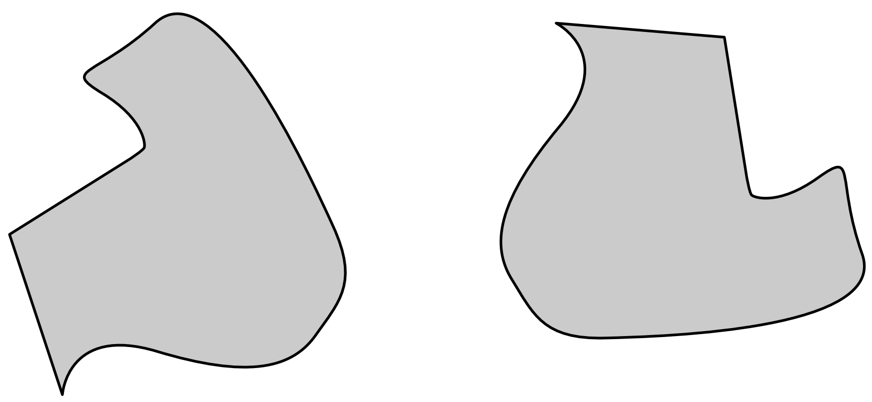
any rigid transform



local registration

nearly aligned

Local vs. Global Matching



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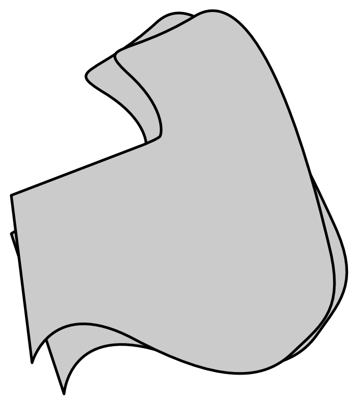

local registration
nearly aligned

Given $M_1, \ldots, M_n$, find $T_2, \ldots, T_n$ such that

$$M_1 \approx T_2(M_2) \cdots \approx T_n(M_n)$$

ICP: Local, partial, rigid transforms

- How many point-pairs are needed to *uniquely* define a rigid transform?

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$$\mathbf{p}_3 \rightarrow \mathbf{q}_3$$

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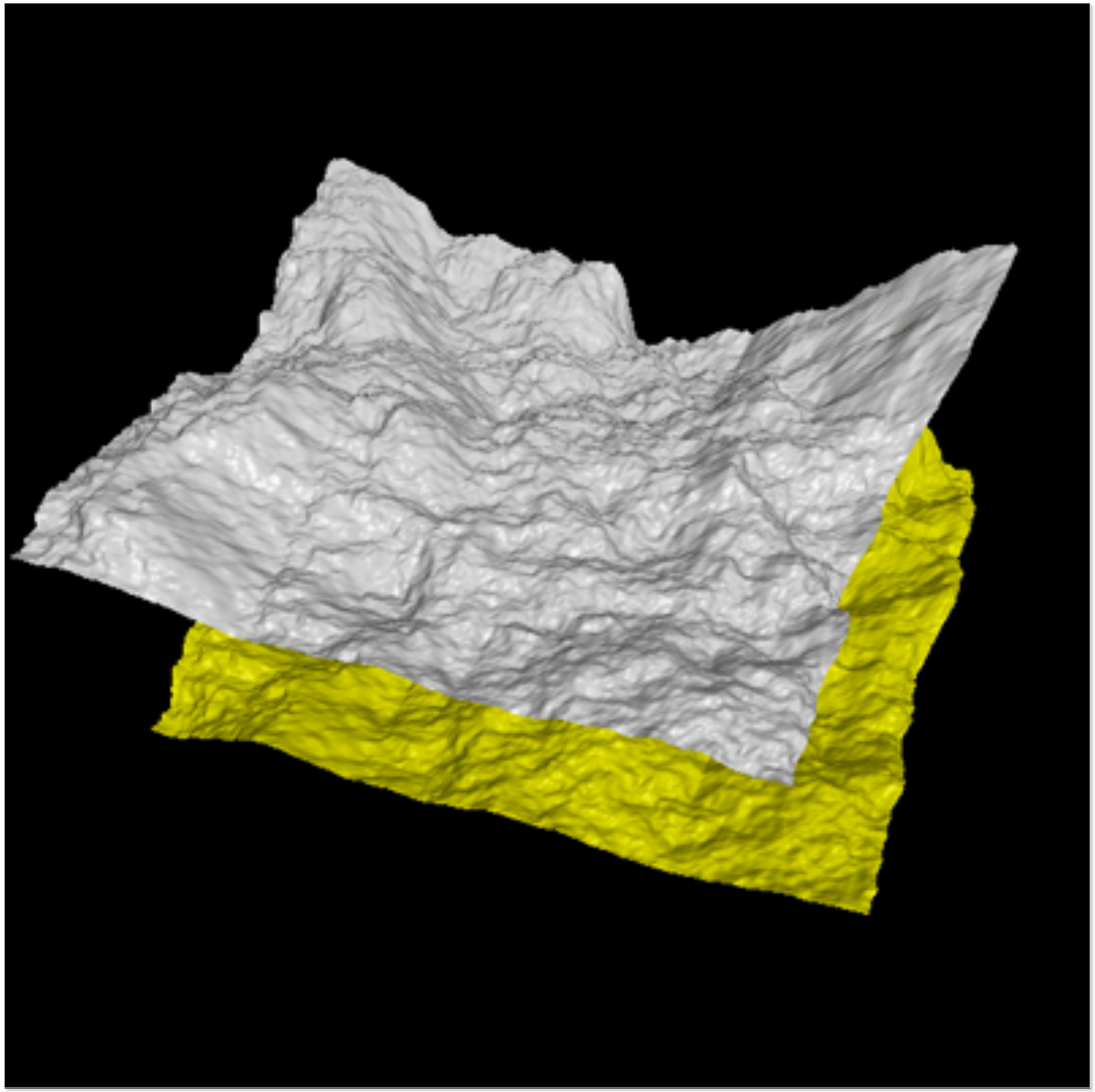
Correspondence problem:

$$\mathbf{p}_i \xrightarrow{?} \mathbf{q}_j$$

Pairwise Rigid Registration Goal

Align two partially-overlapping meshes,
given initial guess for relative transform



[@Rusinkiewicz]

Pairwise Rigid Registration Goal

Align two partially-overlapping meshes,
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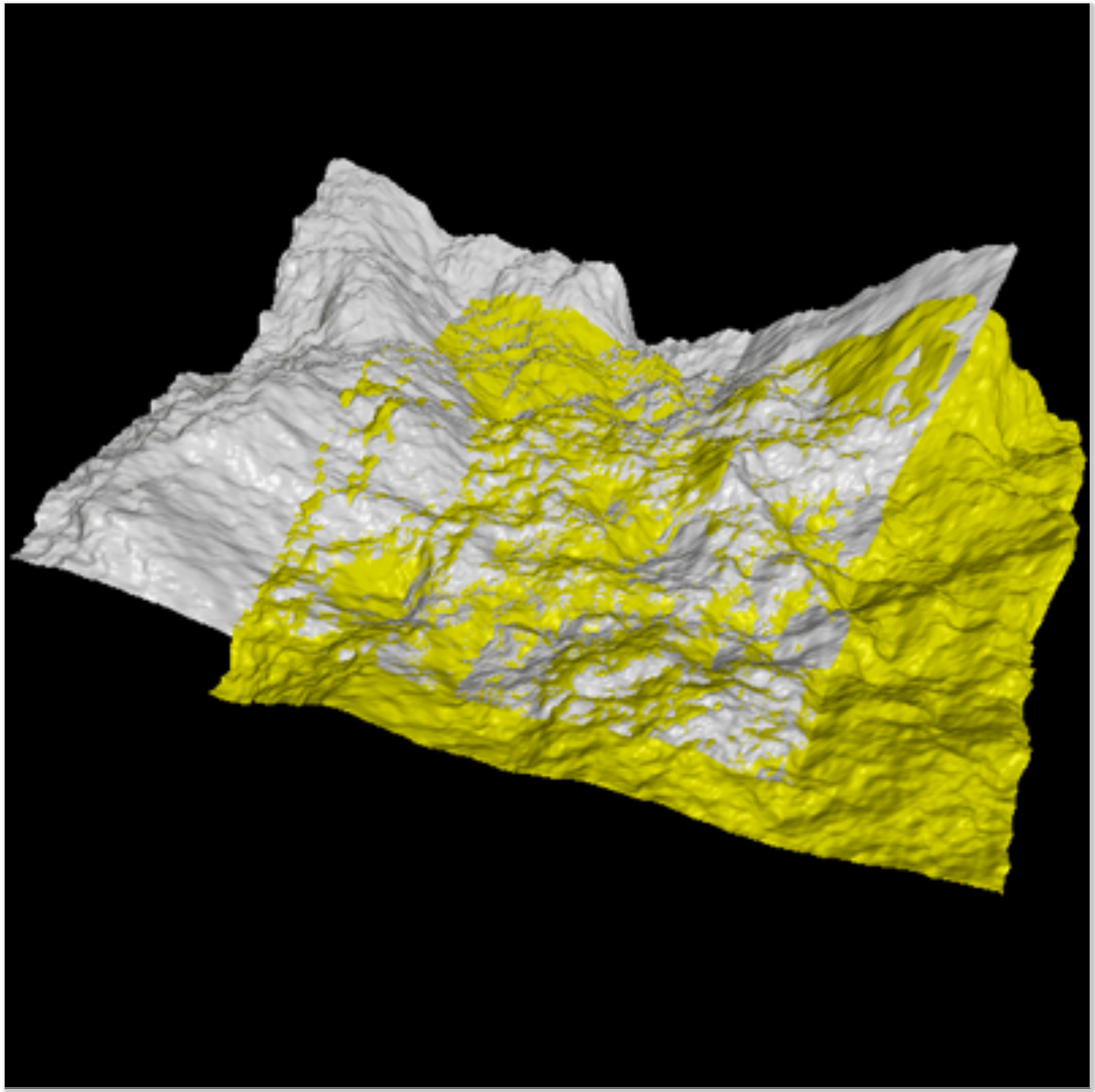
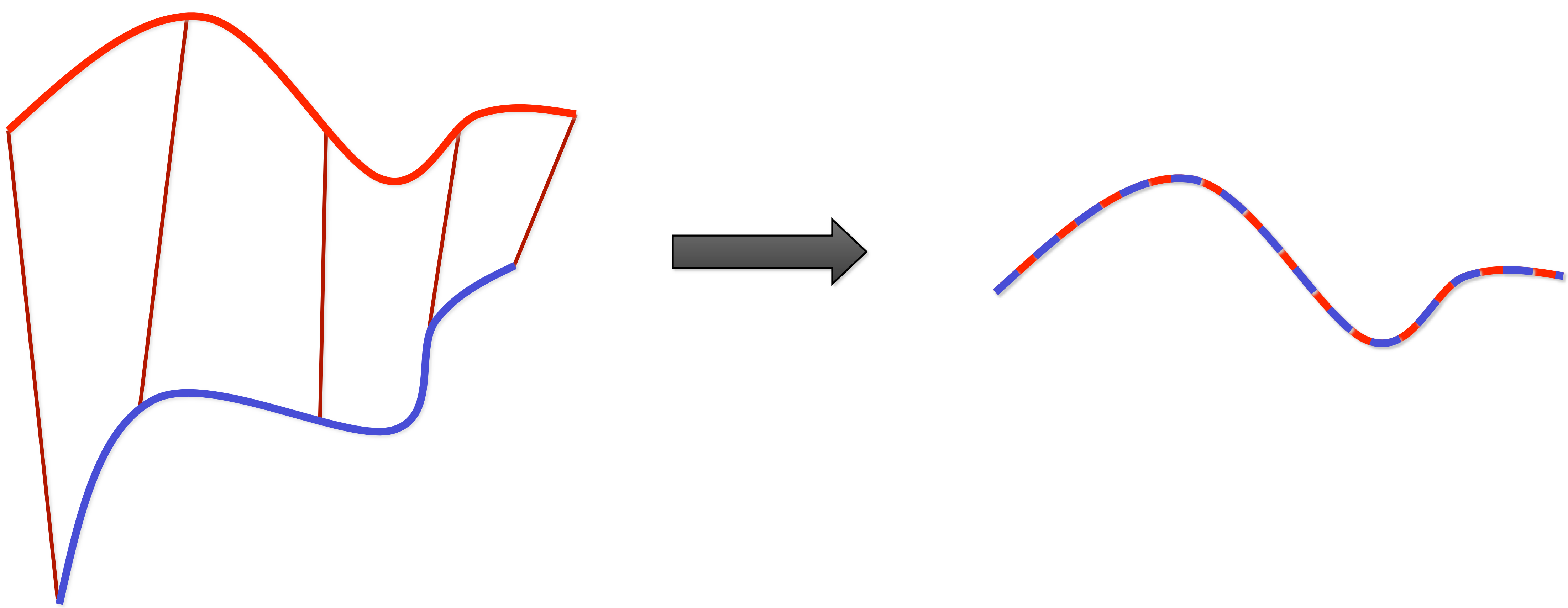


[@Rusinkiewicz]

Aligning 3D Data

If correct correspondences are known,
can find correct relative rotation/translation



Aligning 3D Data

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How to find correspondences:

User input?

Feature detection?

Signatures?

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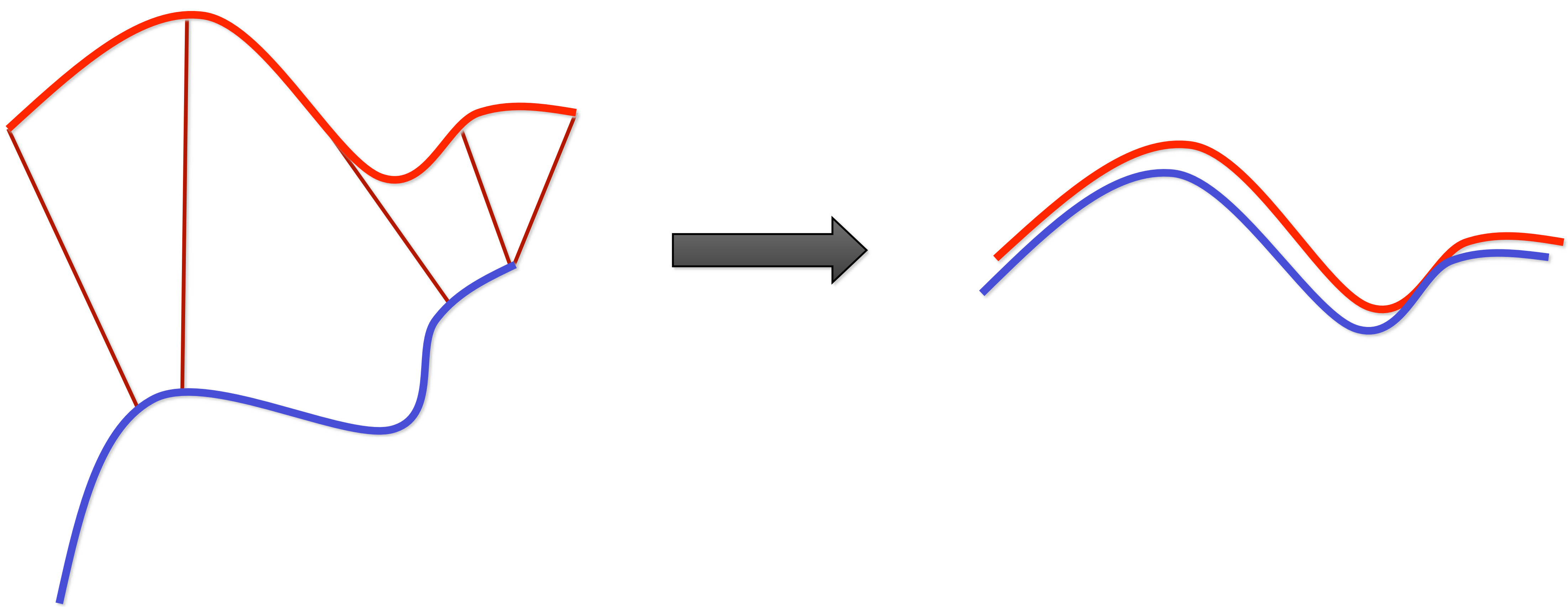
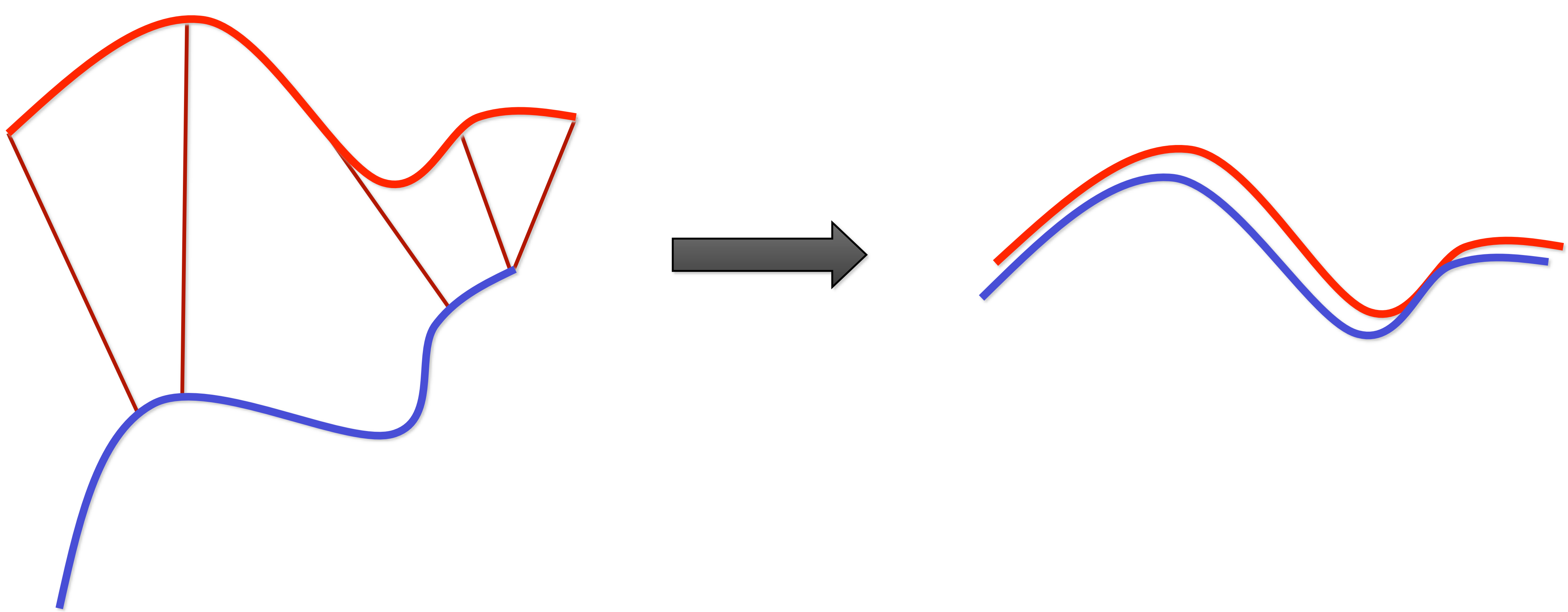


Aligning 3D Data

Assume: Closest points as corresponding

$$\mathbf{p}_i \rightarrow \mathcal{C}(\mathbf{p}_i)$$



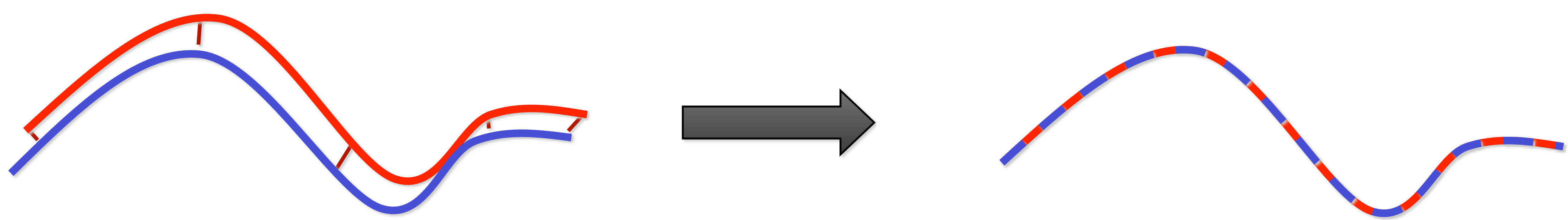
Aligning 3D Data

... and iterate to find alignment

Iterative Closest Points (ICP)

[Besl and McKay 92]

Converges if starting poses are *close enough*



Basic ICP

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- Select (e.g., 1000) random points

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- Reject pairs with distance > k times median
- Construct error function:

$$E := \sum_i (R\mathbf{p}_i + t - \mathbf{q}_i)^2$$

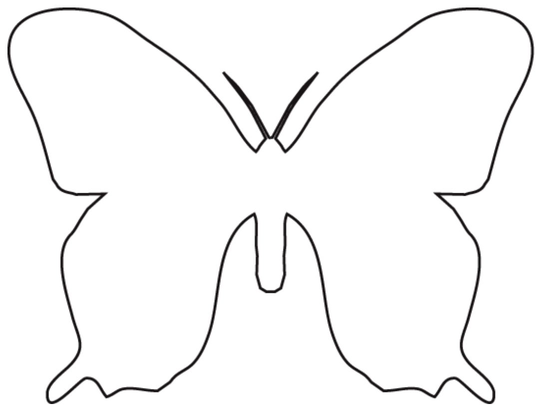
- Minimize (closed form solution in [Horn 87])

ICP Variants

Variants of basic ICP

1. Selecting source points (from one or both meshes)
2. Matching to points in the other mesh
3. Weighting the correspondences
4. Rejecting certain (outlier) point pairs
5. Assigning an error metric to the current transform
6. Minimizing the error metric w.r.t. transformation

Types of Symmetry



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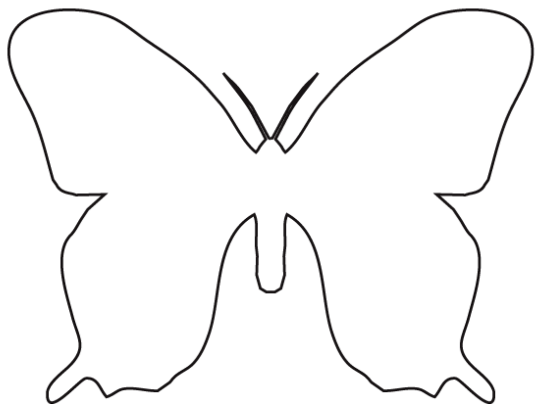
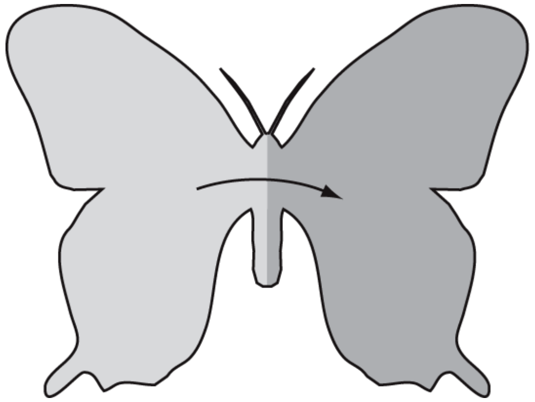
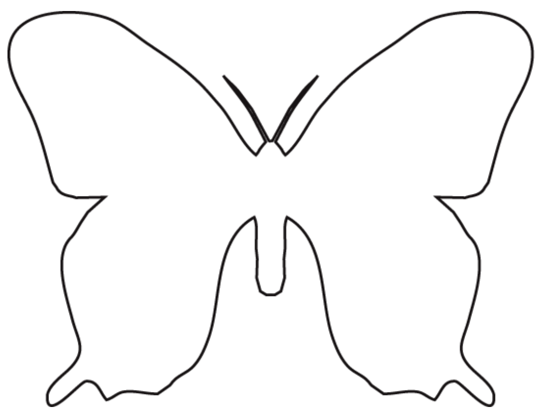
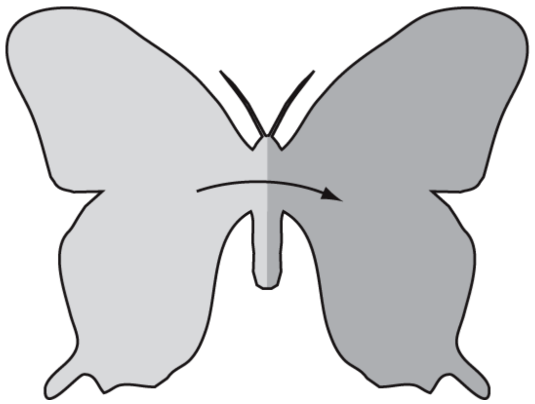


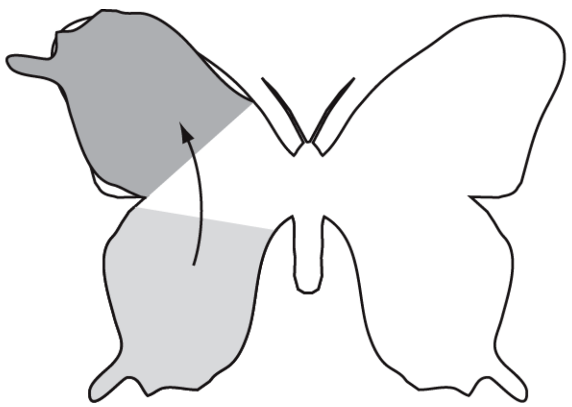


- Reflection

Types of Symmetry

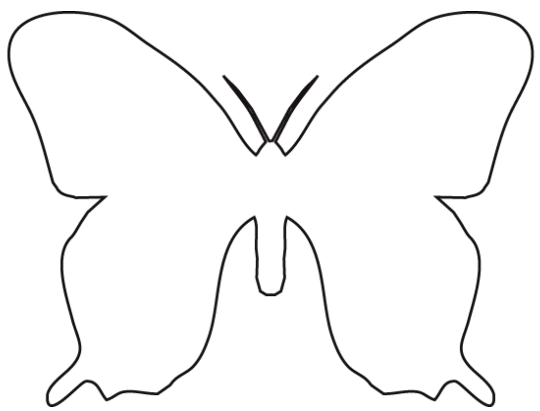


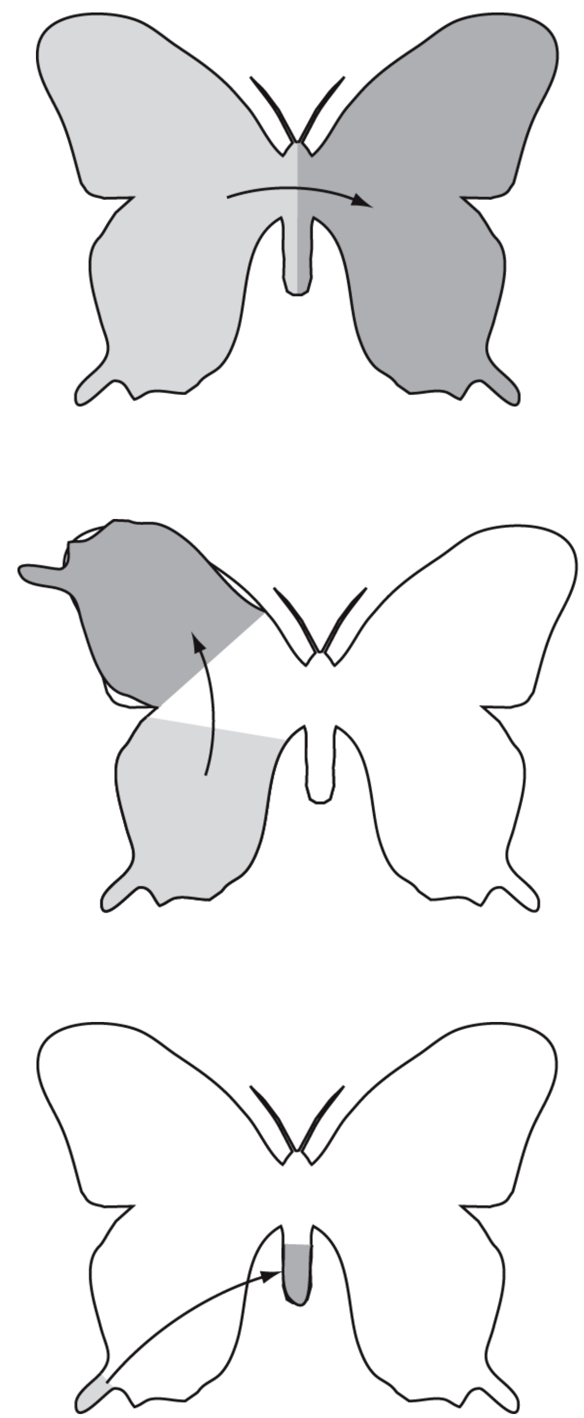




- Reflection
- Rotation + translation

Types of Symmetry





- Reflection
- Rotation + translation
- Uniform scaling

Typical Stages

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- Feature selection

$$\mathcal{F}(M) = \mathcal{F}(T(M))$$

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Type of Features

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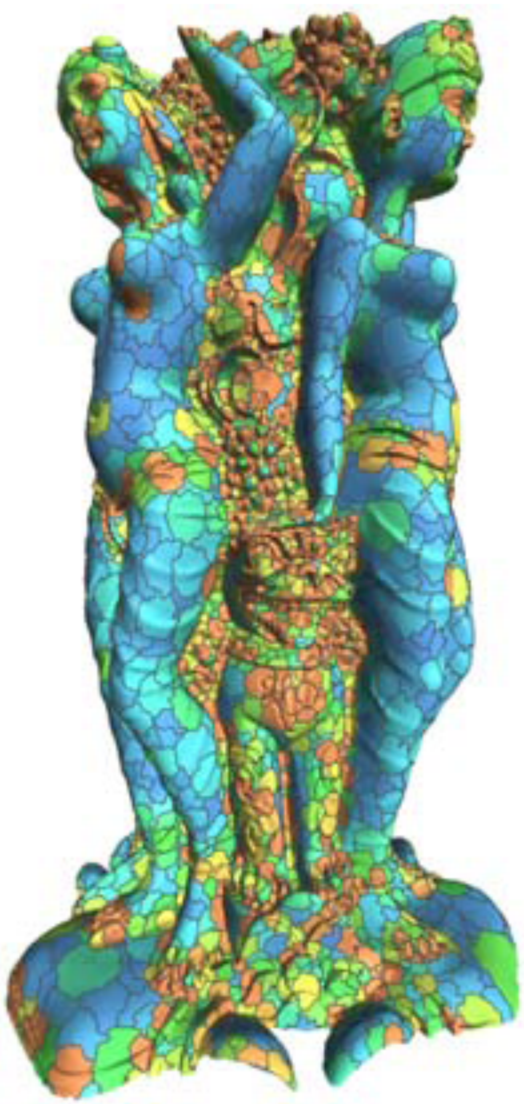
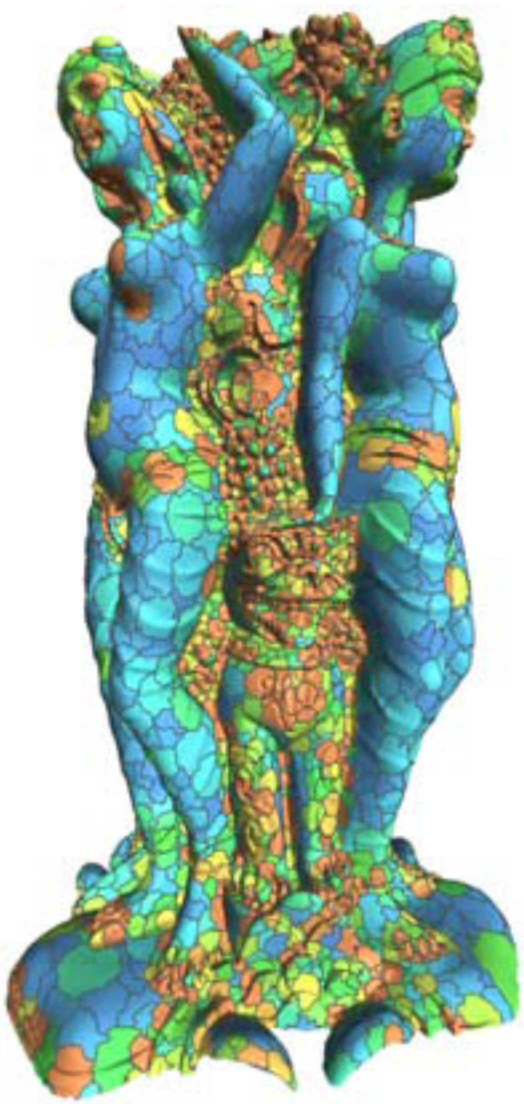


surface curvature
Gal et al. 2006

Type of Features



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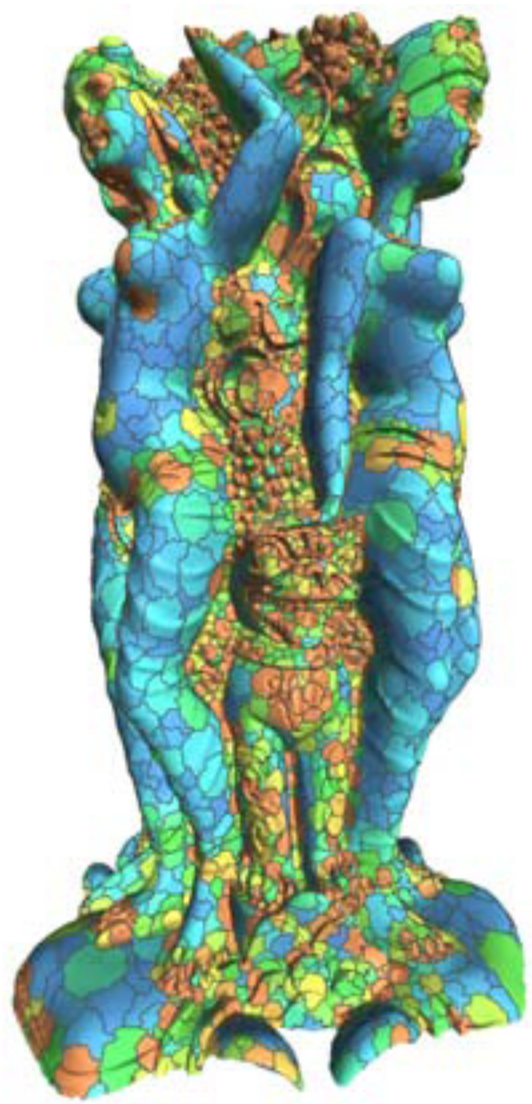

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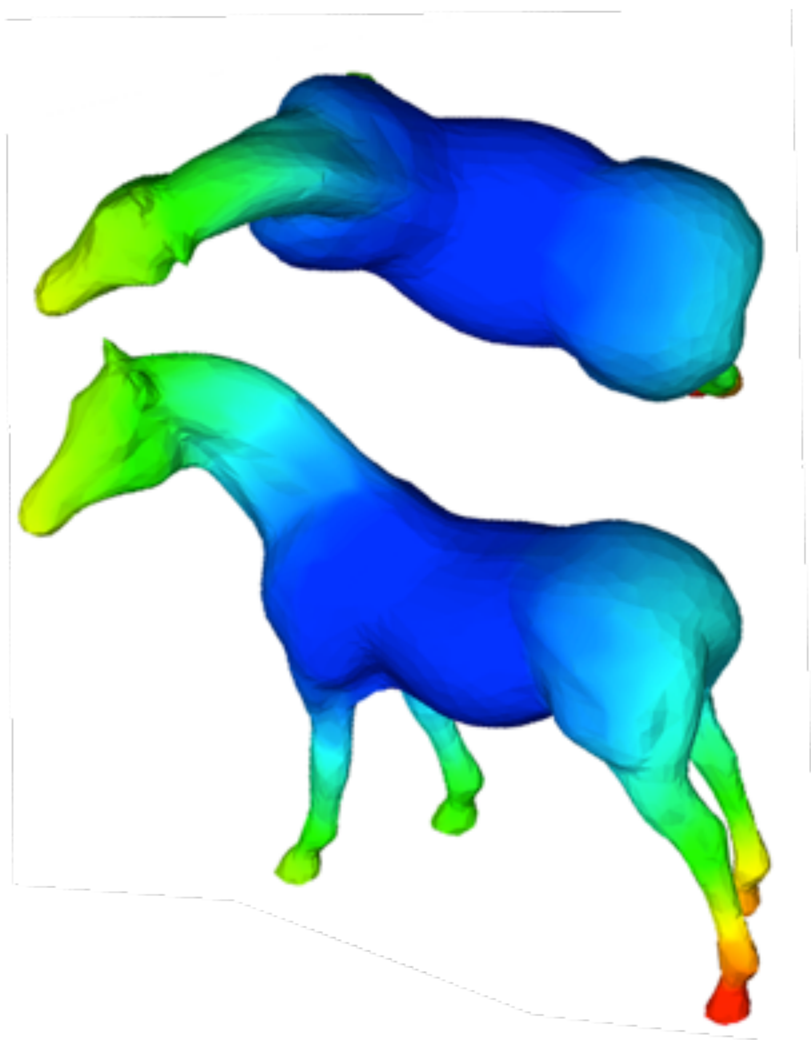

geodesic distances

Kim et al. 2010

Type of Features

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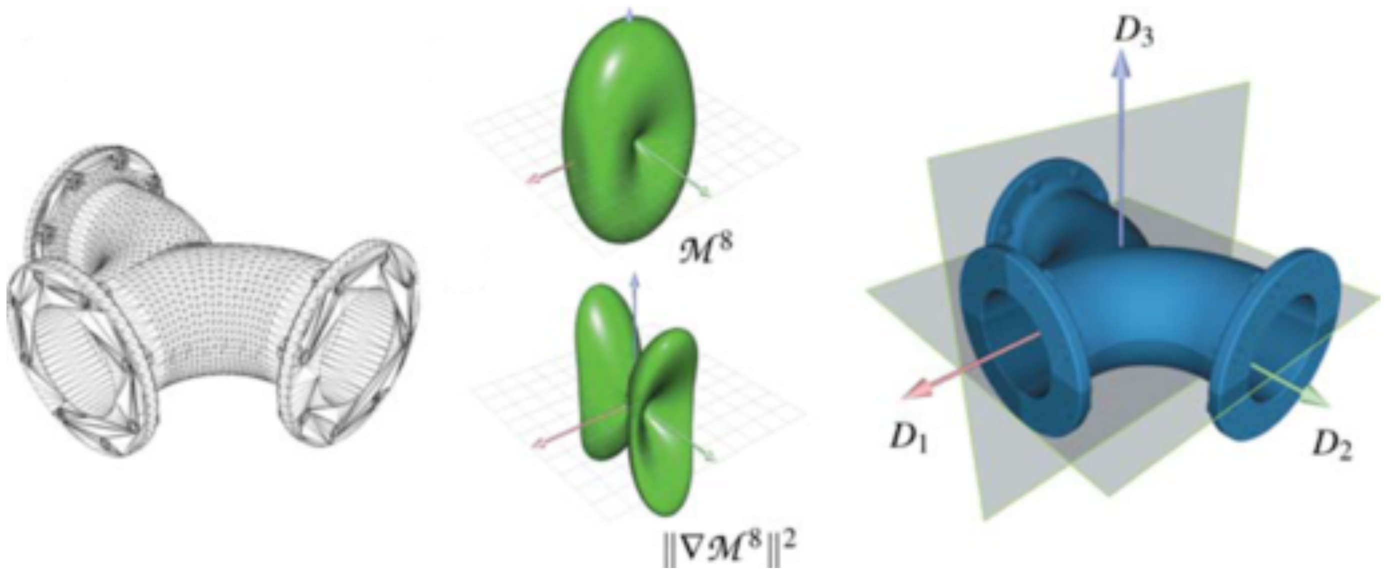

generalized even moments

Martinet et al. 2006

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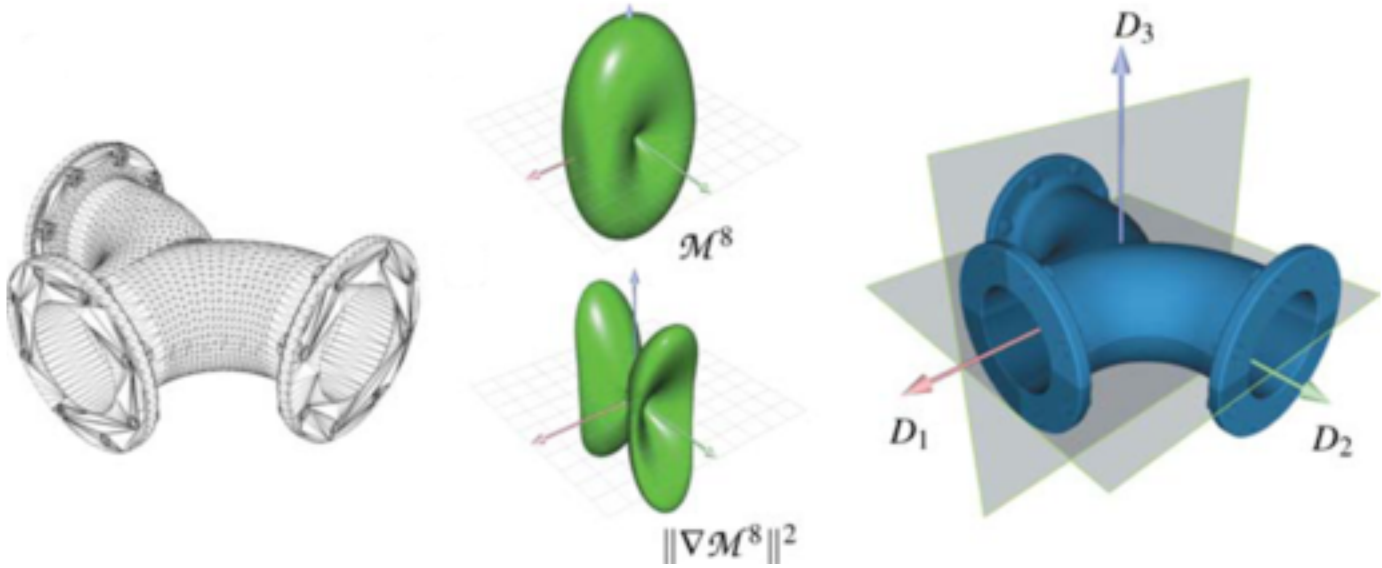

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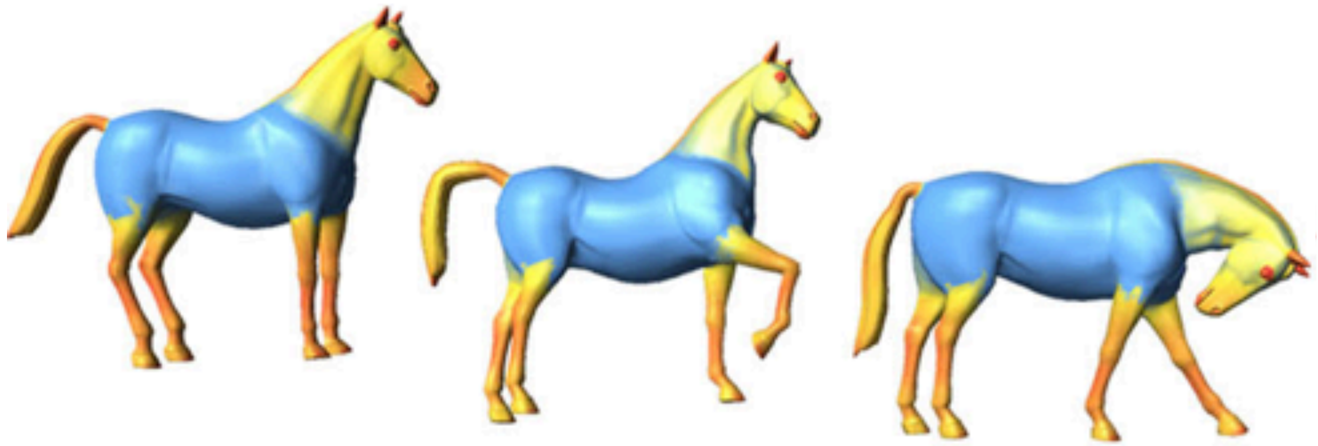
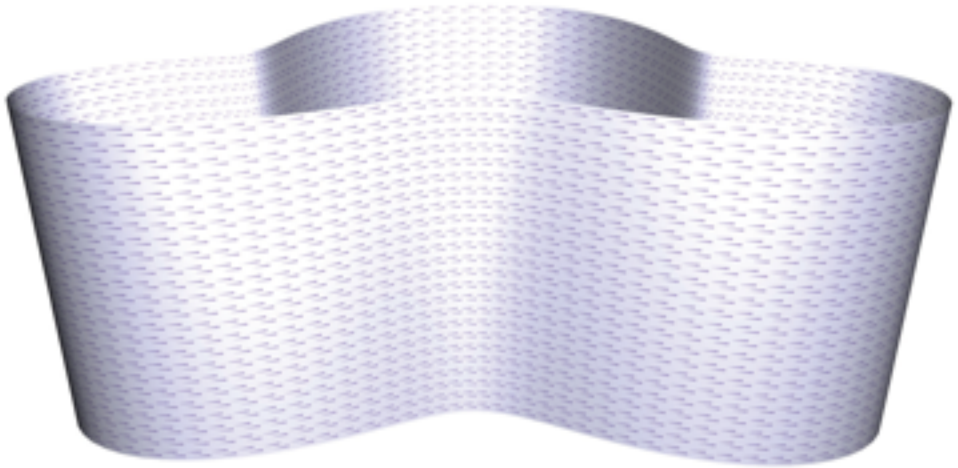


shape diameter functions (SDF)

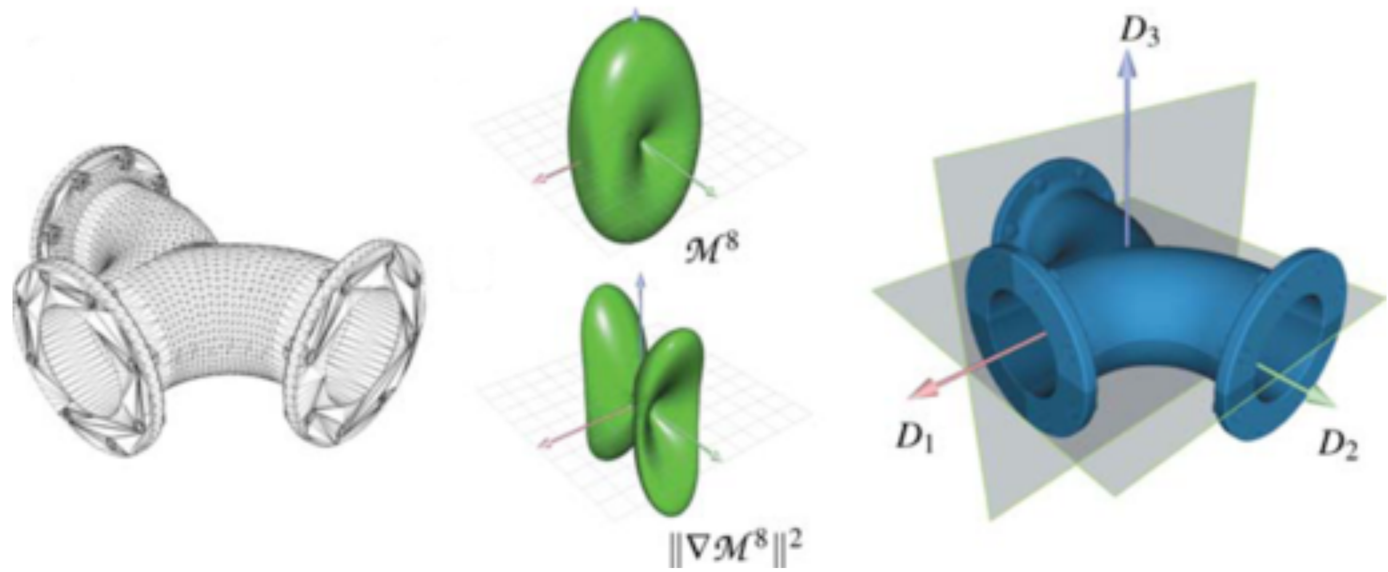
Shapira et al. 2008

Type of Features



Killing vector fields

Ben-Chen et al. 2010



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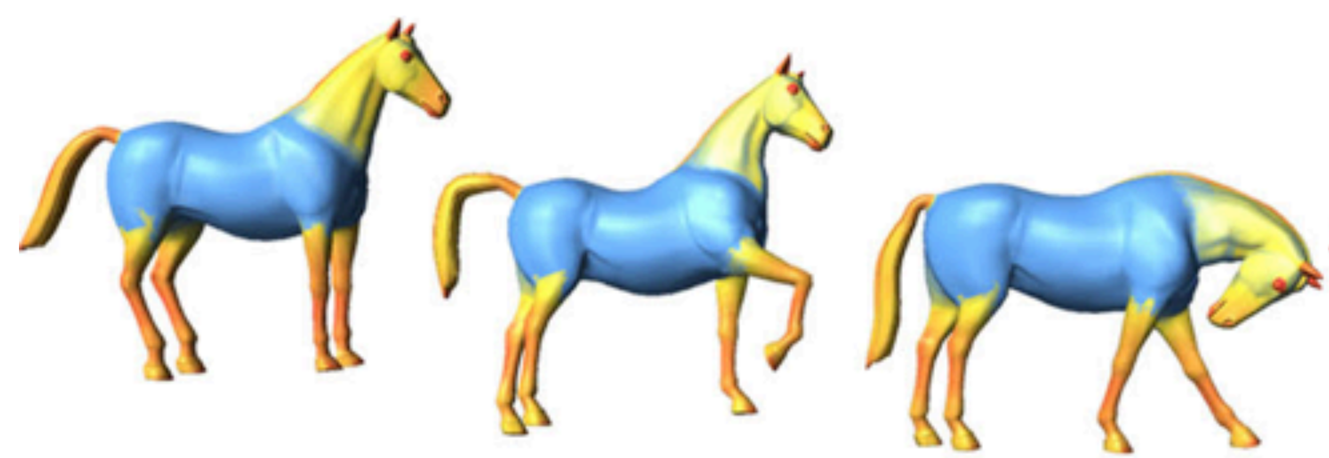

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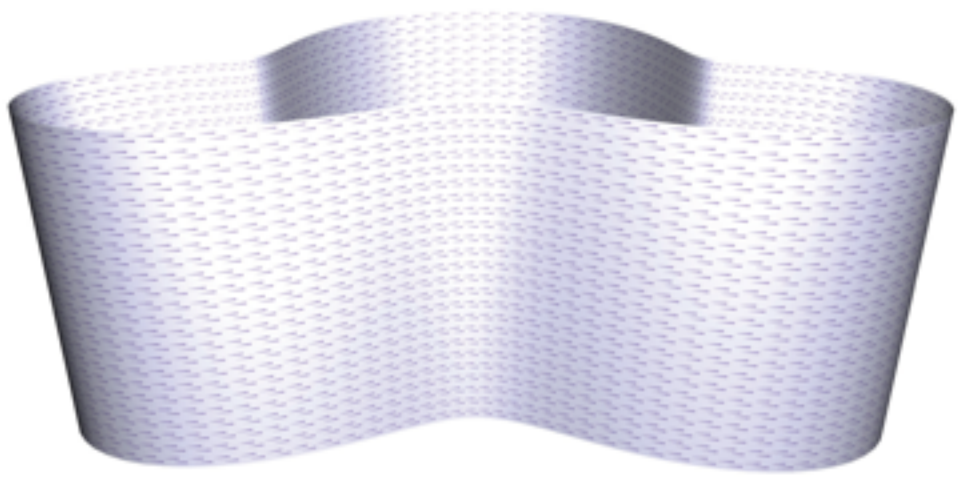

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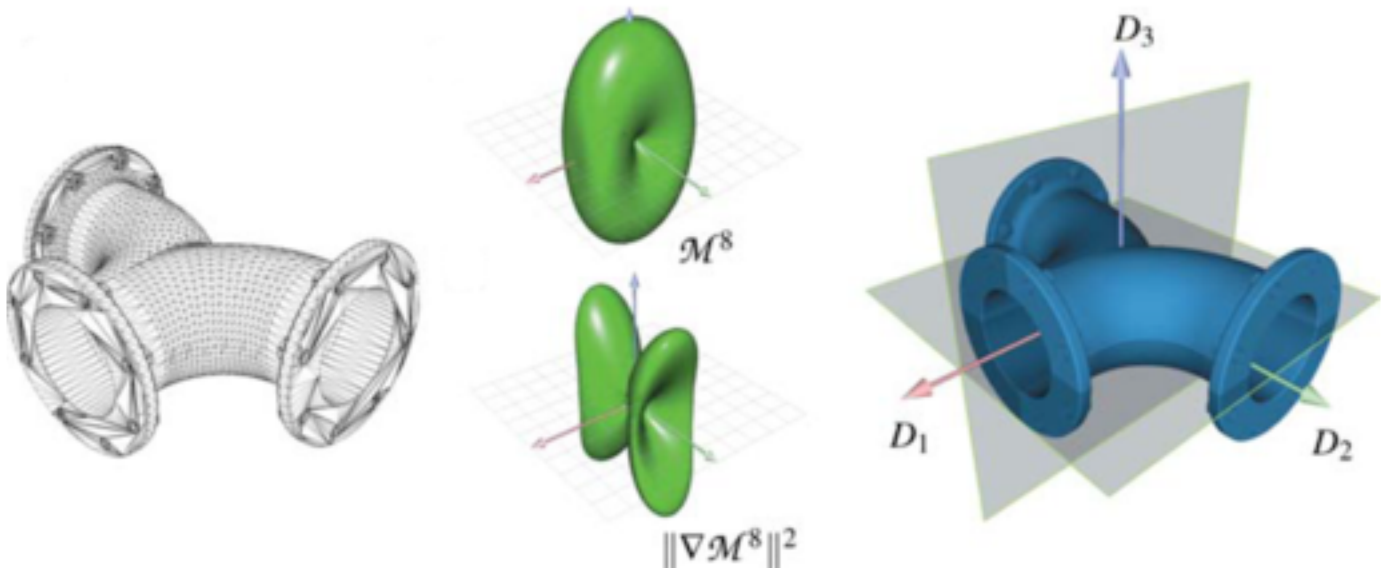

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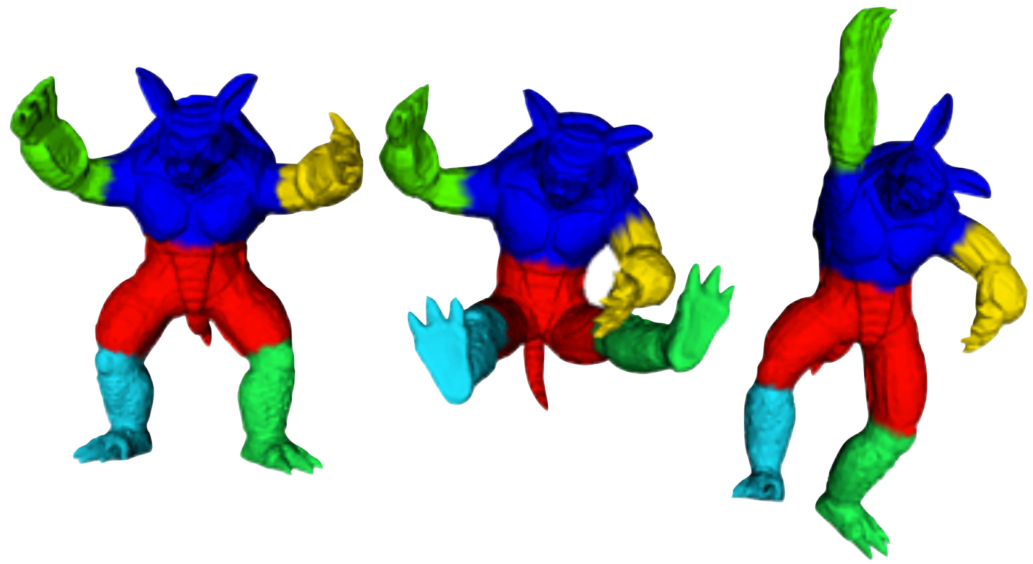
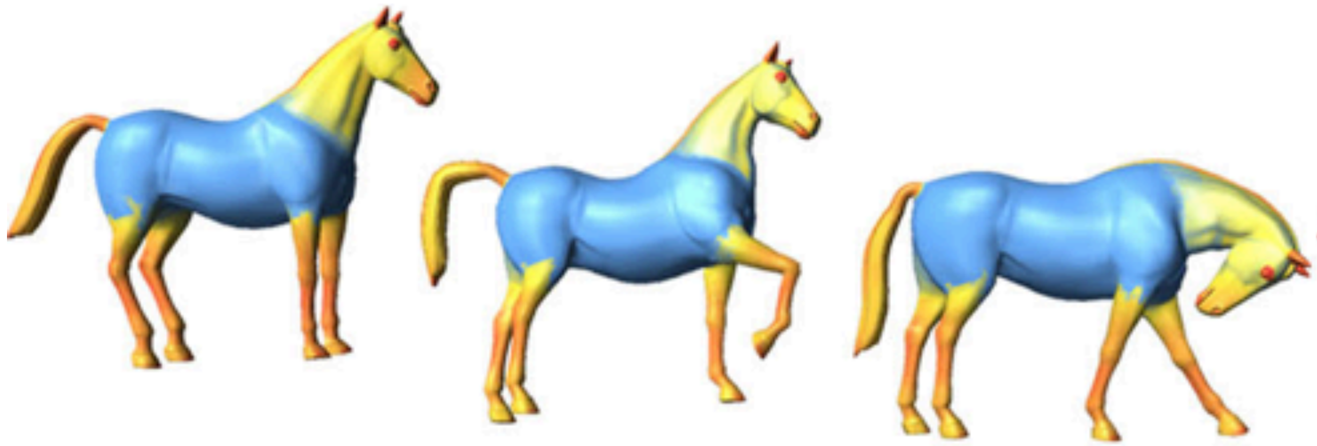


global point signatures (GPS)

Rustamov 2007



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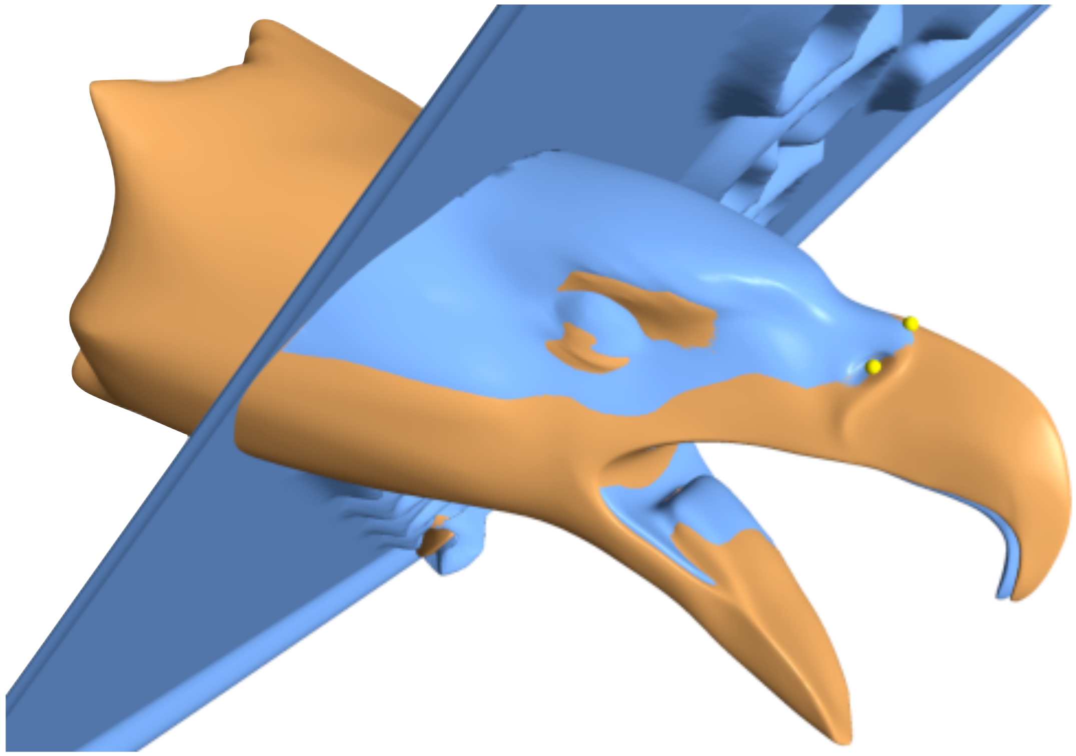
Shapira et al. 2008

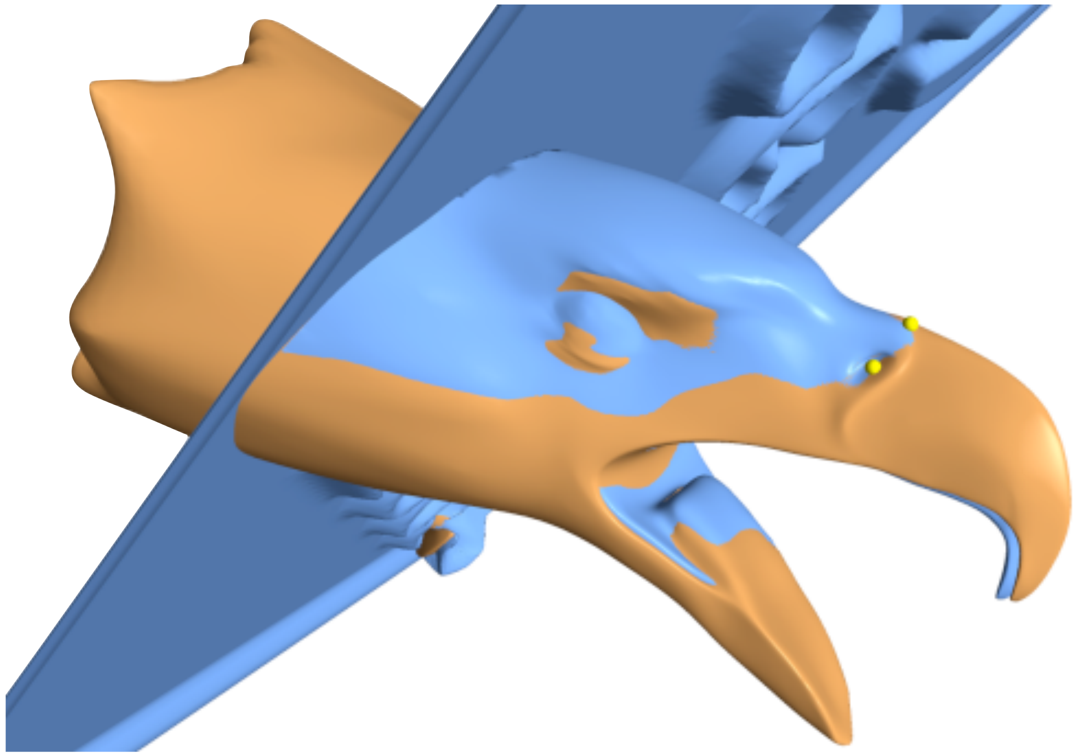
Geometric Hashing



[Gal et al. 2006]

Geometric Hashing

- ***Features:*** quadratic patch parameters



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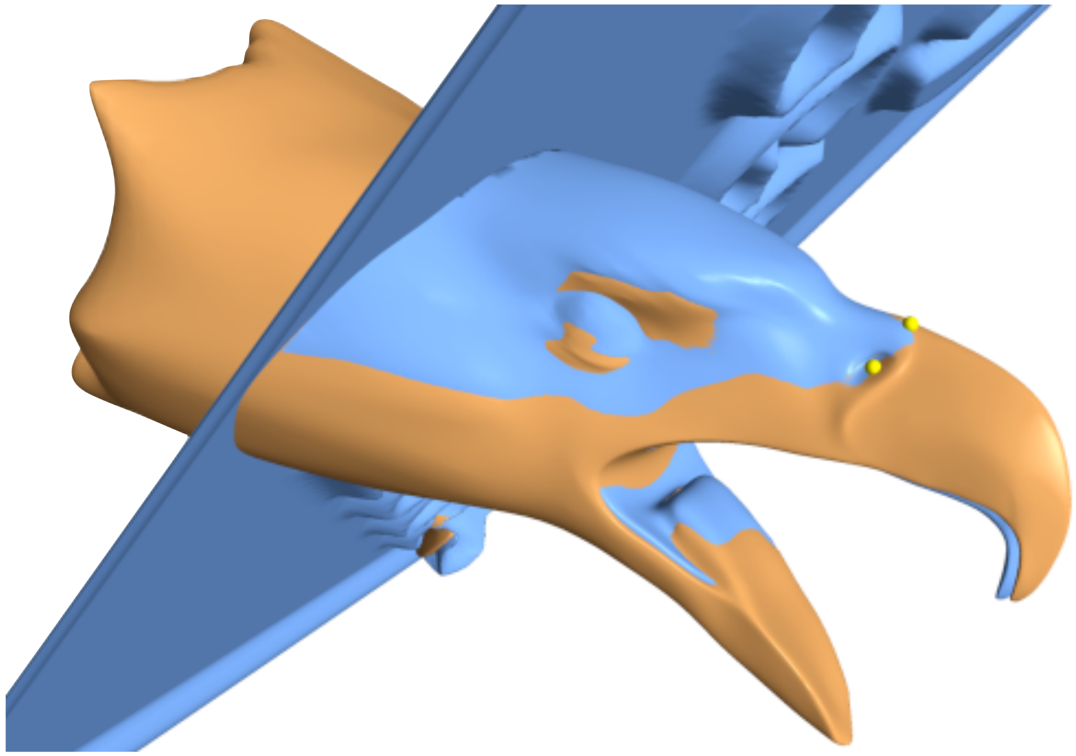

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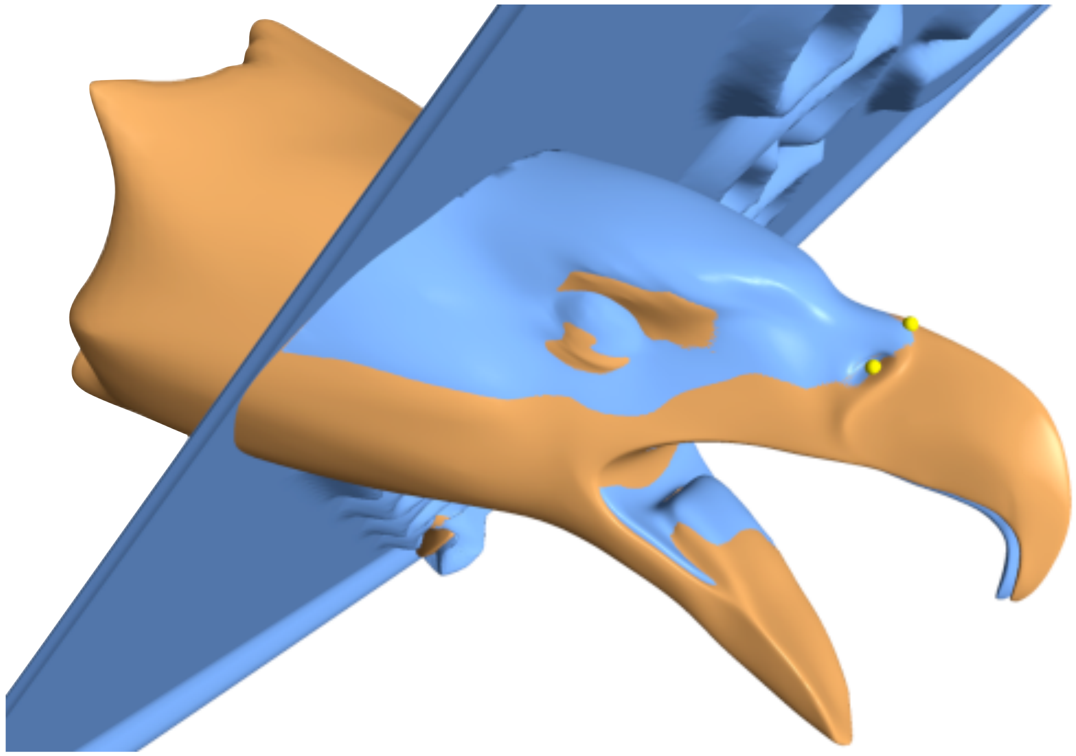

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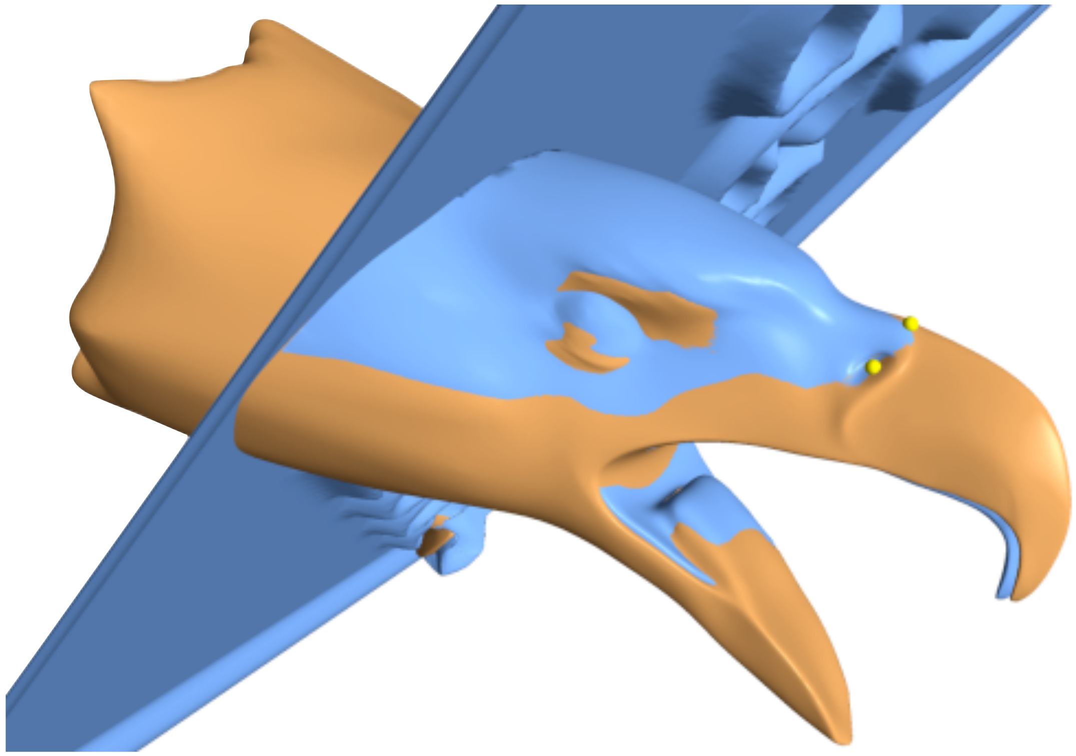
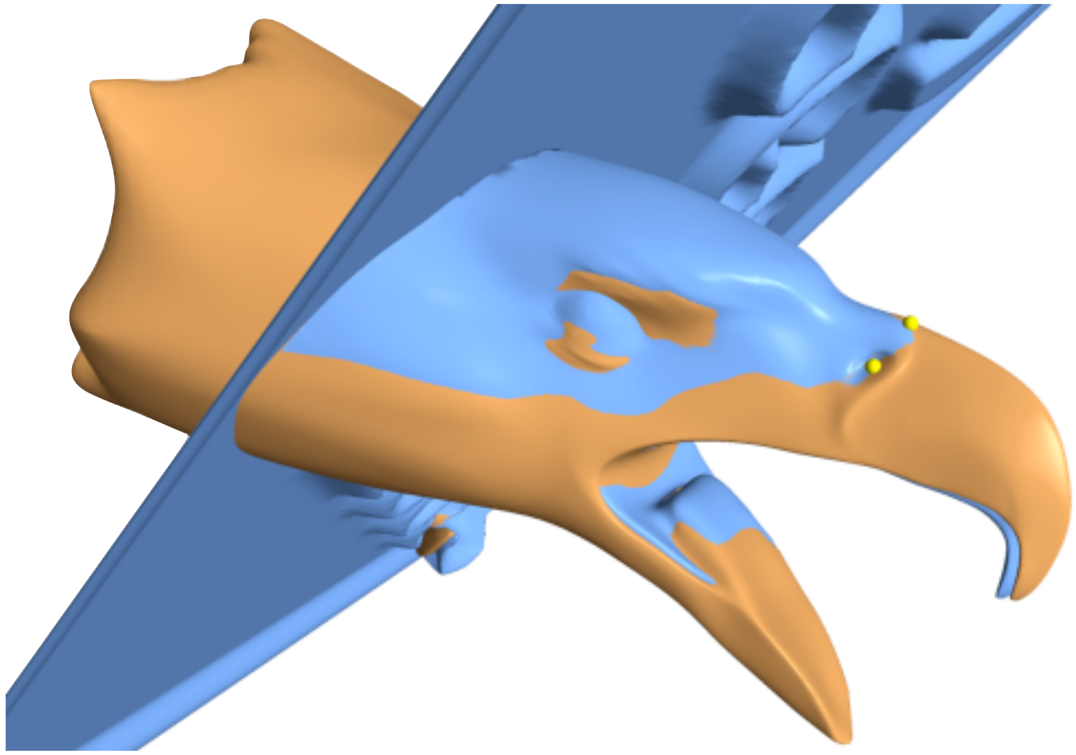


[Gal et al. 2006]

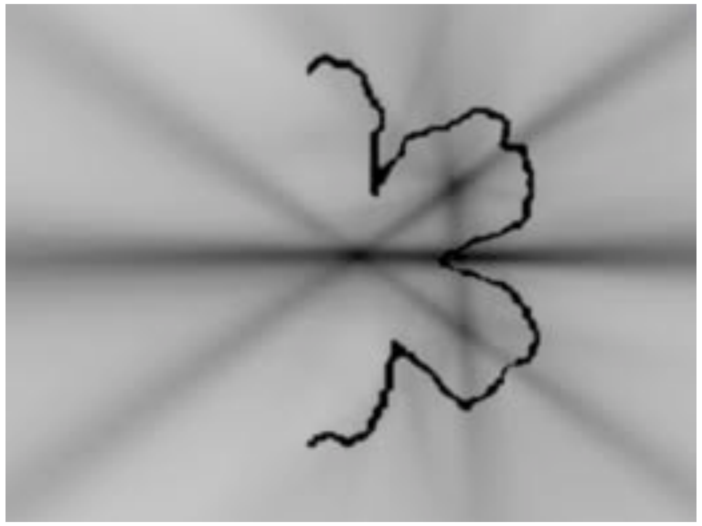
Geometric Hashing

- ***Features:*** quadratic patch parameters
- ***Aggregation:*** geometric hashing
- ***Extraction:*** pre-segmentation



[Gal et al. 2006]

Symmetry Transform



[Podolak et al. 2006]

Symmetry Transform

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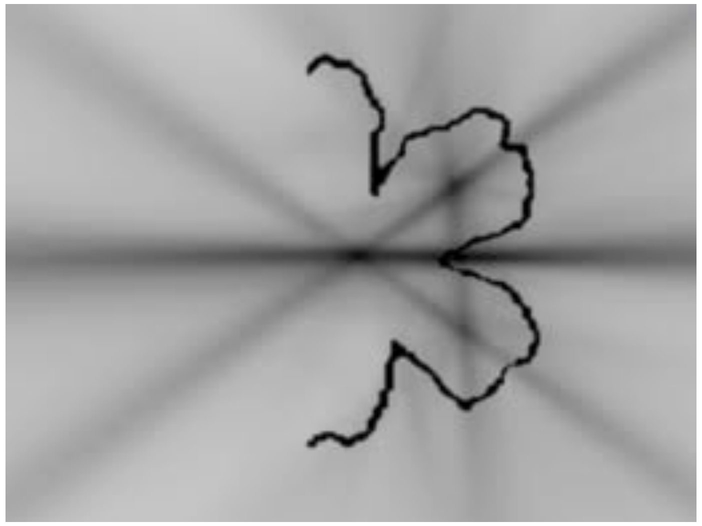

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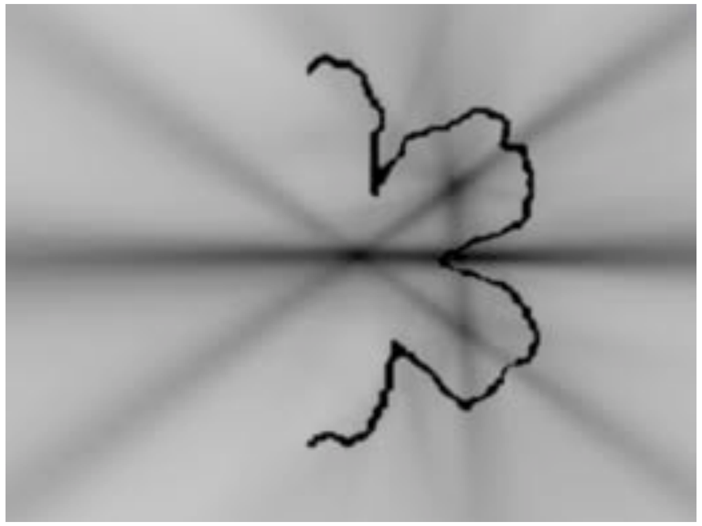
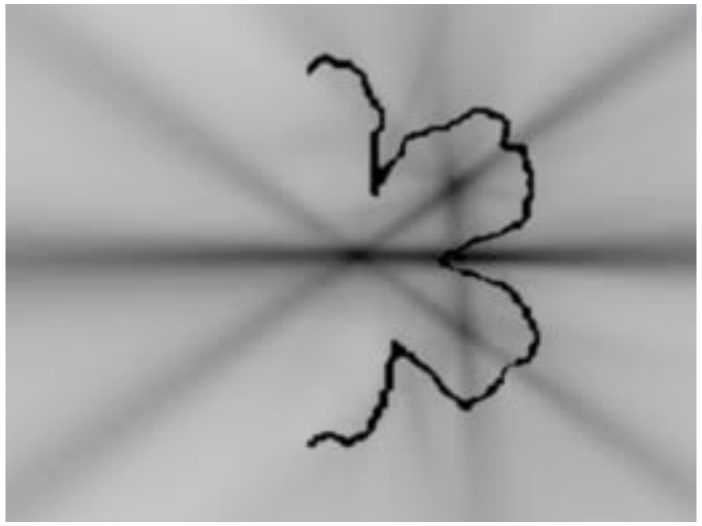


[Podolak et al. 2006]

Symmetry Transform

- ***Features:***

- ***Aggregation:*** FFT in transform domain



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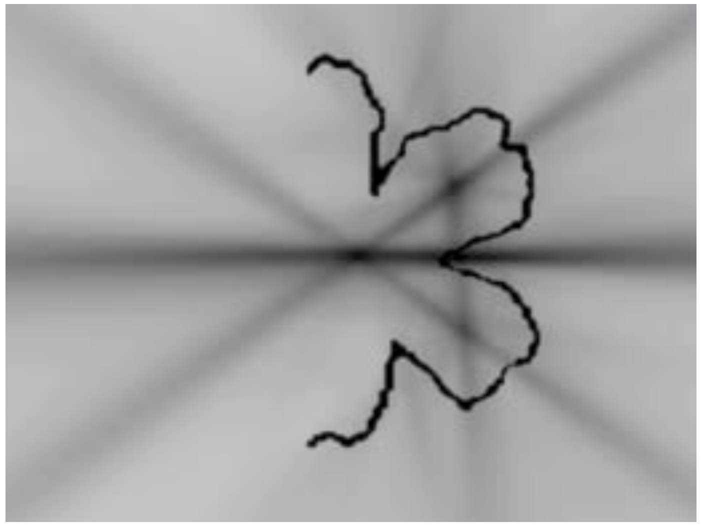
- ***Features:***

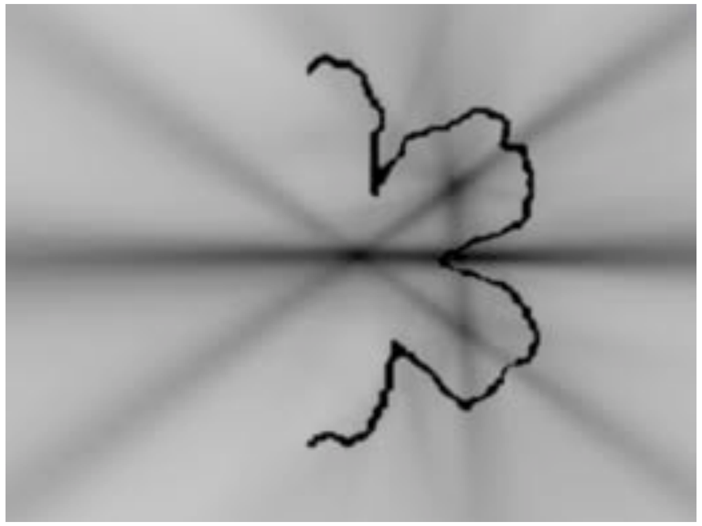
- ***Aggregation:*** FFT in transform domain



[Podolak et al. 2006]

Symmetry Transform

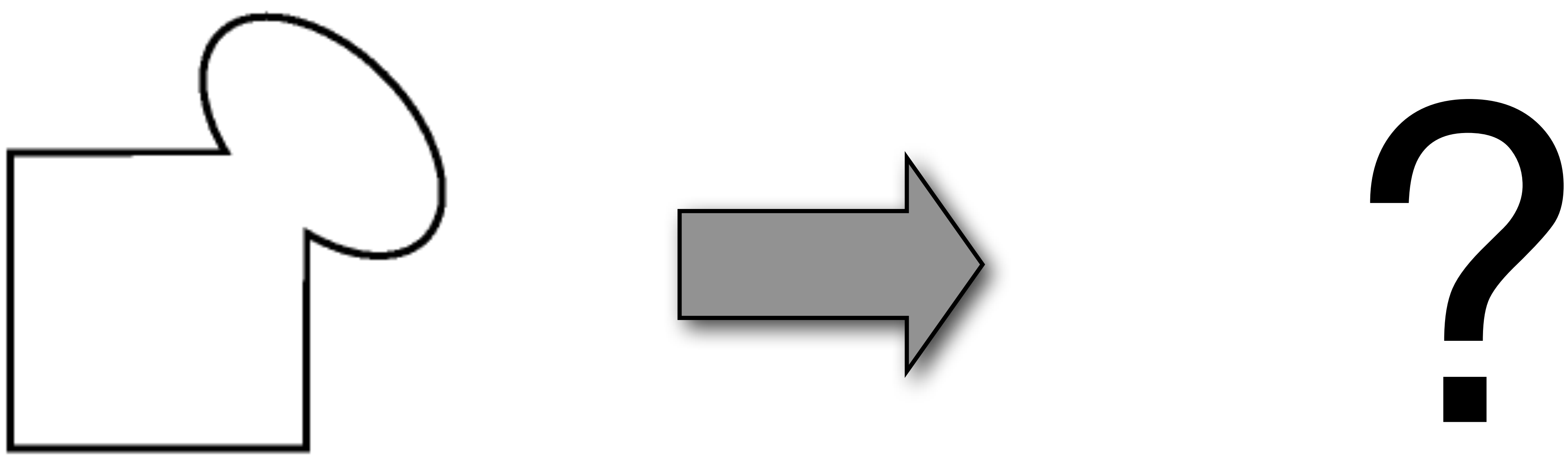
- ***Features:***
- ***Aggregation:*** FFT in transform domain
- ***Extraction:*** clustering, region growing



[Podolak et al. 2006]

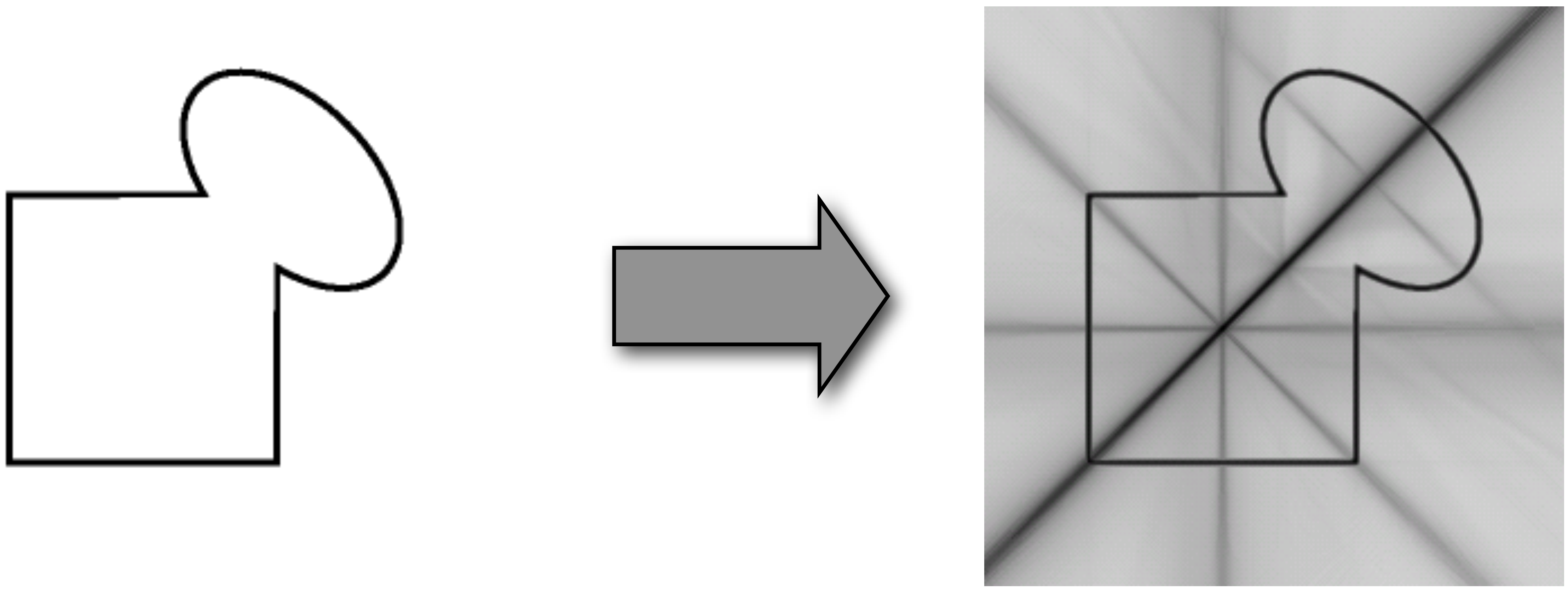
Goal

A computational representation that describes all planar symmetries of a shape



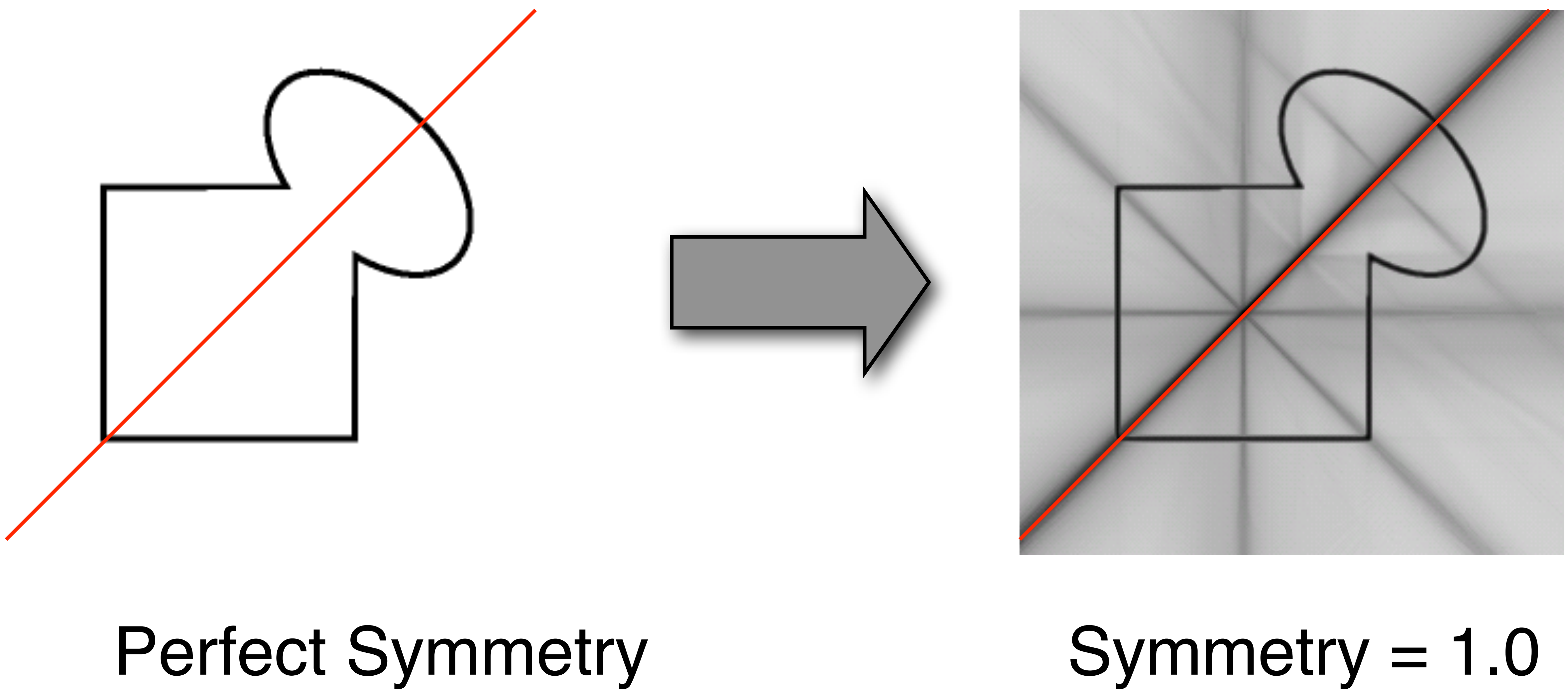
Symmetry Transform

A computational representation that describes all planar symmetries of a shape



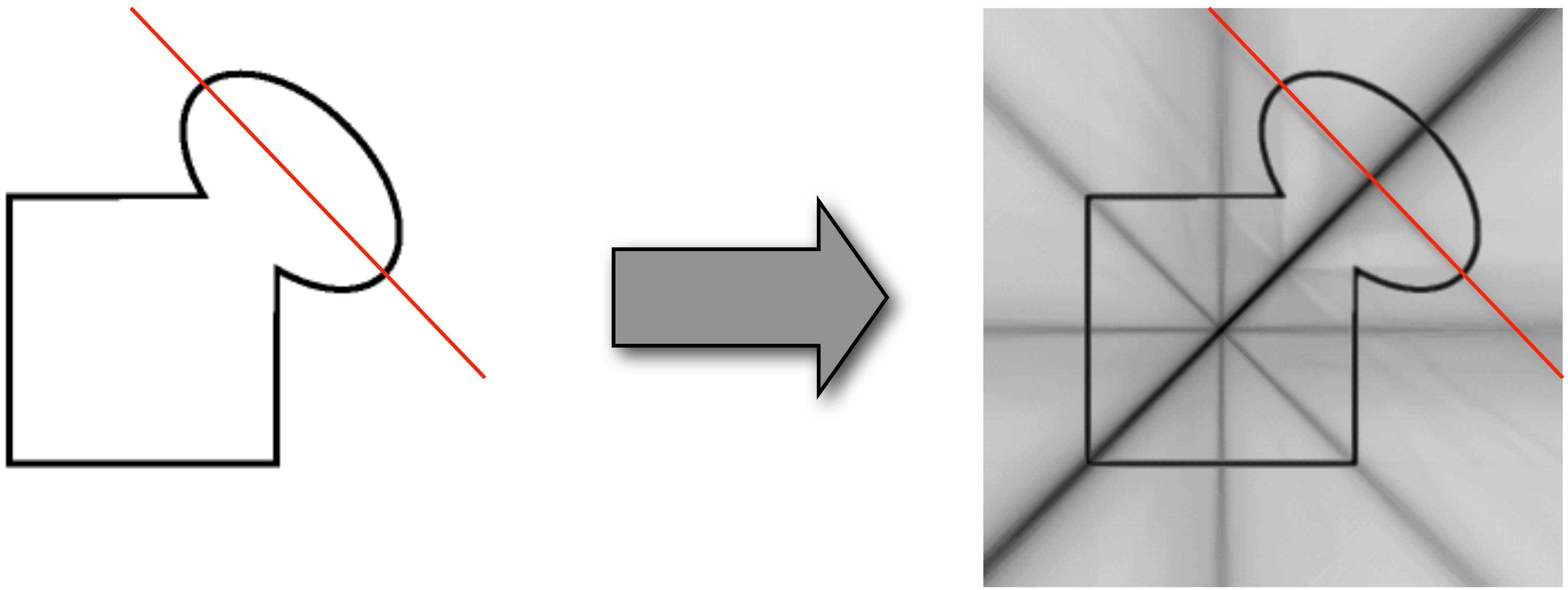
Symmetry Transform

A computational representation that describes all planar symmetries of a shape



Symmetry Transform

A computational representation that describes all planar symmetries of a shape

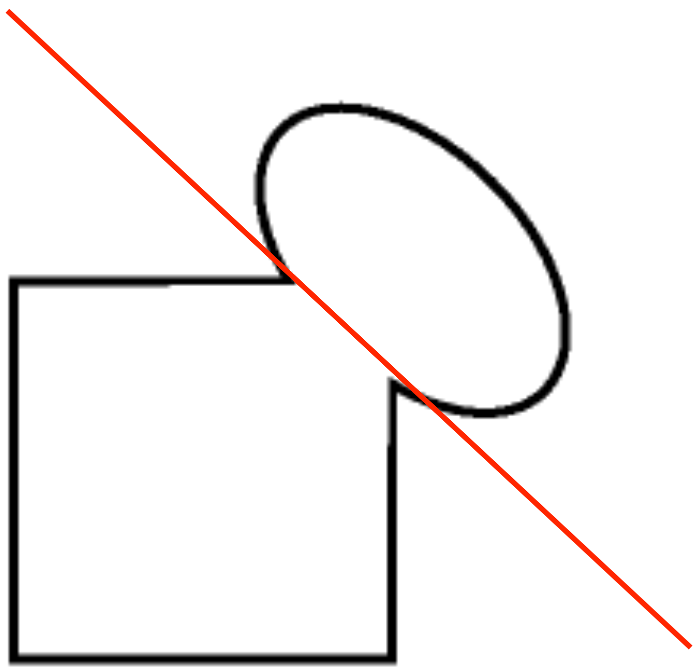


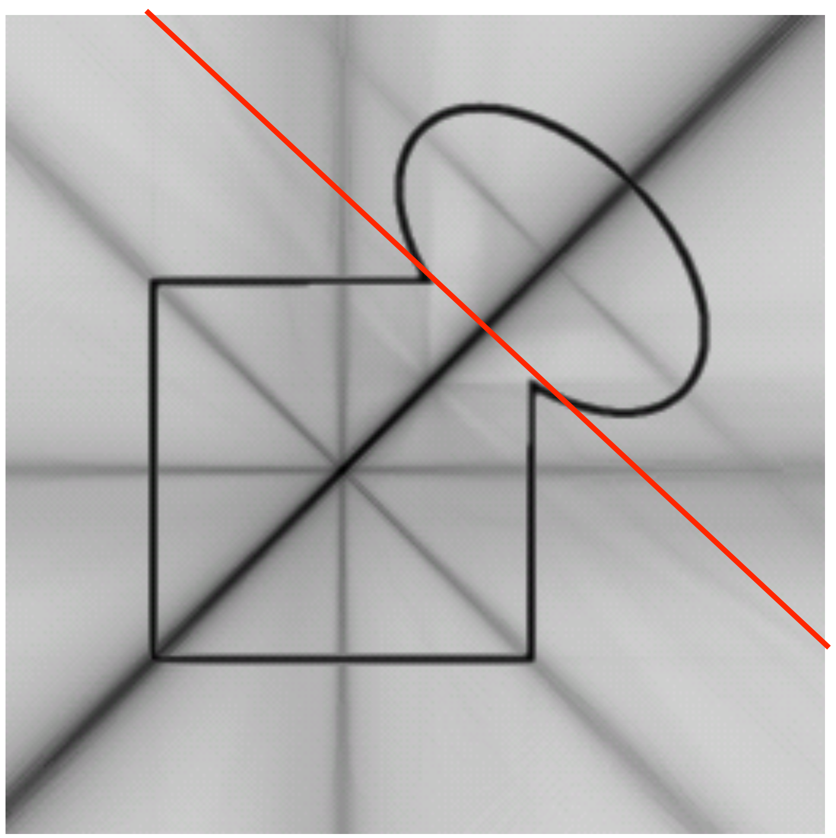
Local Symmetry

Symmetry = 0.3

Symmetry Transform

A computational representation that describes all planar symmetries of a shape





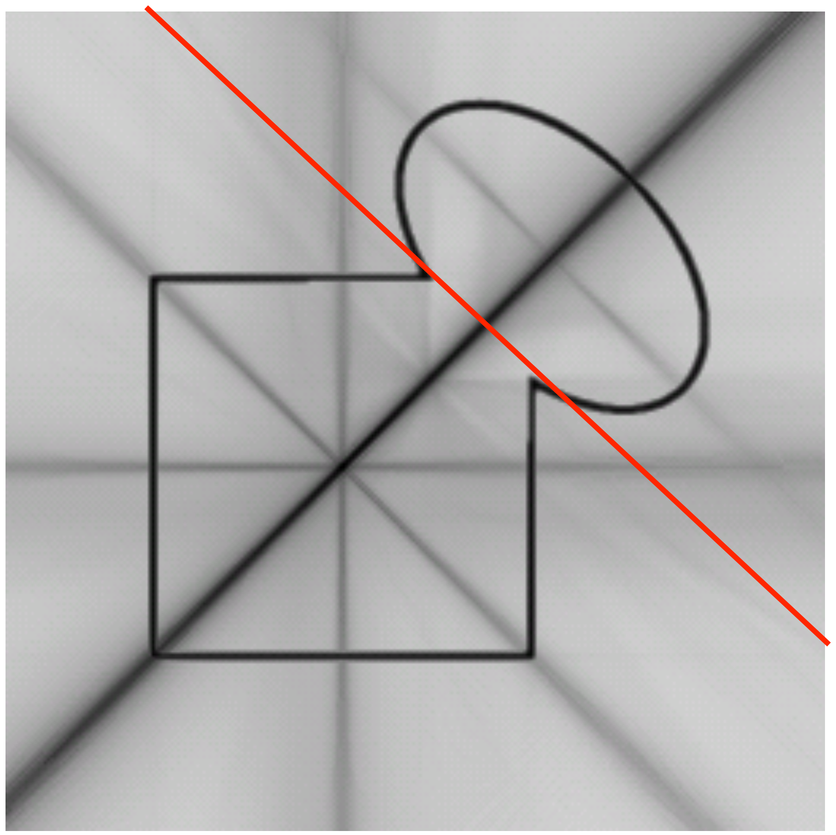
Partial Symmetry

Symmetry = 0.2

Symmetry Transform

A computational representation that describes all planar symmetries of a shape

$$d(M,T) = \left\| \frac{M - T(M)}{2} \right\|$$



Symmetry = 0.2

Transform Domain Analysis

[Mitra et al. 2006]

Transform Domain Analysis

- ***Features:*** curvatures

[Mitra et al. 2006]

Transform Domain Analysis

- ***Features:*** curvatures

[Mitra et al. 2006]

Transform Domain Analysis

- ***Features:*** curvatures
- ***Aggregation:*** transform domain analysis

[Mitra et al. 2006]

Transform Domain Analysis

- ***Features:*** curvatures

- ***Aggregation:*** transform domain analysis

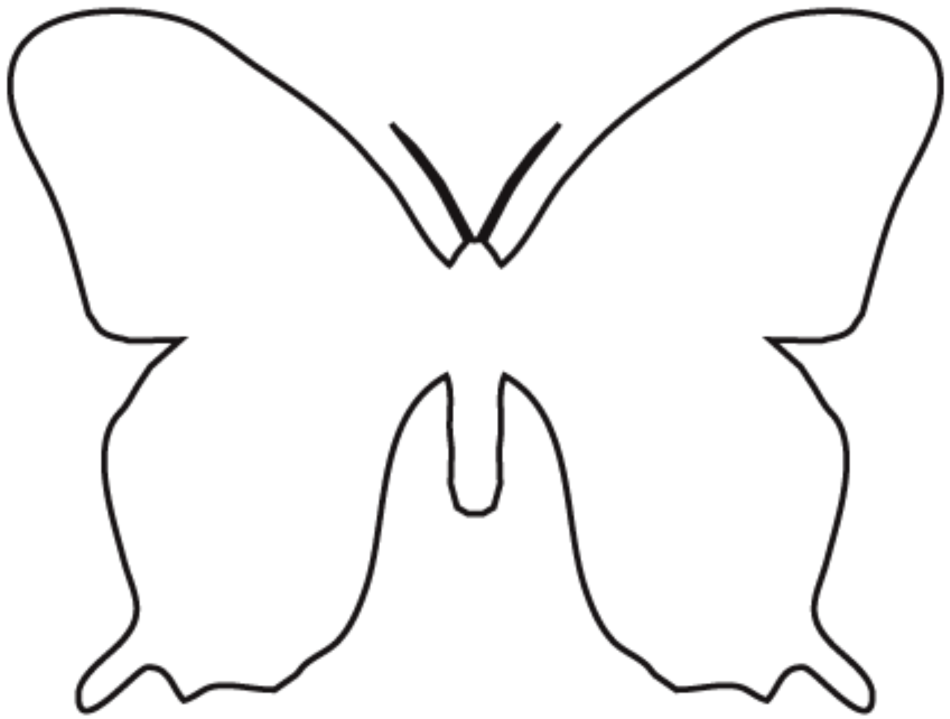
[Mitra et al. 2006]

Transform Domain Analysis

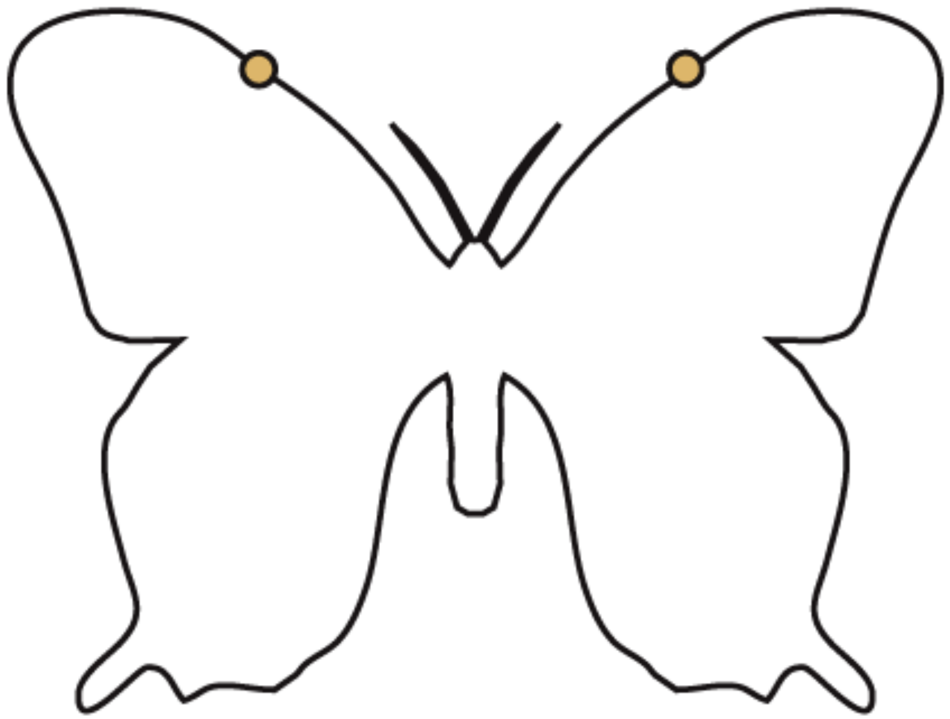
- ***Features:*** curvatures
- ***Aggregation:*** transform domain analysis
- ***Extraction:*** region growing

[Mitra et al. 2006]

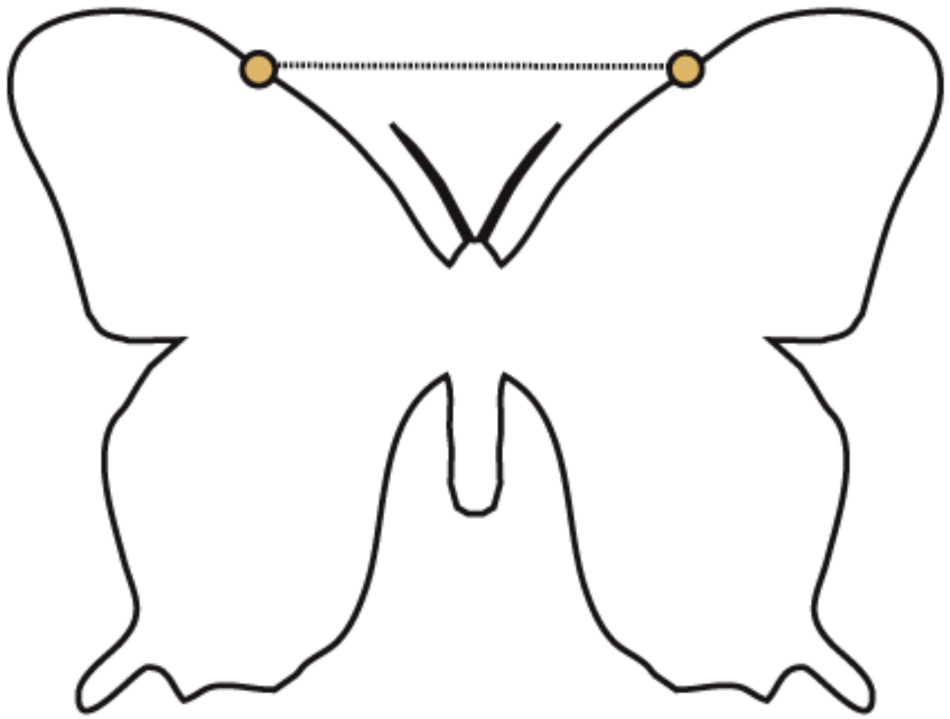
Reflective Symmetry



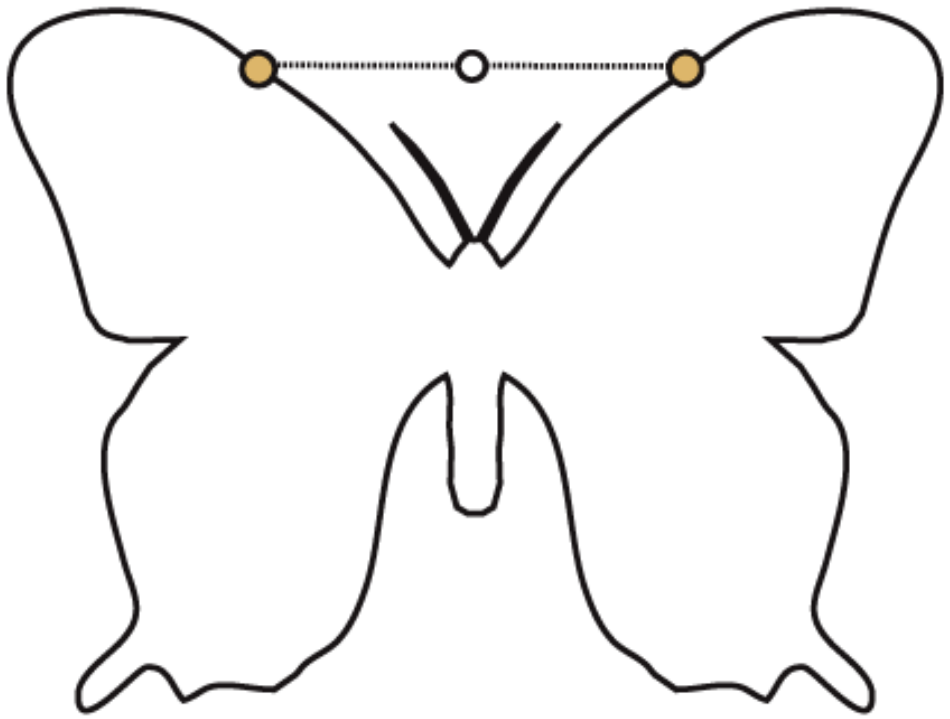
Reflective Symmetry



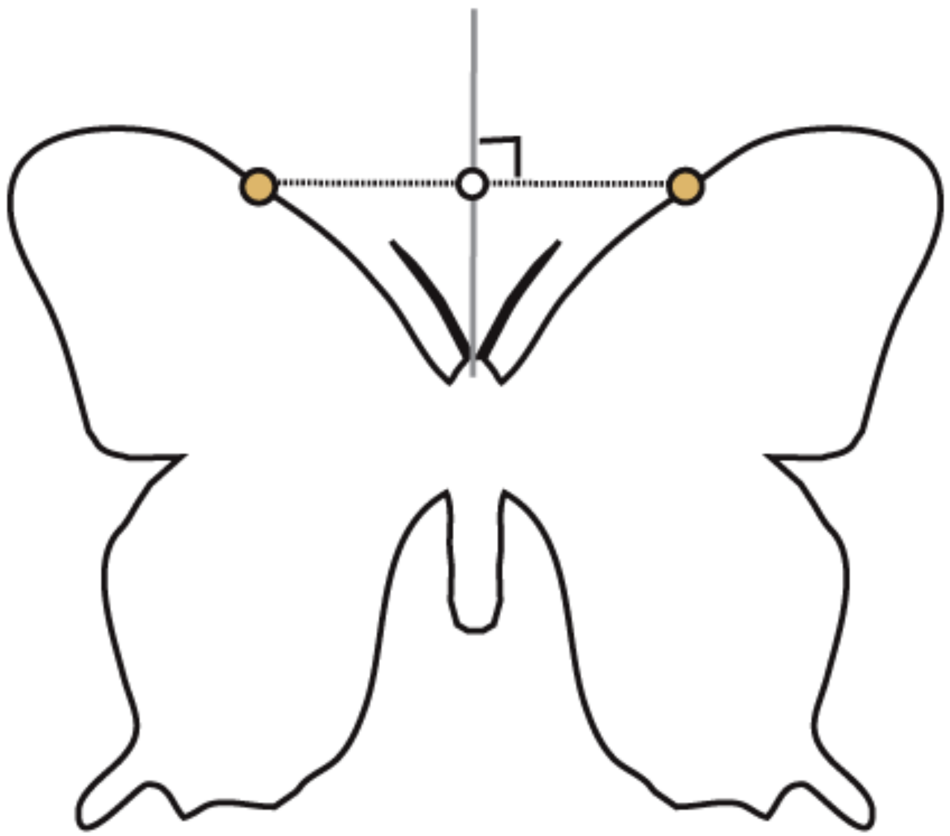
Reflective Symmetry: A Pair Votes



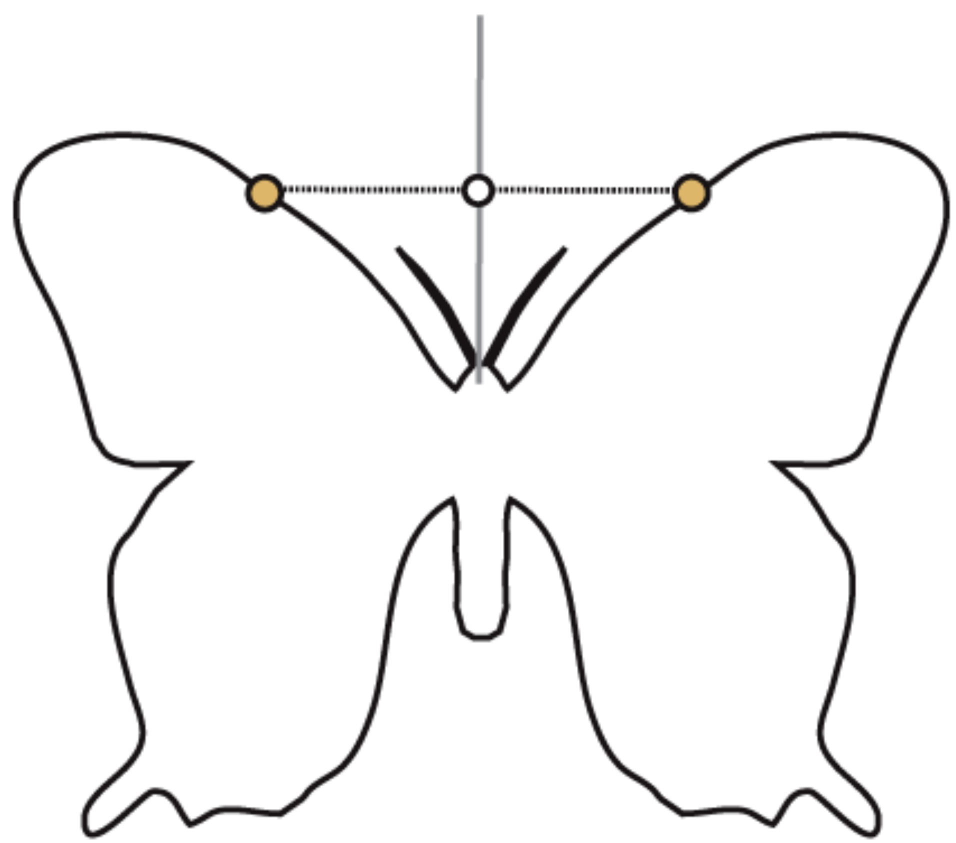
Reflective Symmetry: A Pair Votes

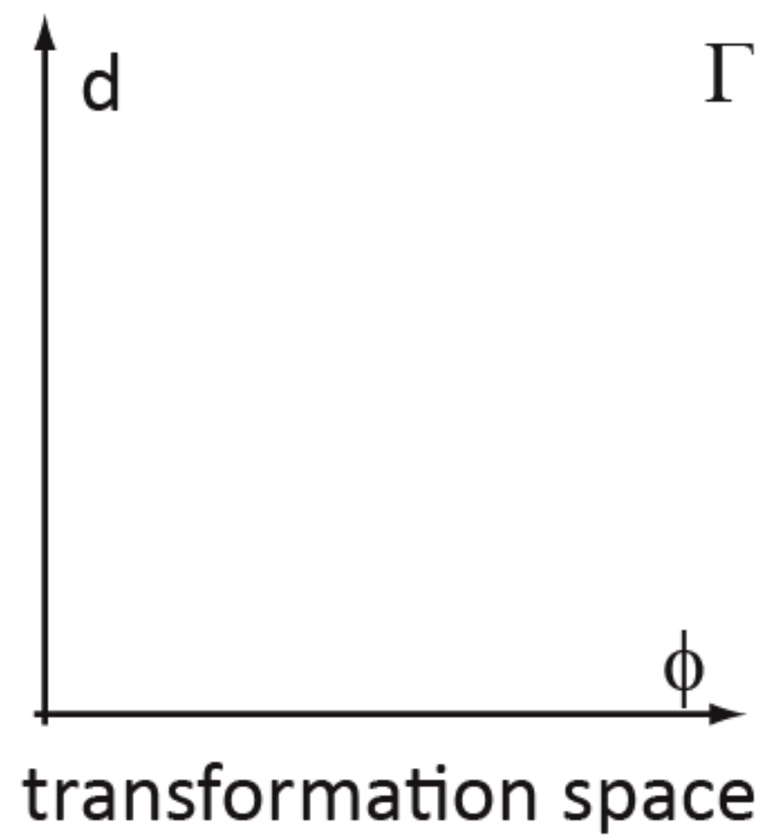


Reflective Symmetry: A Pair Votes



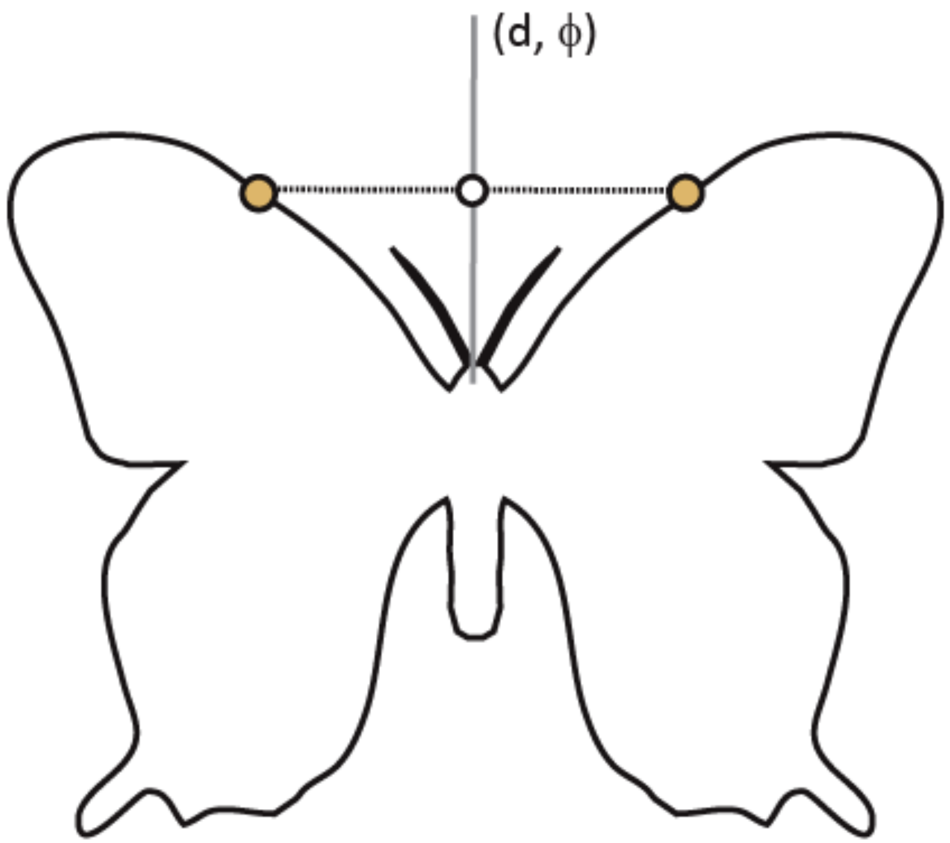
Reflective Symmetry: A Pair Votes

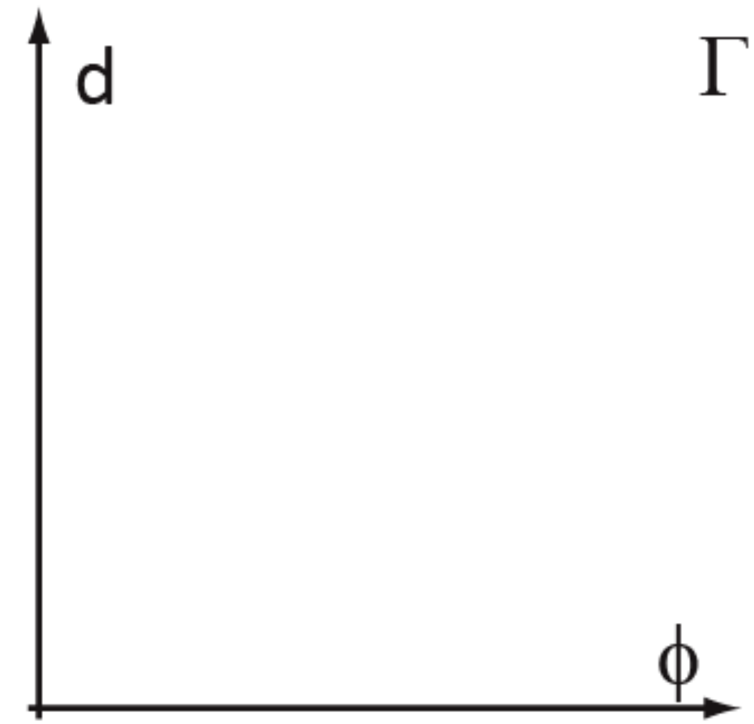




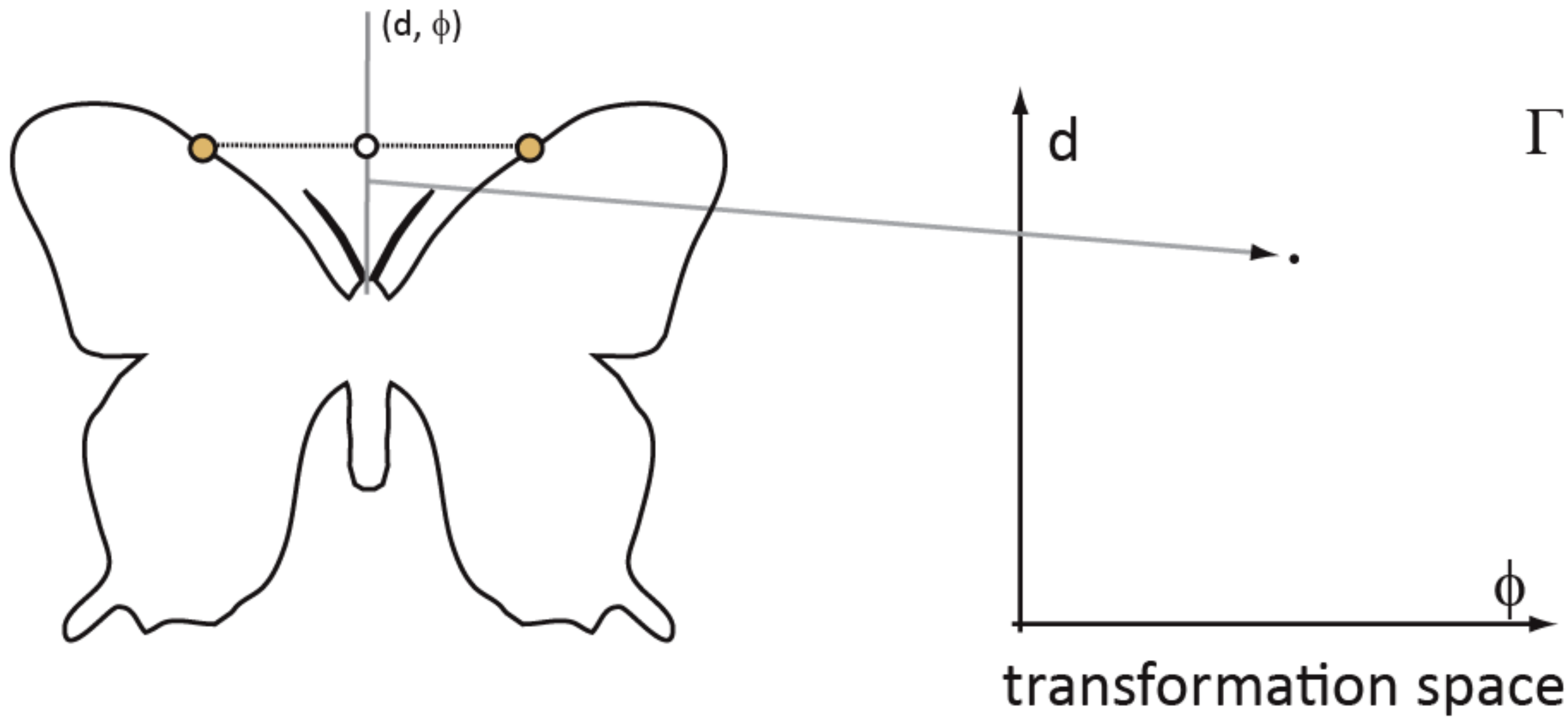
Thursday, 16 May 13

Reflective Symmetry: A Pair Votes

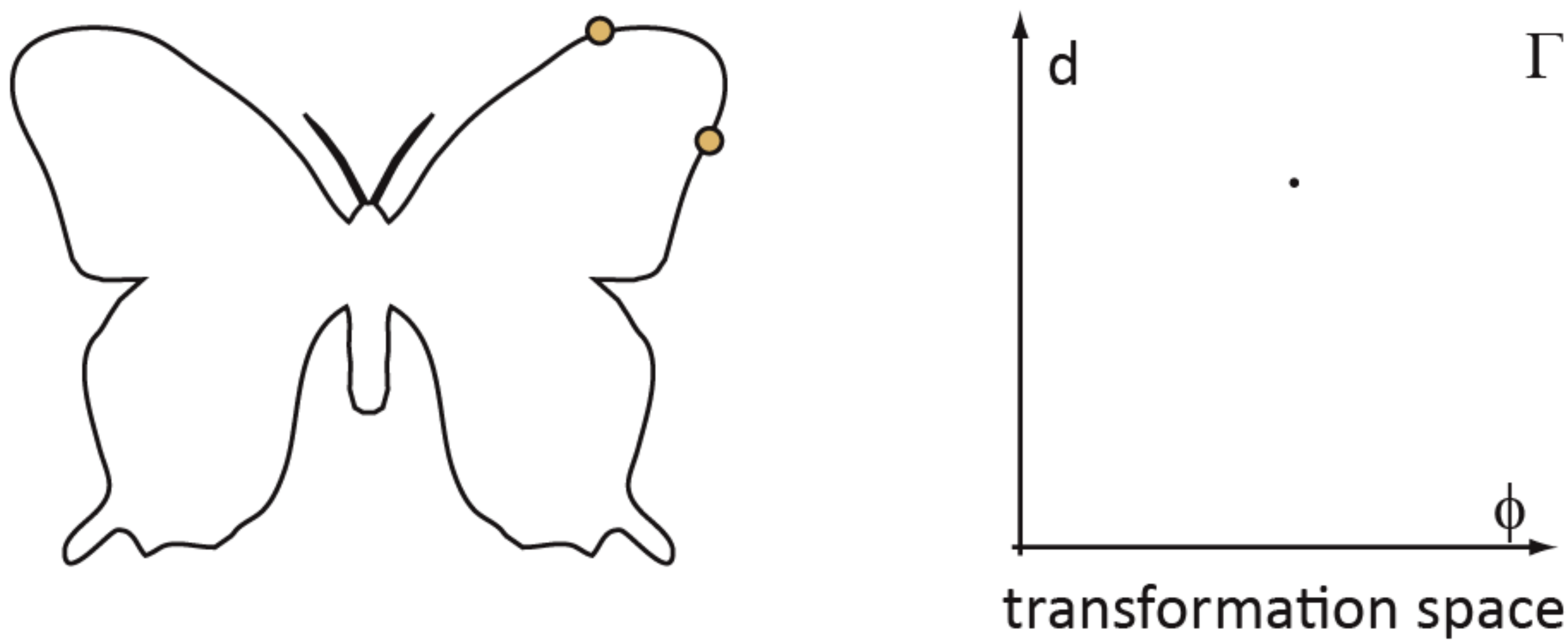




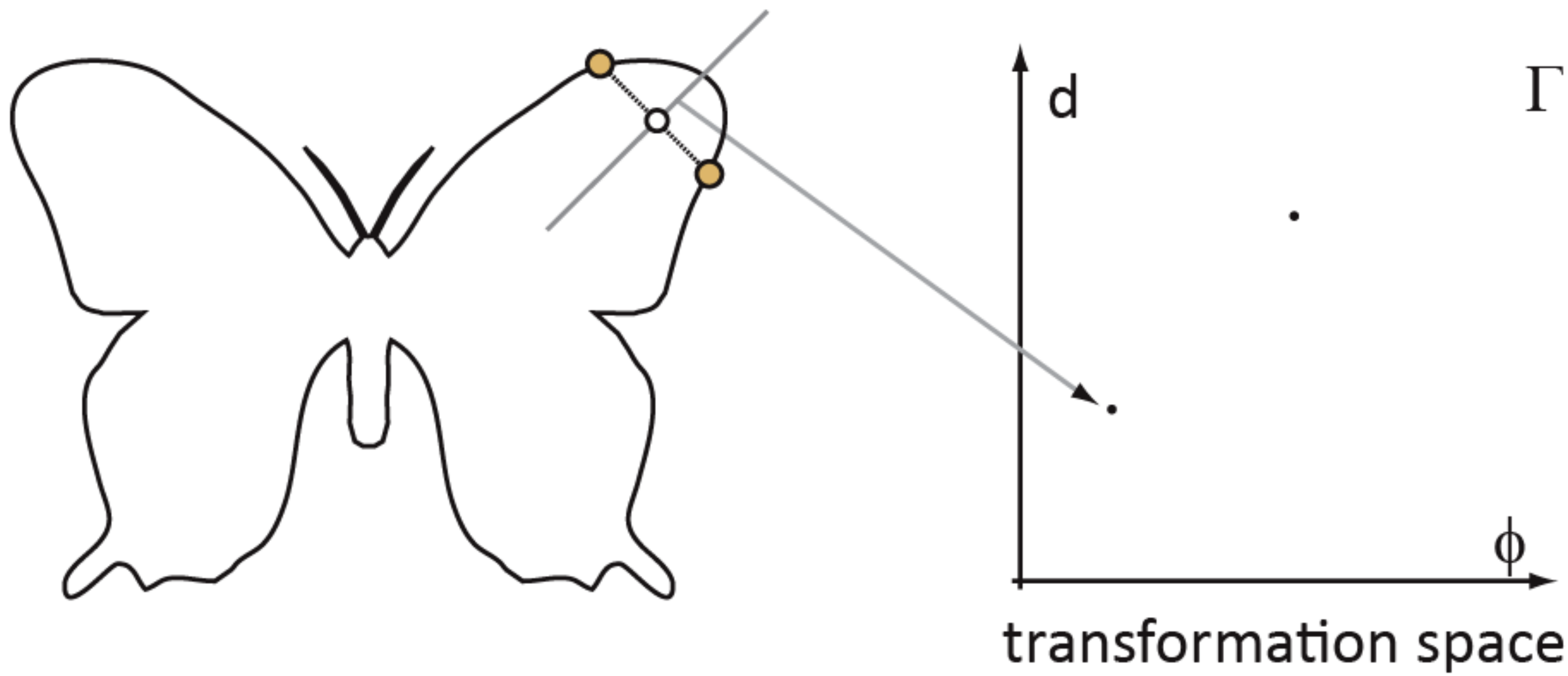
Reflective Symmetry: A Pair Votes



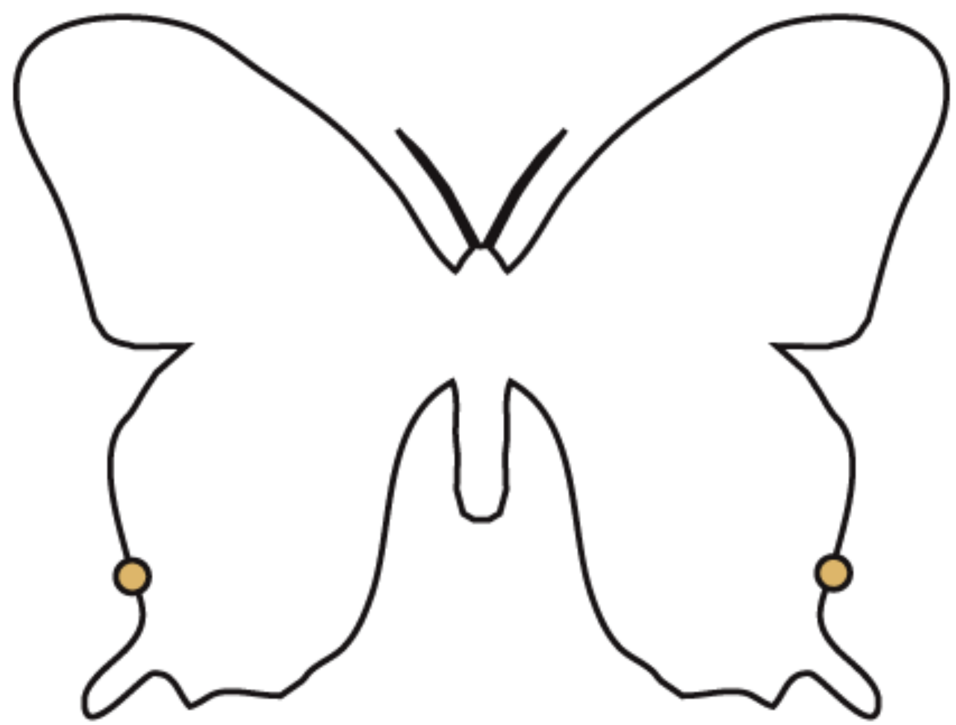
Reflective Symmetry: Voting Continues

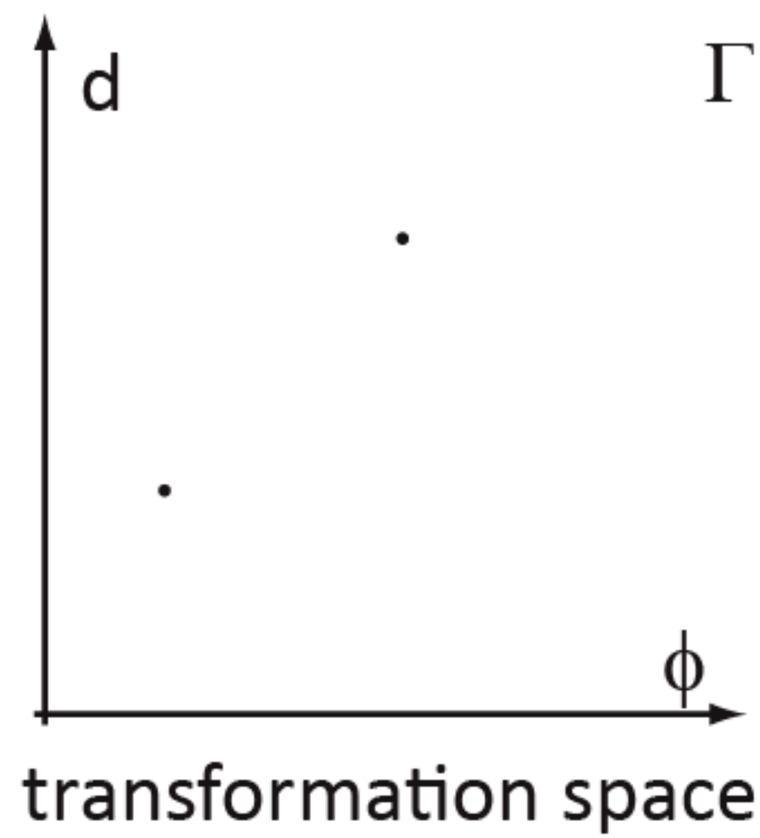


Reflective Symmetry: Voting Continues

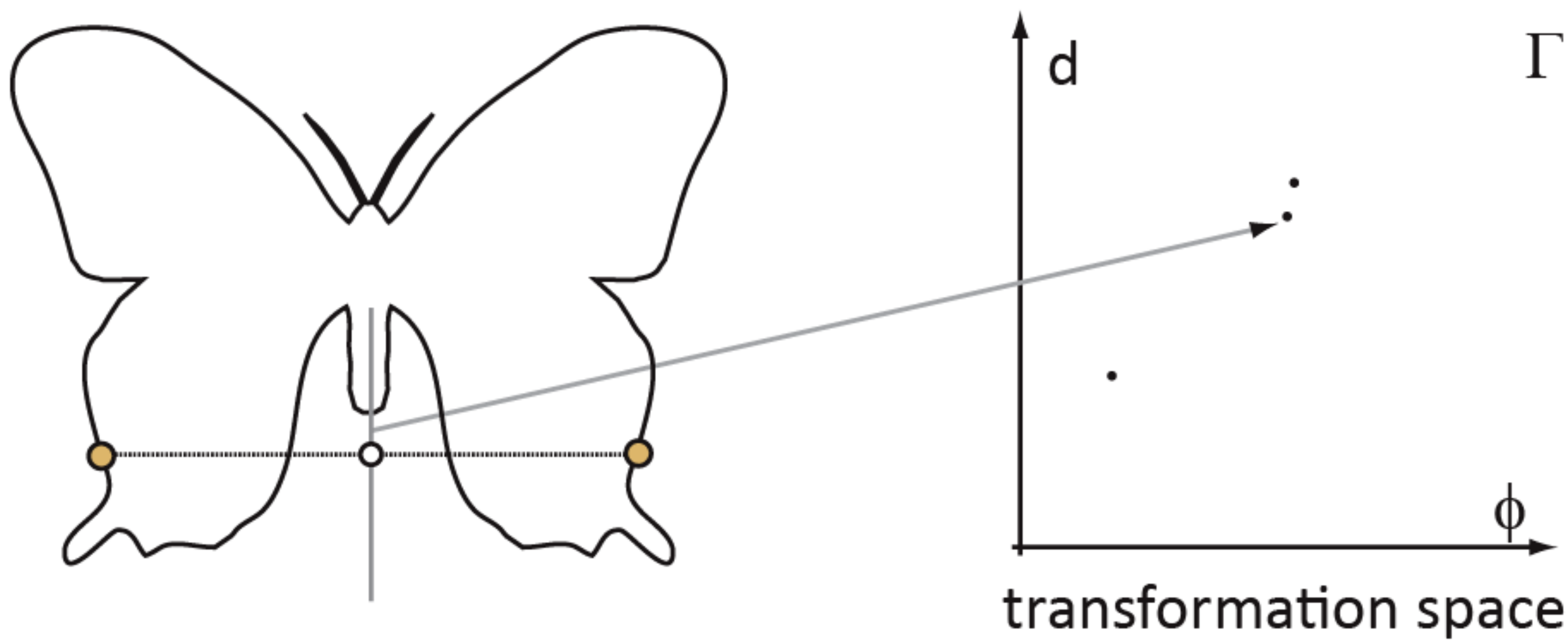


Reflective Symmetry: Voting Continues

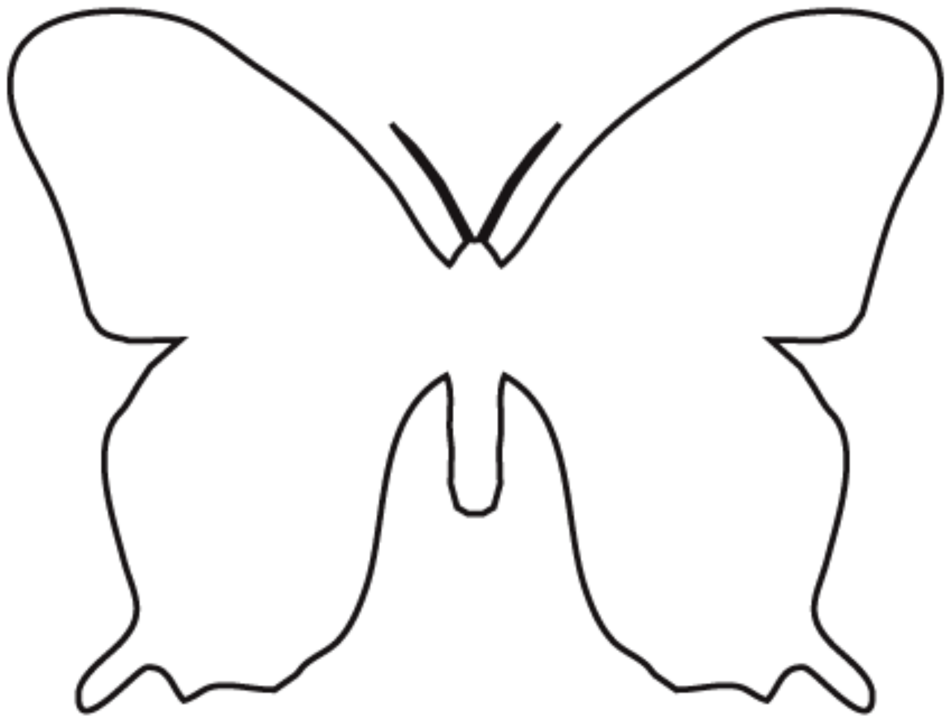


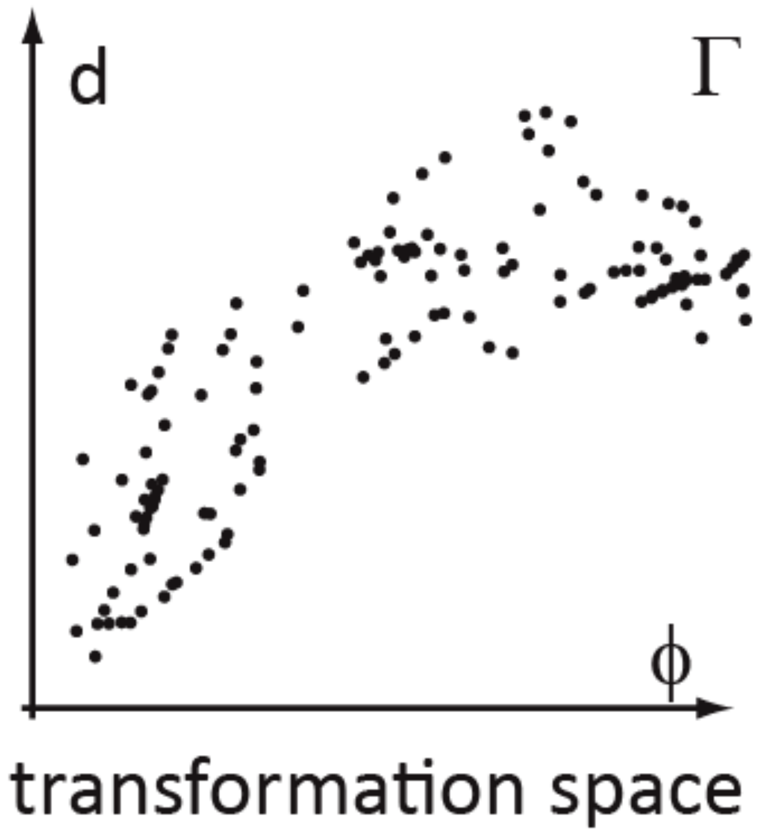


Reflective Symmetry: Voting Continues

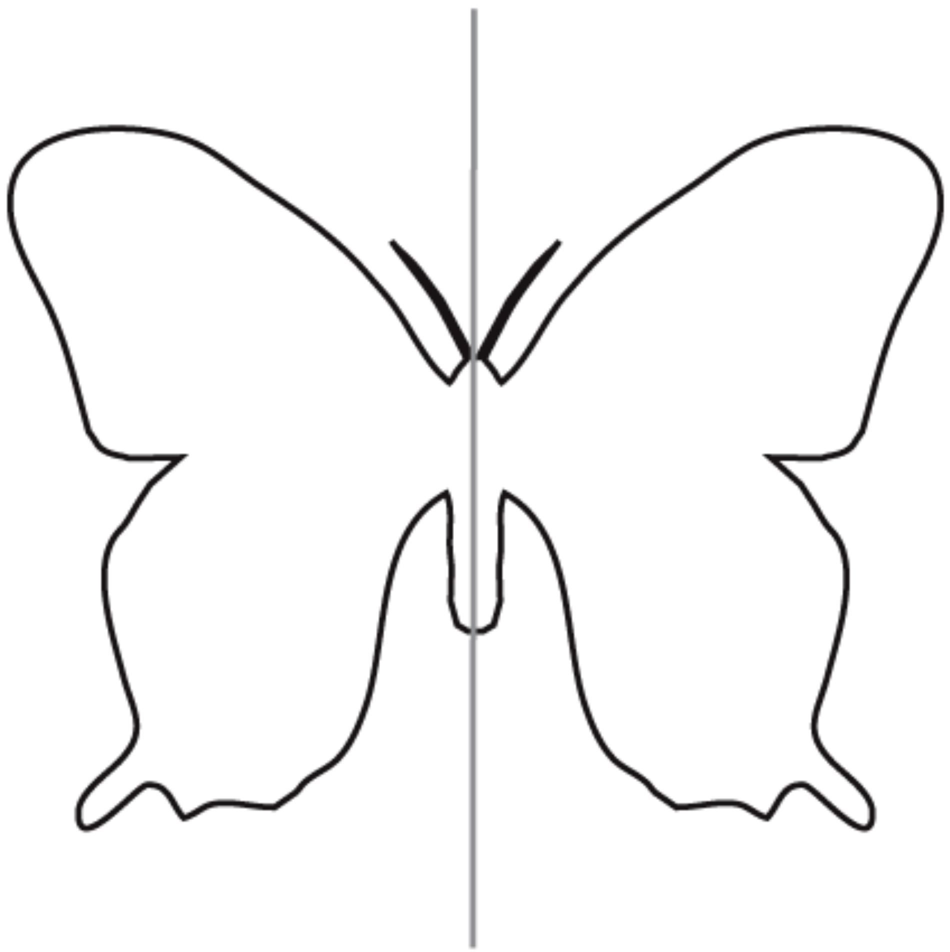


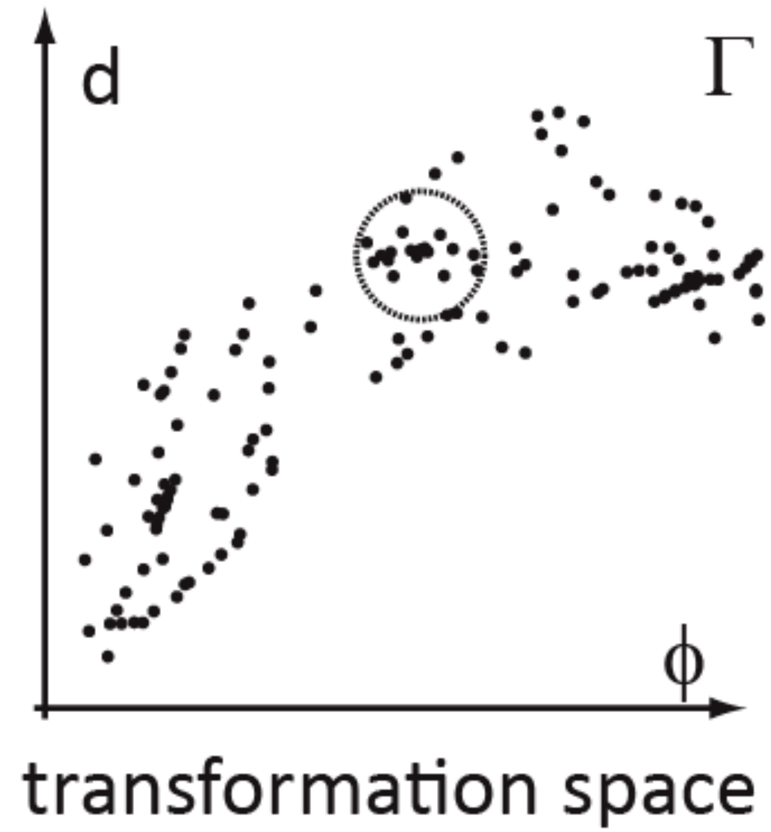
Reflective Symmetry: Largest Cluster



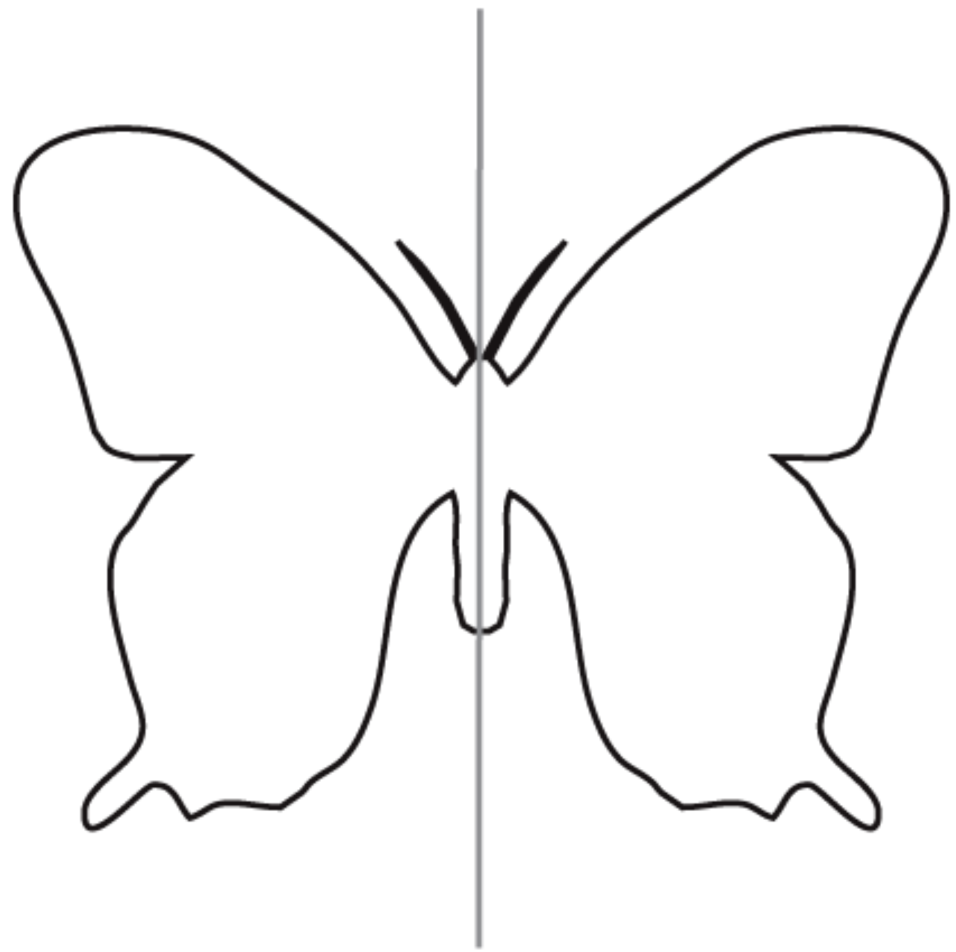


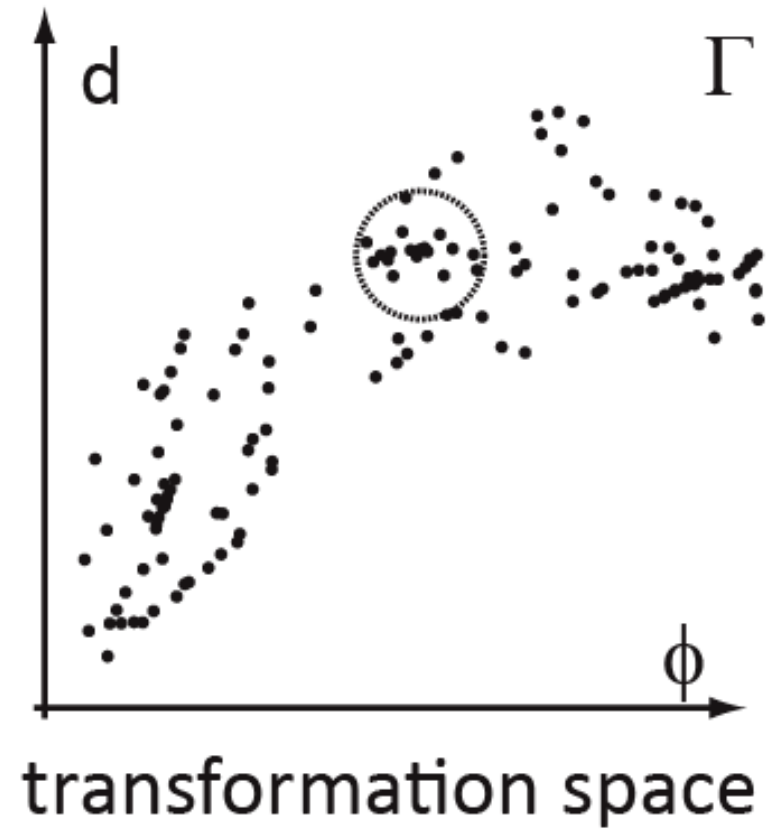
Reflective Symmetry: Largest Cluster





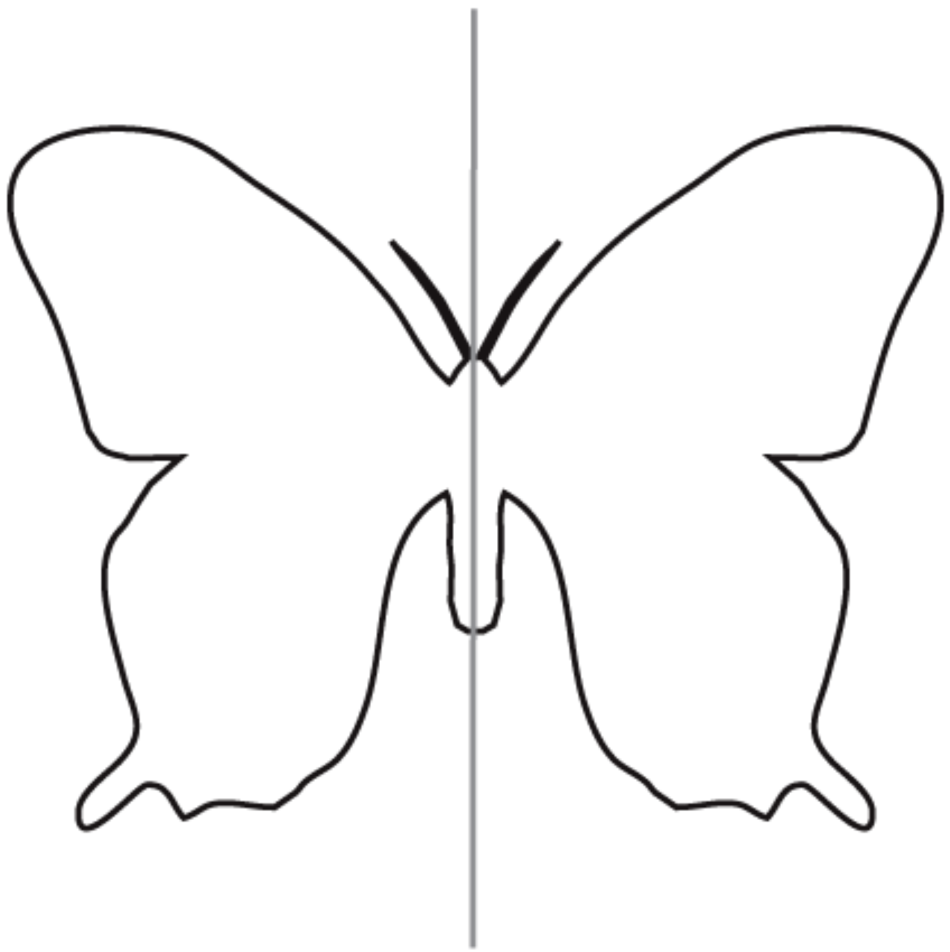
Reflective Symmetry: Largest Cluster

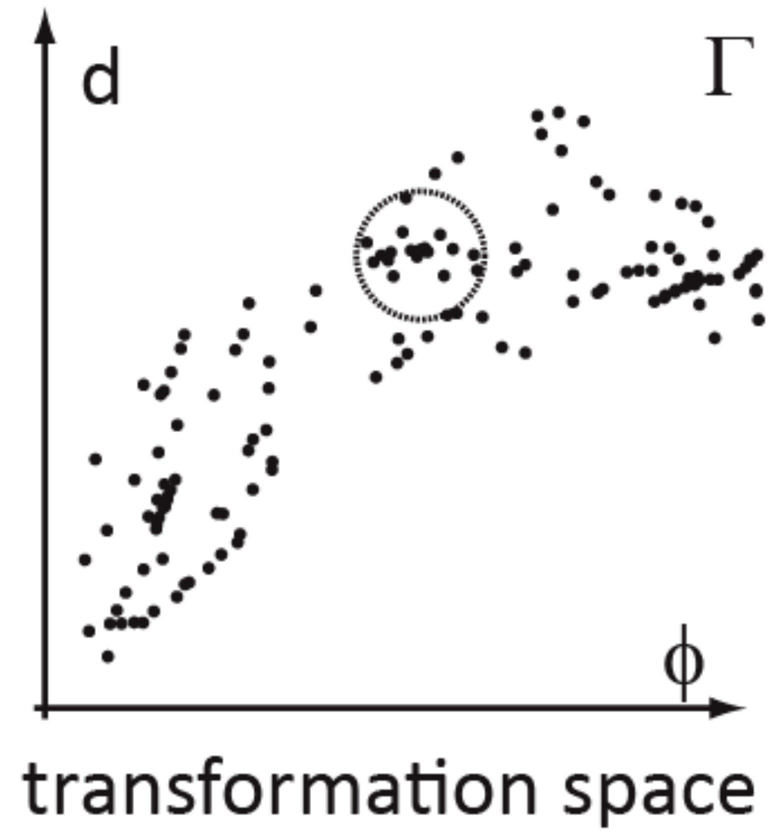




- Height of cluster ⟶ size of patch

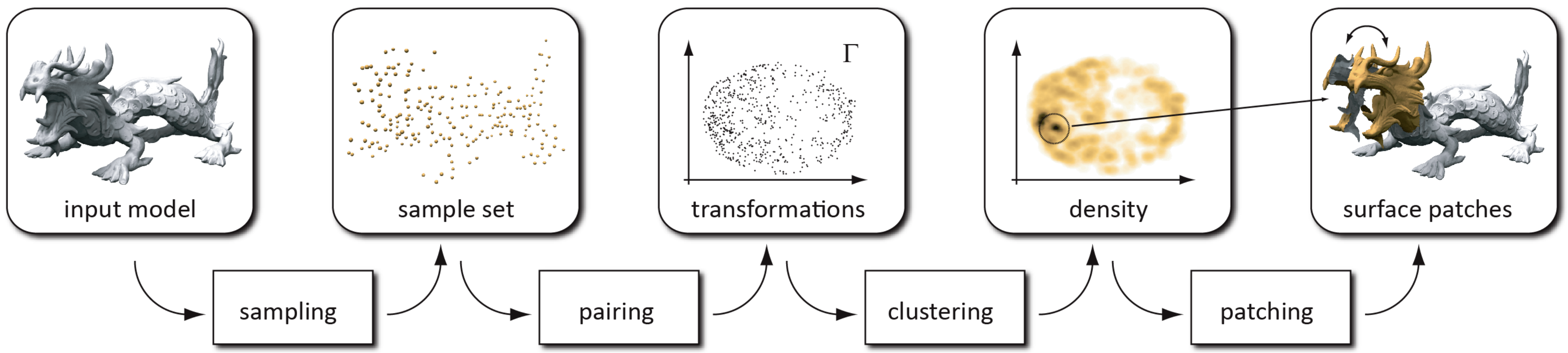
Reflective Symmetry: Largest Cluster





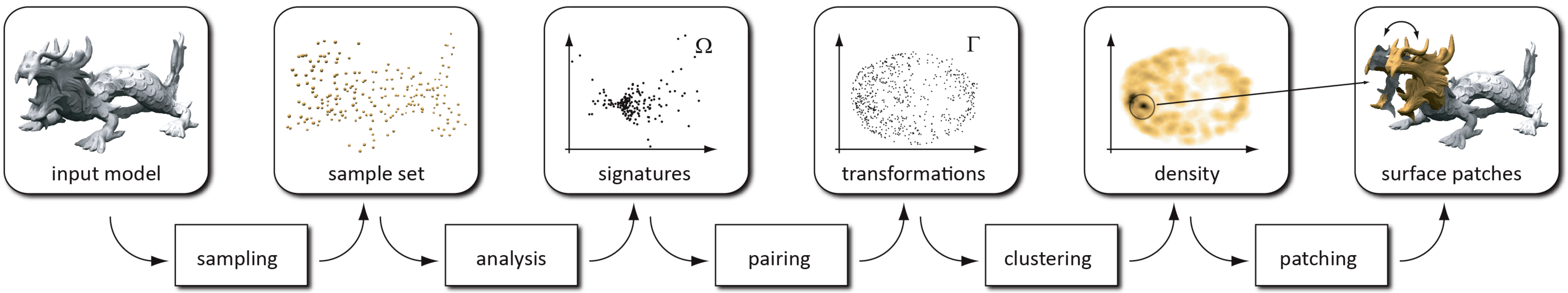
- Height of cluster ⟶ size of patch
- Spread of cluster ⟶ level of approximation

Pipeline



Γ
input model
sample set
transformations
density
surface patches
sampling
pairing
clustering
patching

Pipeline



input model
sample set
signatures
Ω
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density
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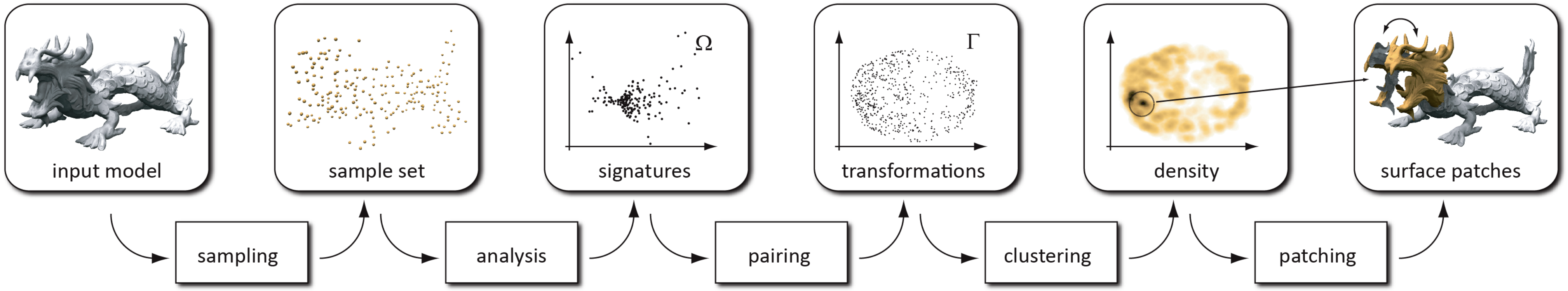
Pipeline


input model
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signatures
Ω
transformations
Γ
density
surface patches
sampling
analysis
pairing
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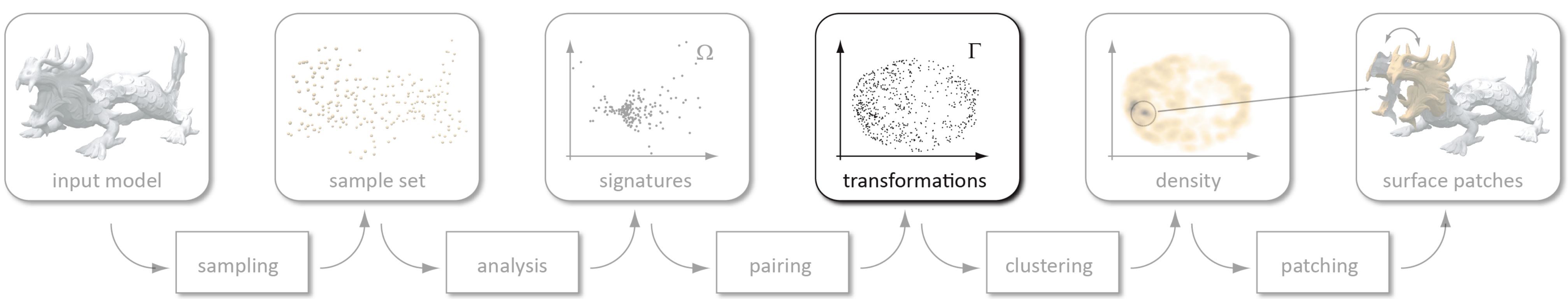
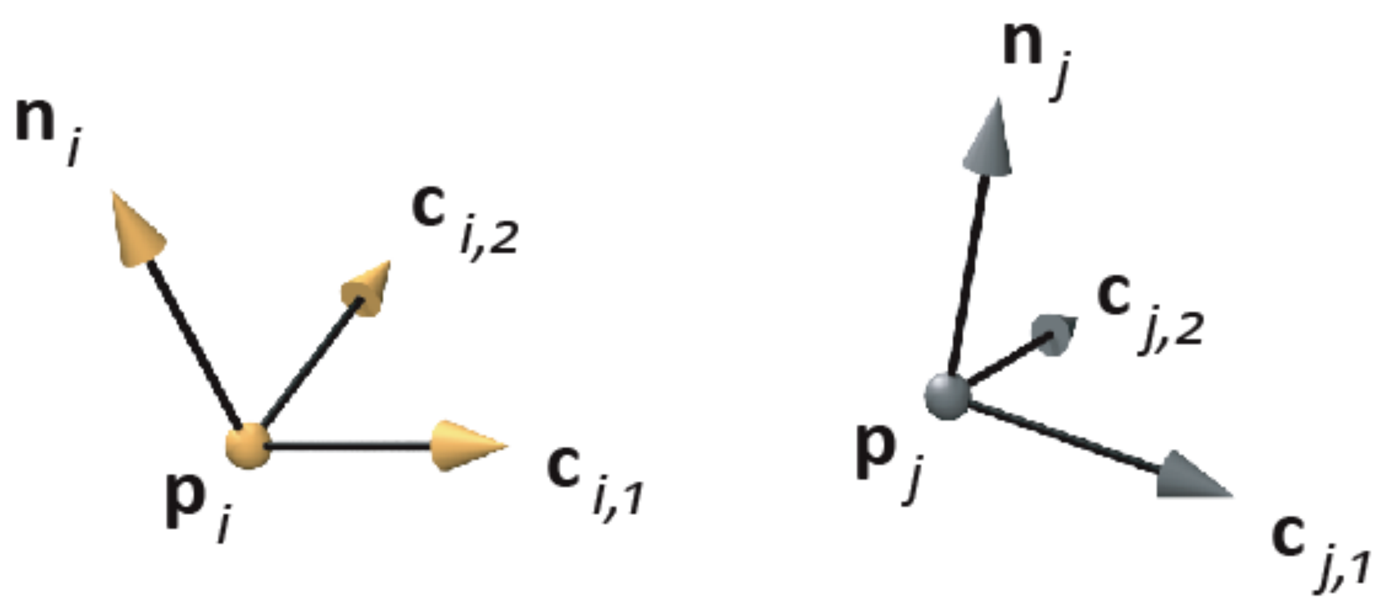
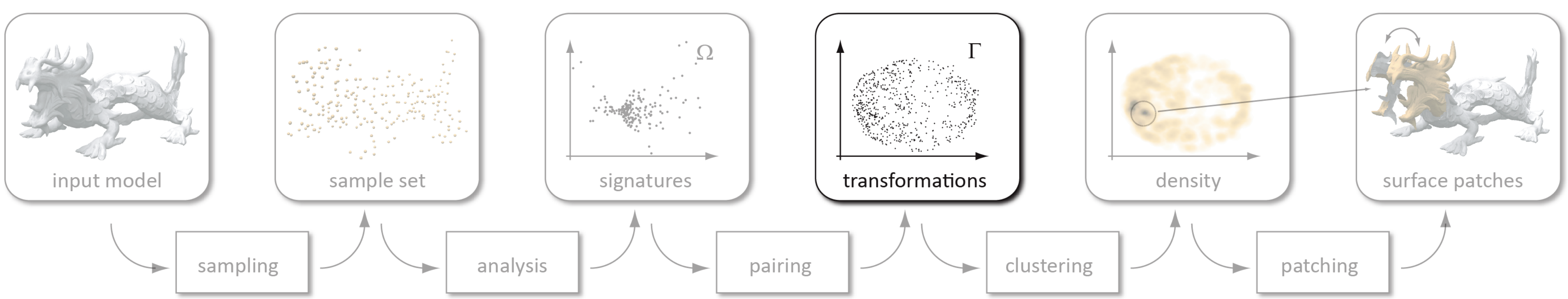
patching

Rigid Transformations

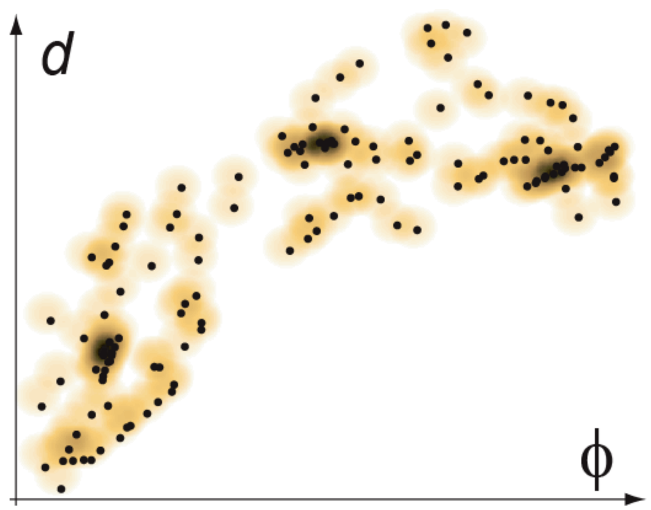


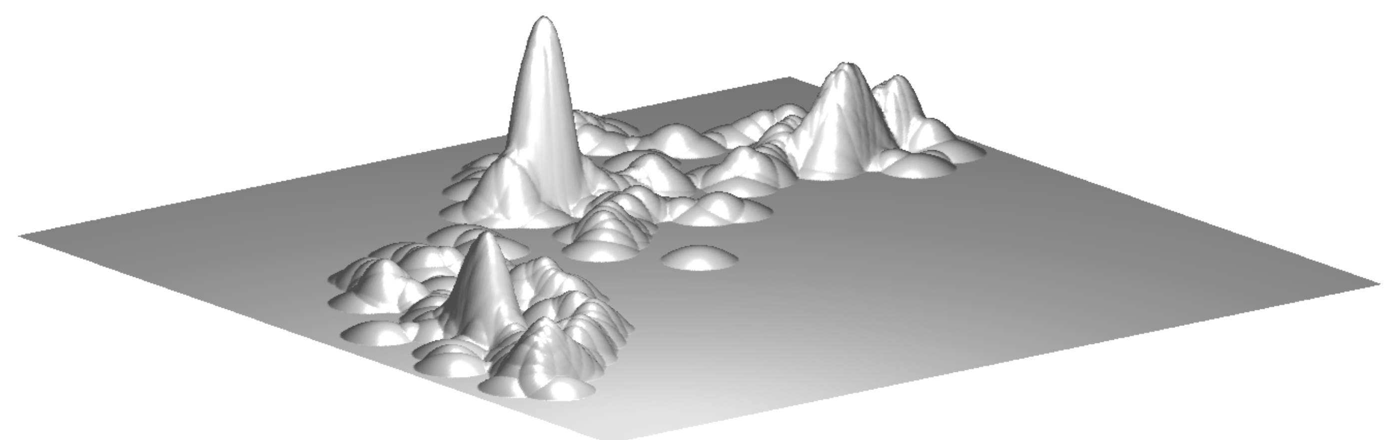
Rigid Transformations

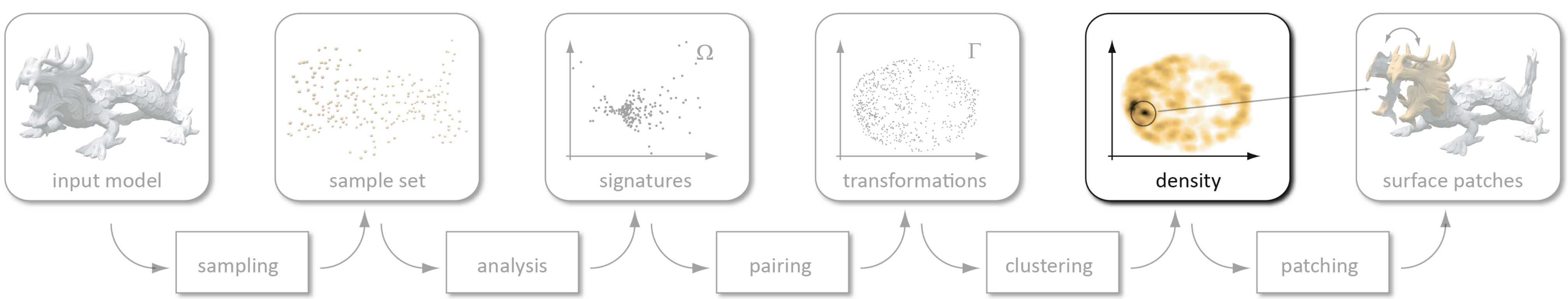




Mean-Shift Clustering







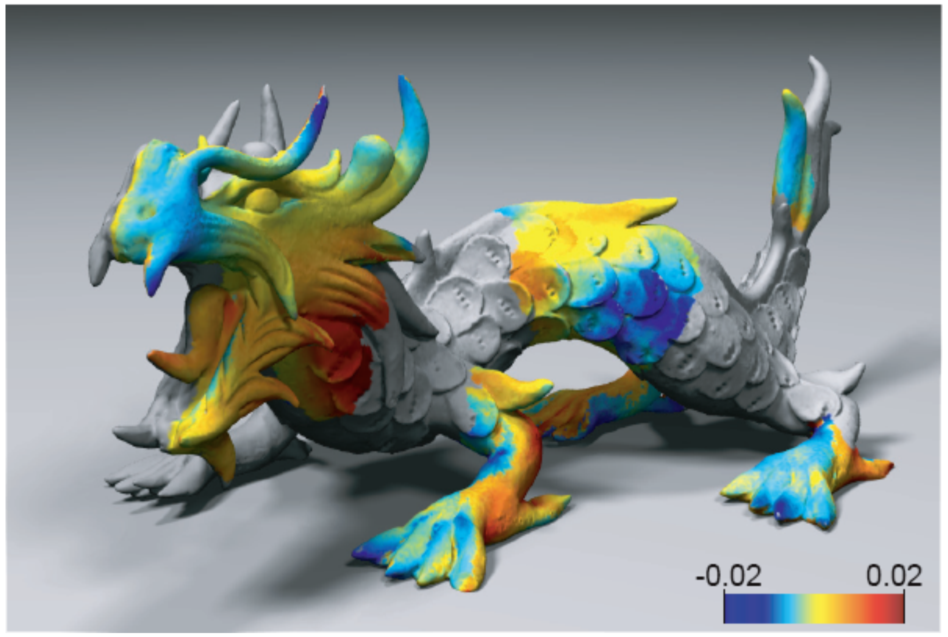
Detection Results: Dragon

Detection Results: Dragon

detected symmetries

Detection Results: Dragon

detected symmetries



correction field

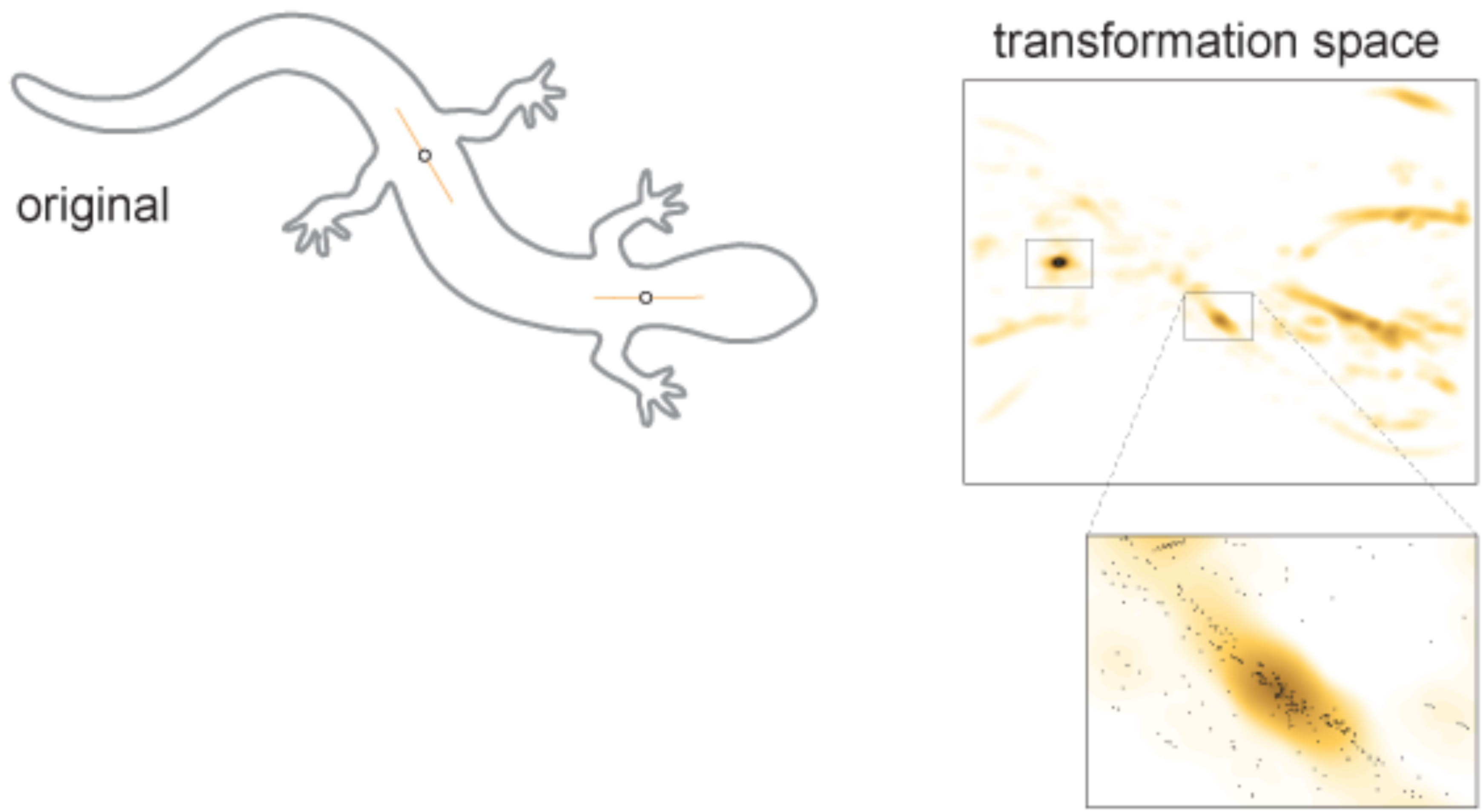
Insight: Global to Local Problem

(Euclidean) symmetry in spatial domain

↓

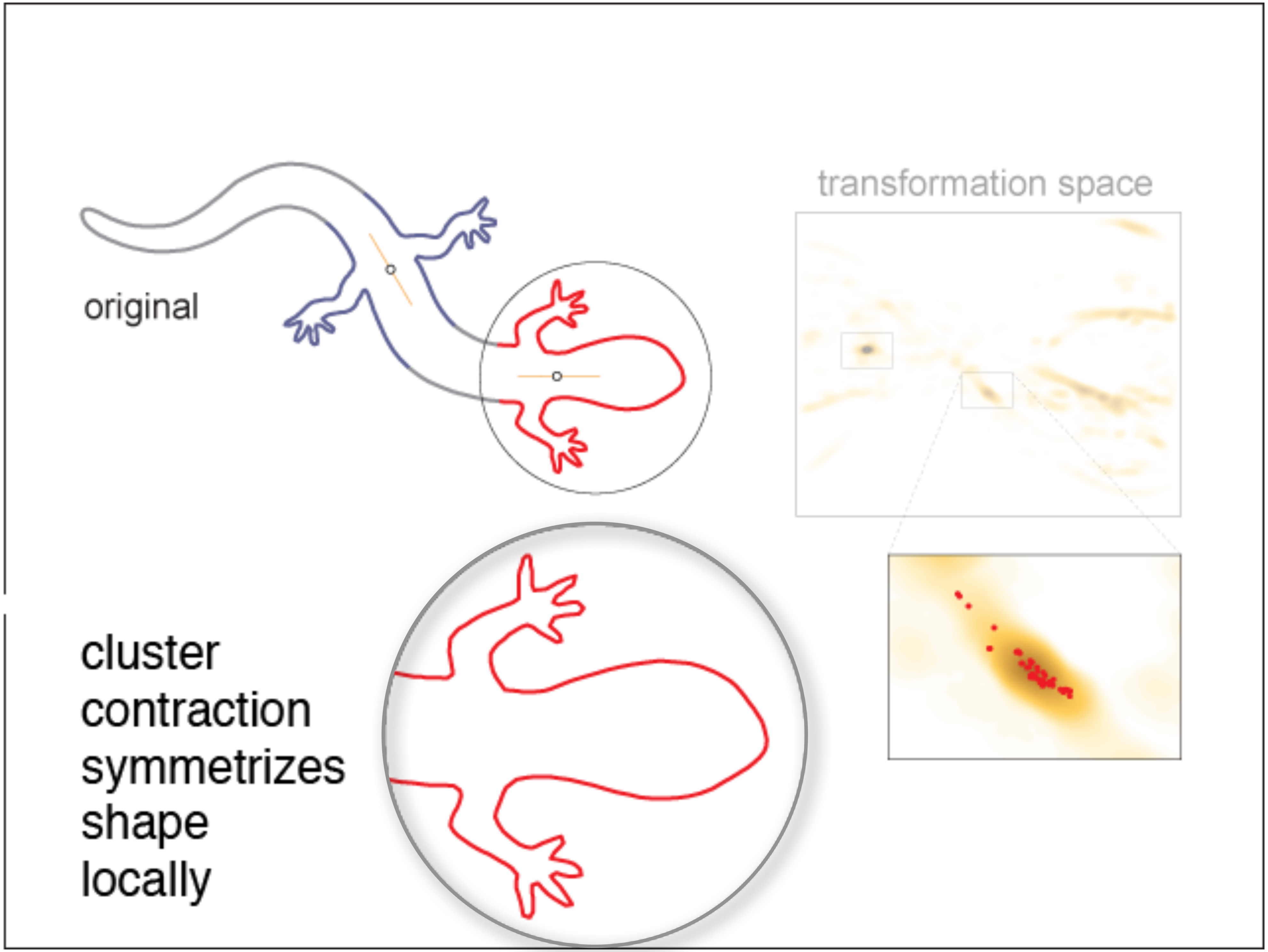
cluster(s) in transform domain

2D Example: Symmetrization

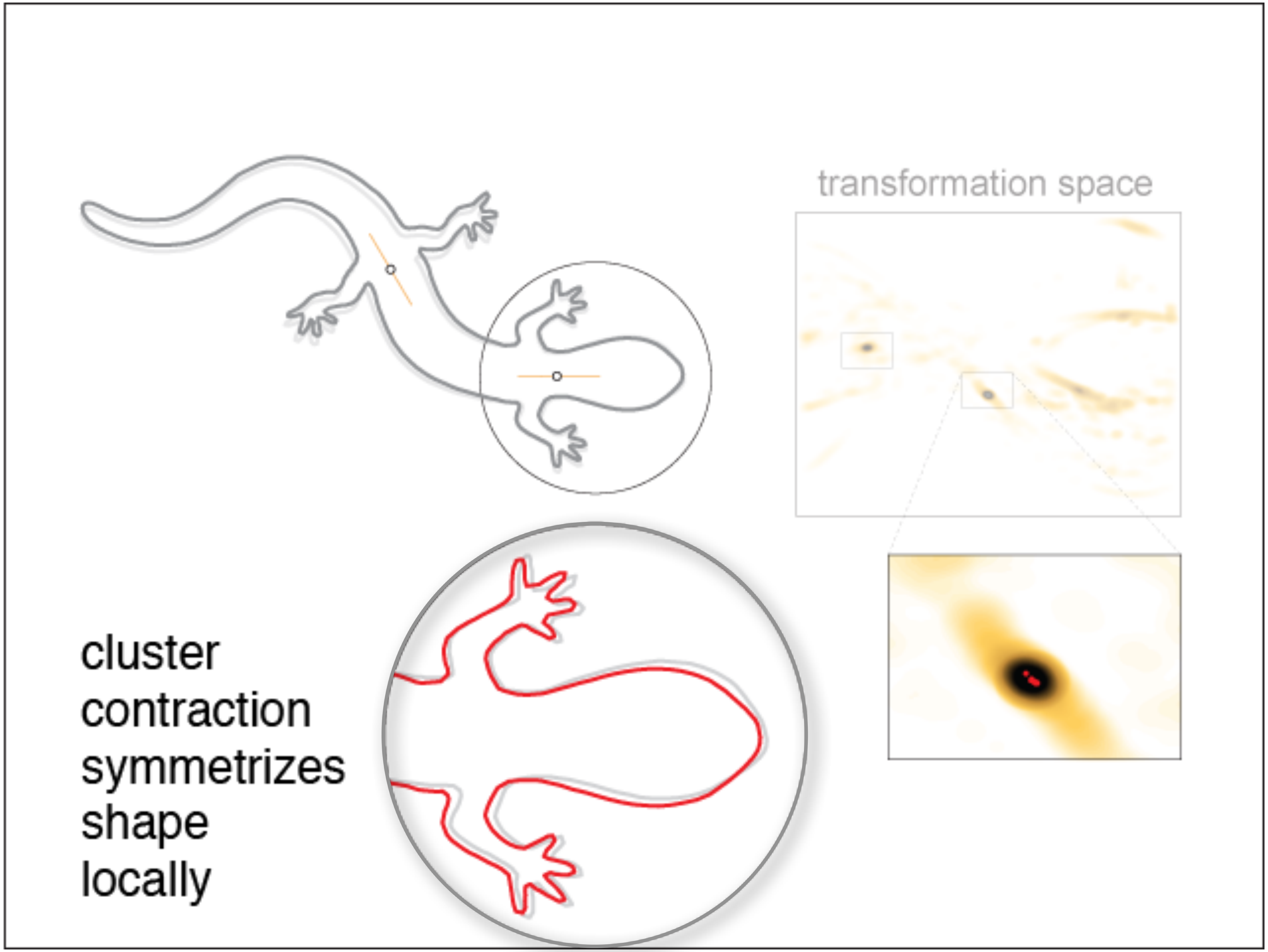


[Mitra et al. 2007]

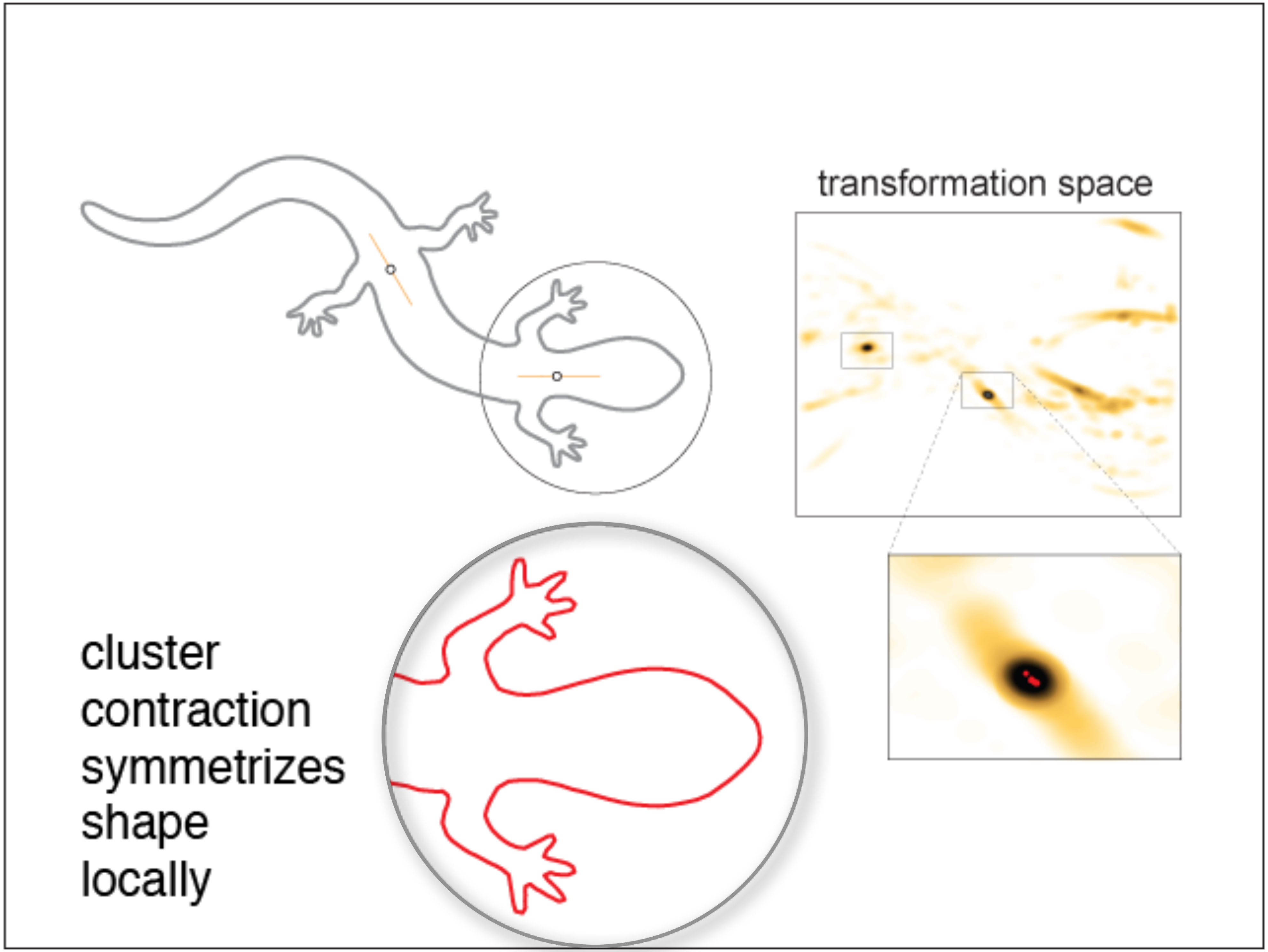
2D Example: Symmetrization



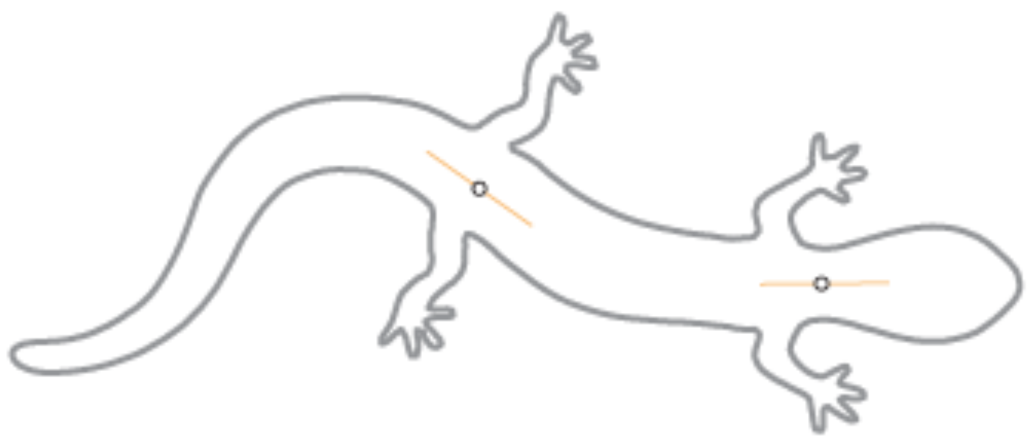
2D Example: Symmetrization



2D Example: Symmetrization



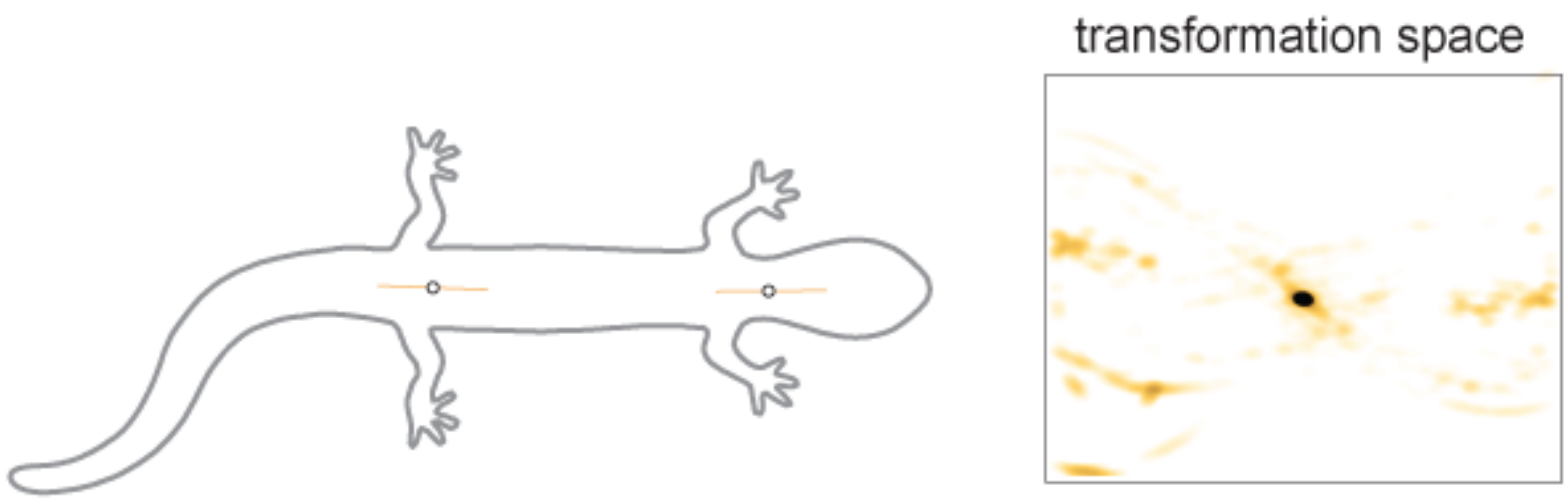
2D Example: Symmetrization



transformation space

cluster merging achieves
global symmetry

2D Example: Symmetrization



cluster merging achieves
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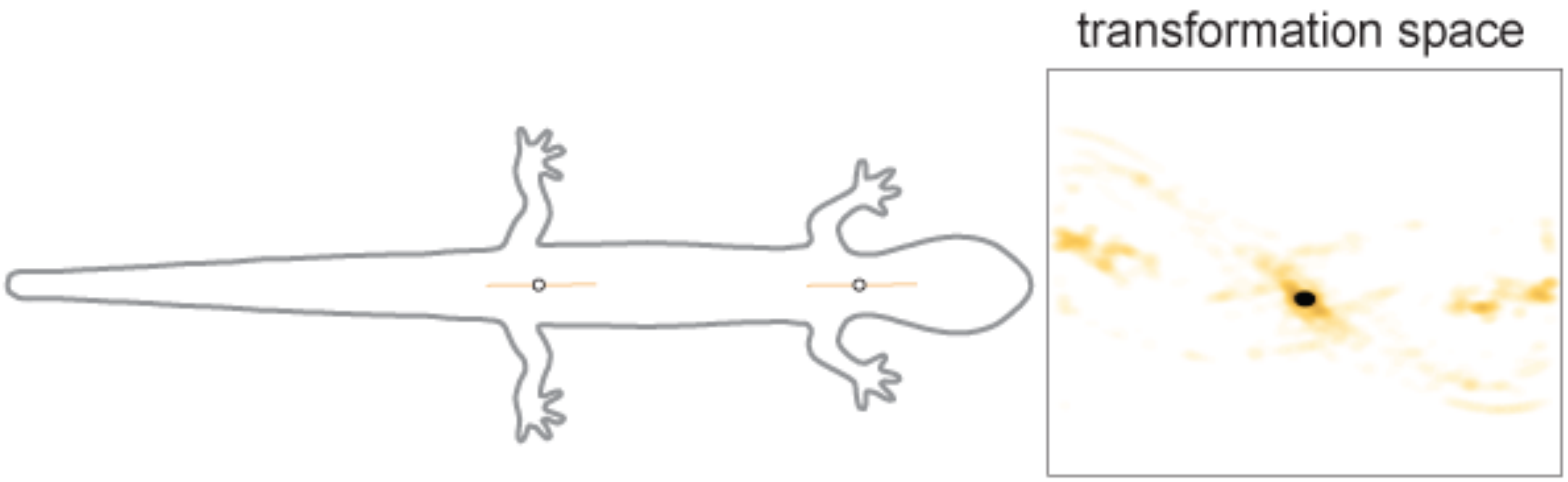
2D Example: Symmetrization



cluster merging achieves
global symmetry

Symmetrization: Bunny

Cluster Contraction

Transformation Space

Graph-based Symmetries

[Bokeloh et al. 2009]

Graph-based Symmetries

- ***Features:*** slippage analysis

[Bokeloh et al. 2009]

Graph-based Symmetries

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[Bokeloh et al. 2009]

Graph-based Symmetries

- ***Features:*** slippage analysis
- ***Aggregation:*** locally coherent line arrangements

[Bokeloh et al. 2009]

Graph-based Symmetries

- ***Features:*** slippage analysis

- ***Aggregation:*** locally coherent line arrangements

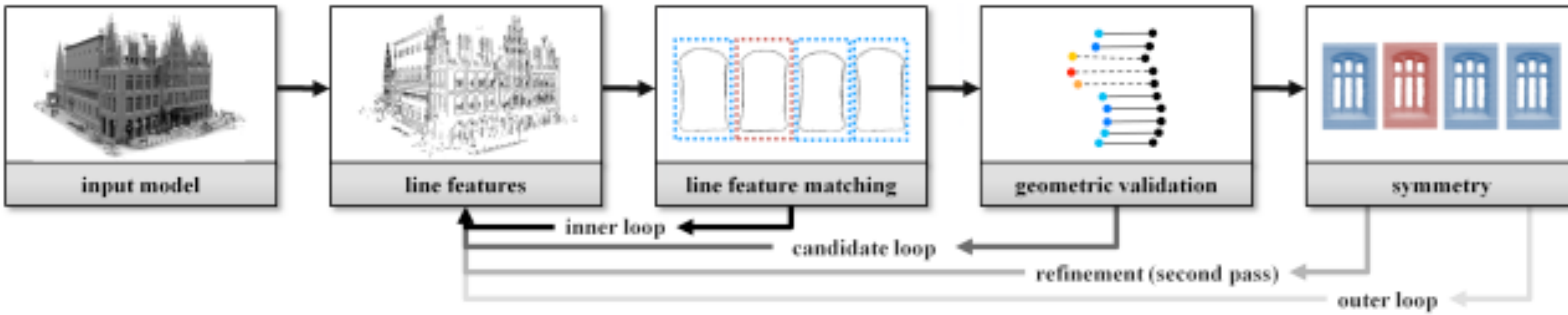
[Bokeloh et al. 2009]

Graph-based Symmetries

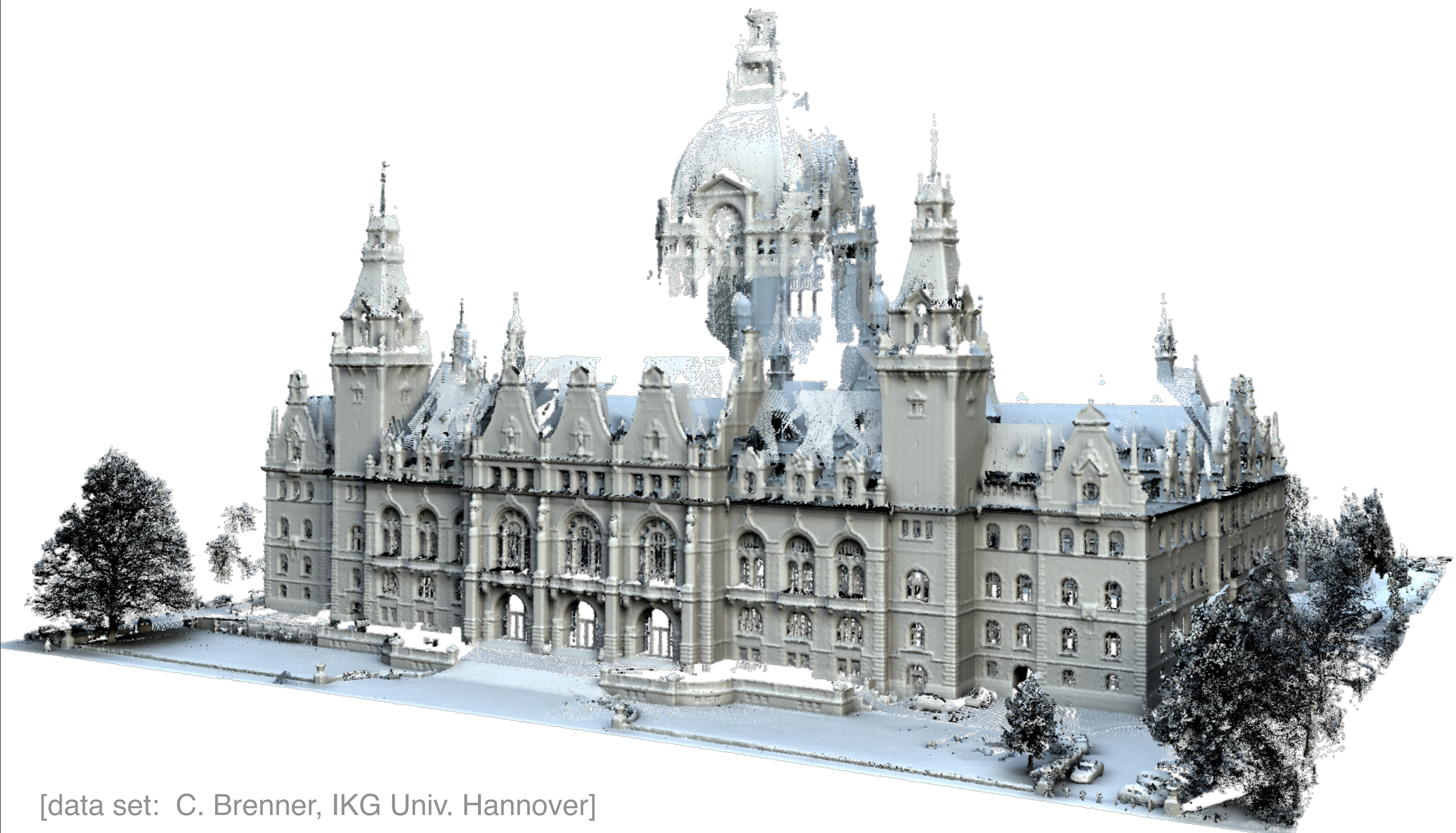
- ***Features:*** slippage analysis
- ***Aggregation:*** locally coherent line arrangements
- ***Extraction:*** simultaneous refinement

[Bokeloh et al. 2009]

Algorithm Pipeline

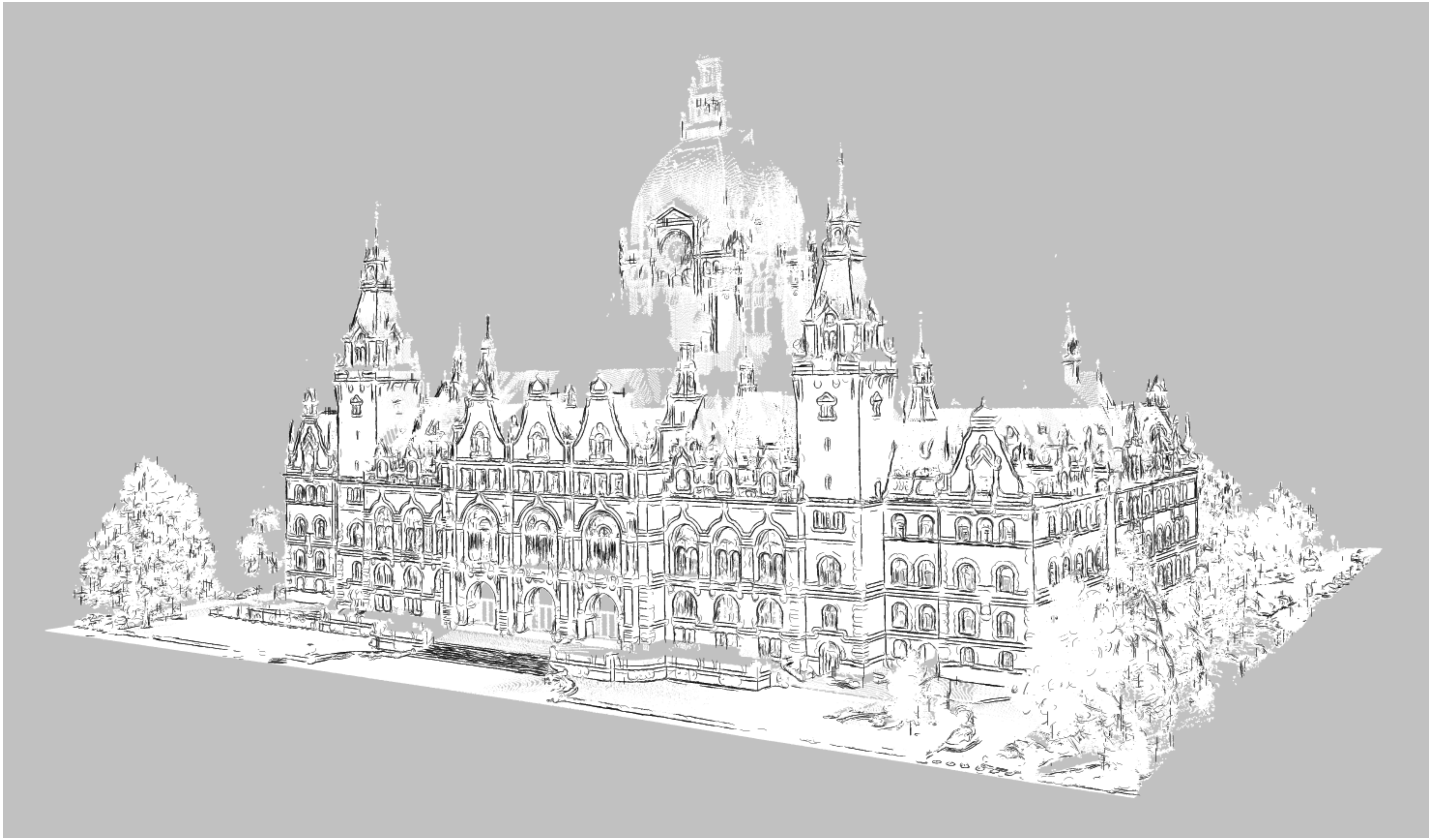


Symmetry Detection

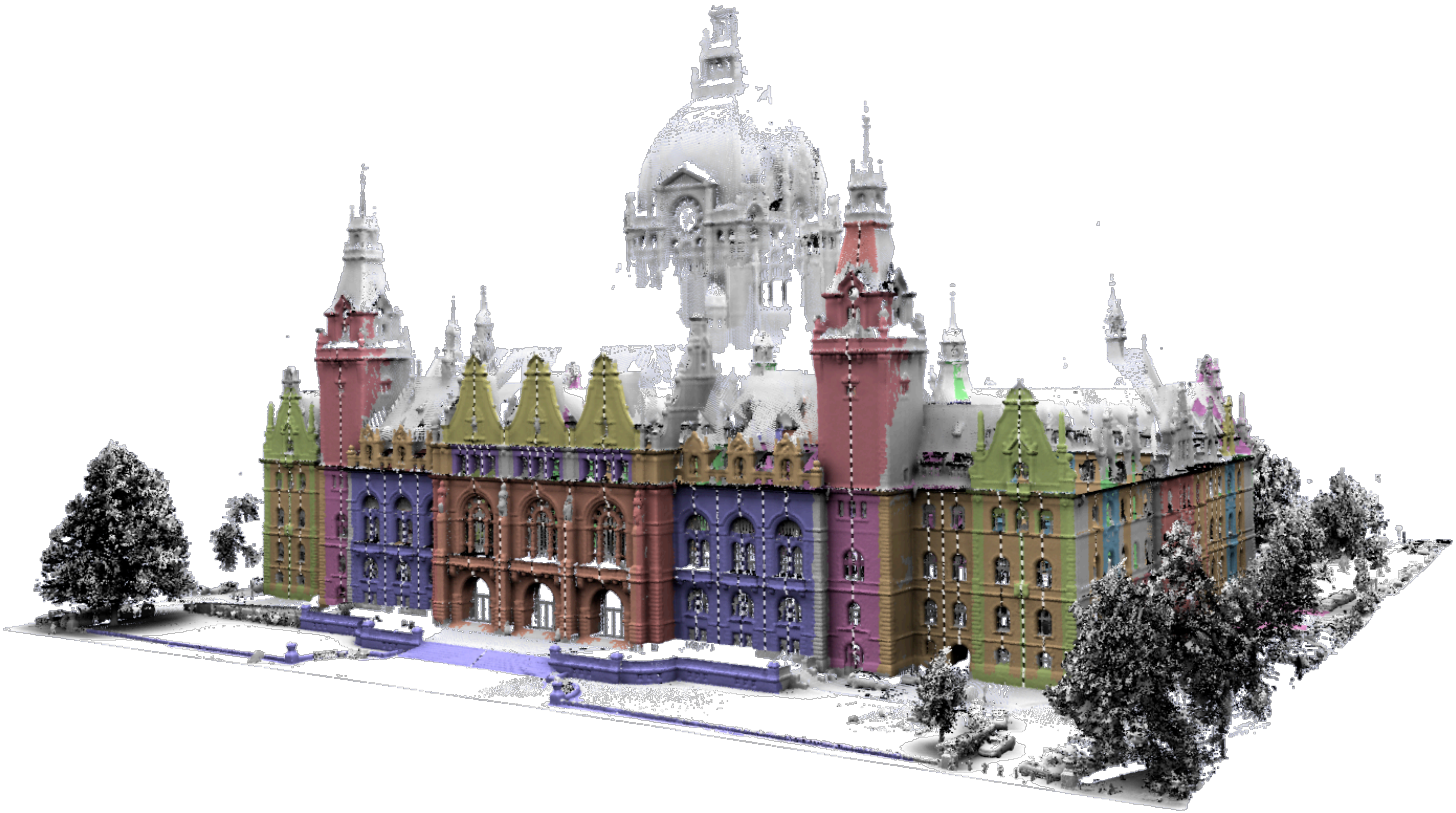


[data set: C. Brenner, IKG Univ. Hannover]

Line Features

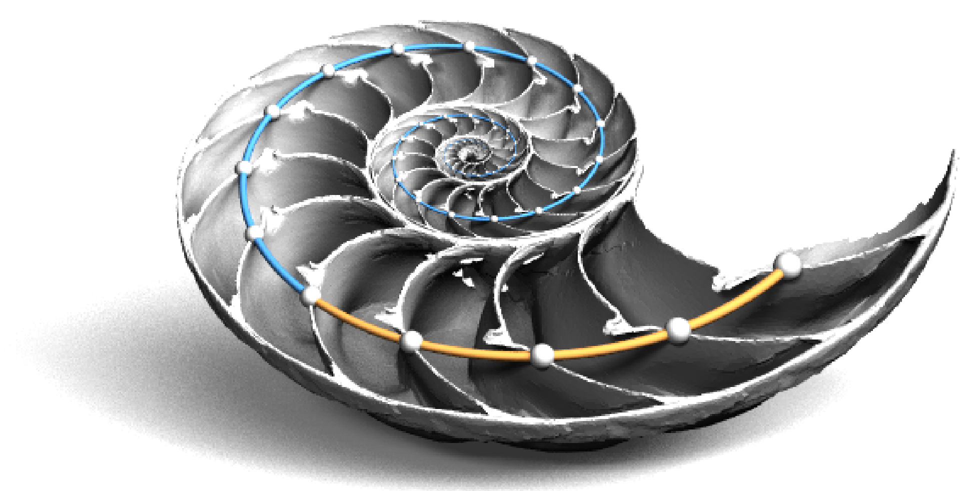


Symmetry Detection



Symmetry Detection

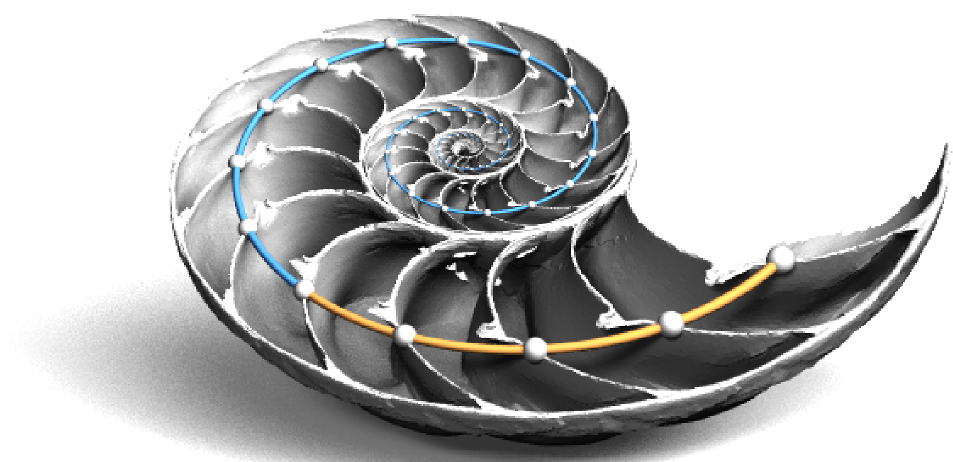
Symmetry of Symmetries



[Pauly et al. 2008]

Symmetry of Symmetries

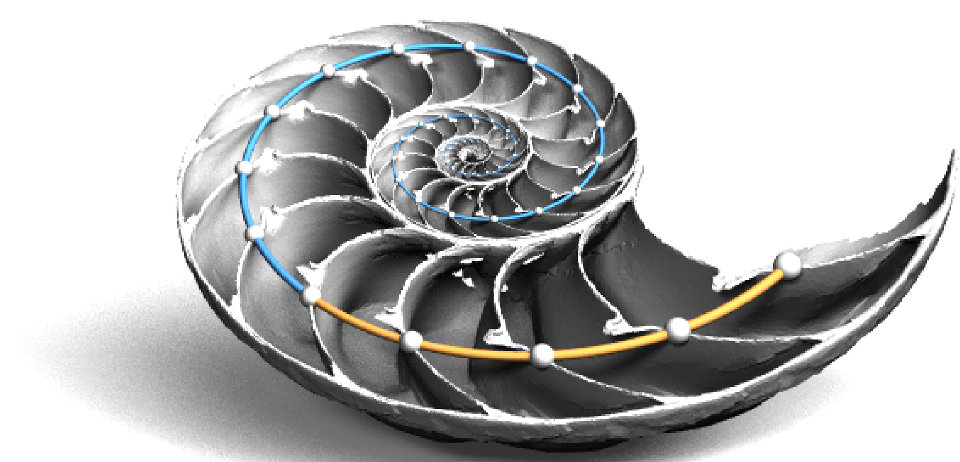
- ***Features:*** curvatures



[Pauly et al. 2008]

Symmetry of Symmetries

- ***Features:*** curvatures

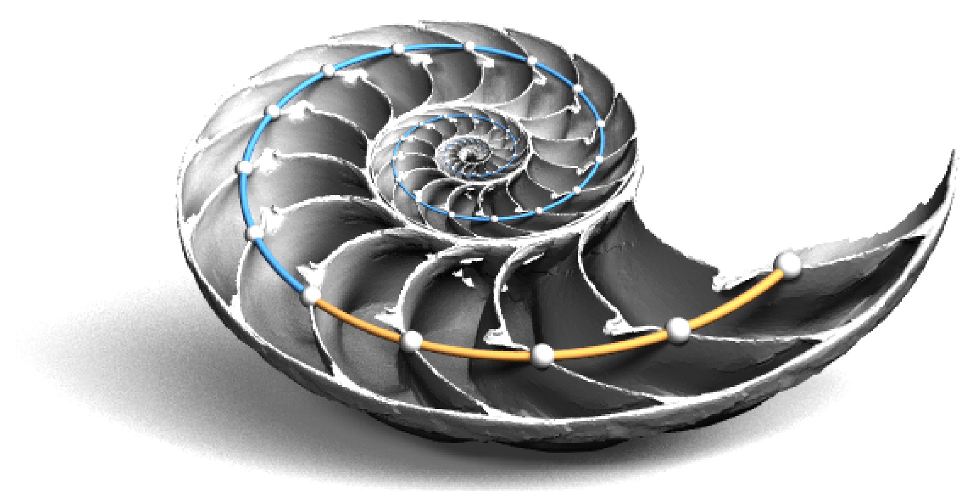


[Pauly et al. 2008]

Symmetry of Symmetries

- ***Features:*** curvatures

- ***Aggregation:*** transform domain model extraction



[Pauly et al. 2008]

Symmetry of Symmetries

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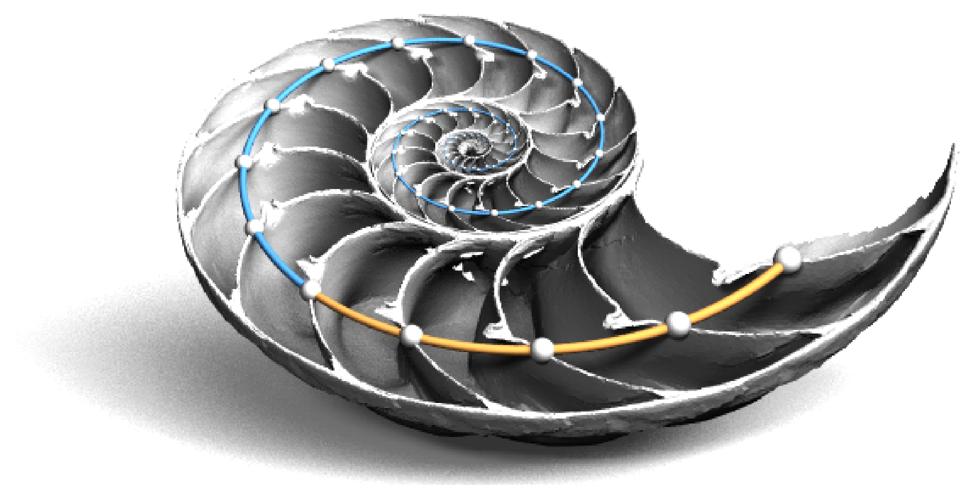
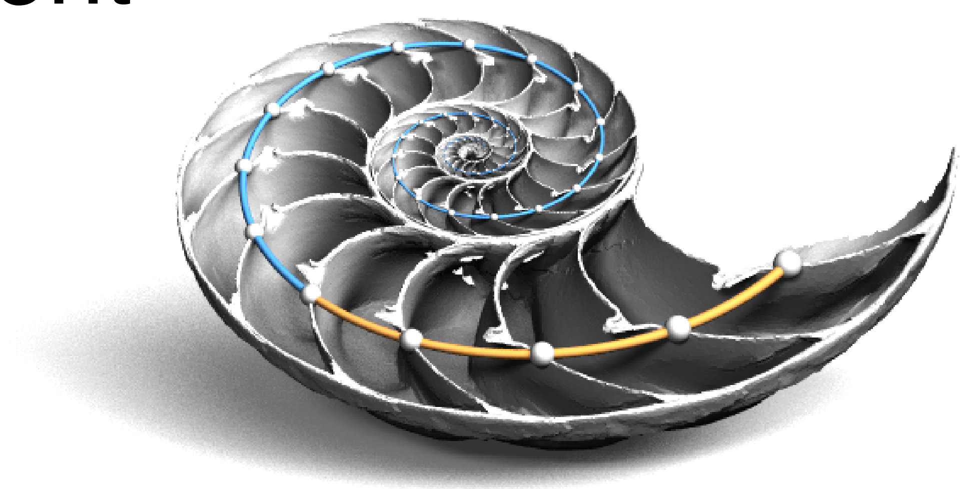


[Pauly et al. 2008]

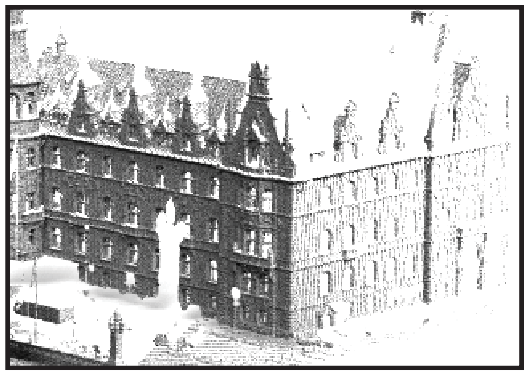
Symmetry of Symmetries

- ***Features:*** curvatures
- ***Aggregation:*** transform domain model extraction
- ***Extraction:*** simultaneous refinement



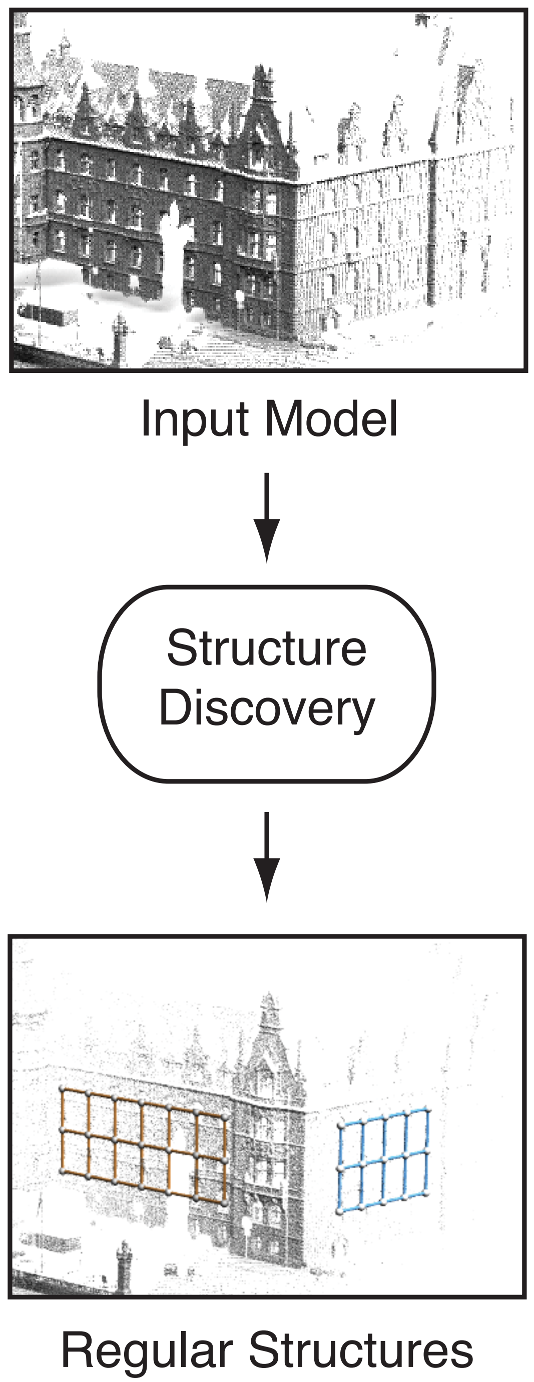
[Pauly et al. 2008]

Structure Discovery

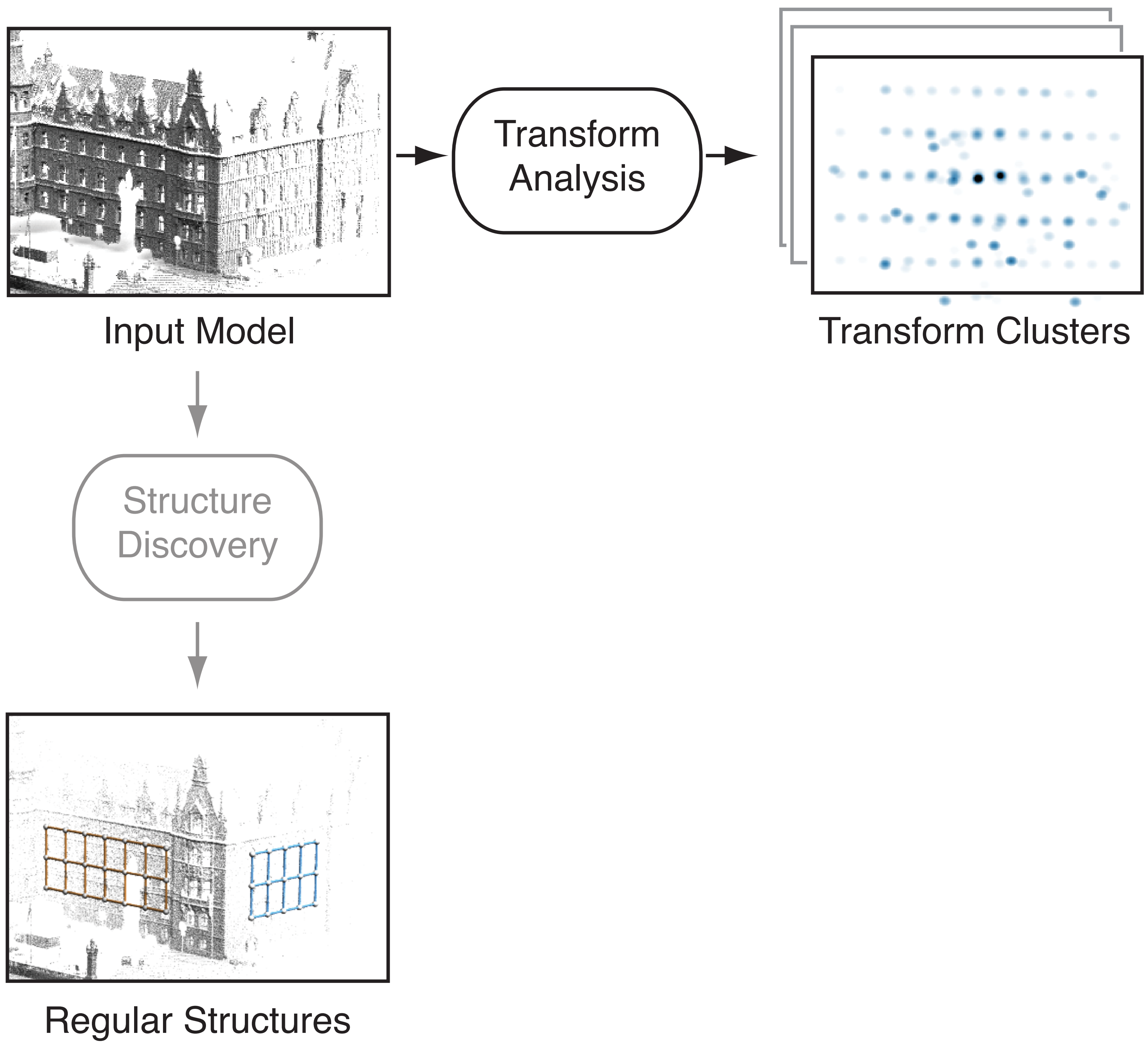


Input Model

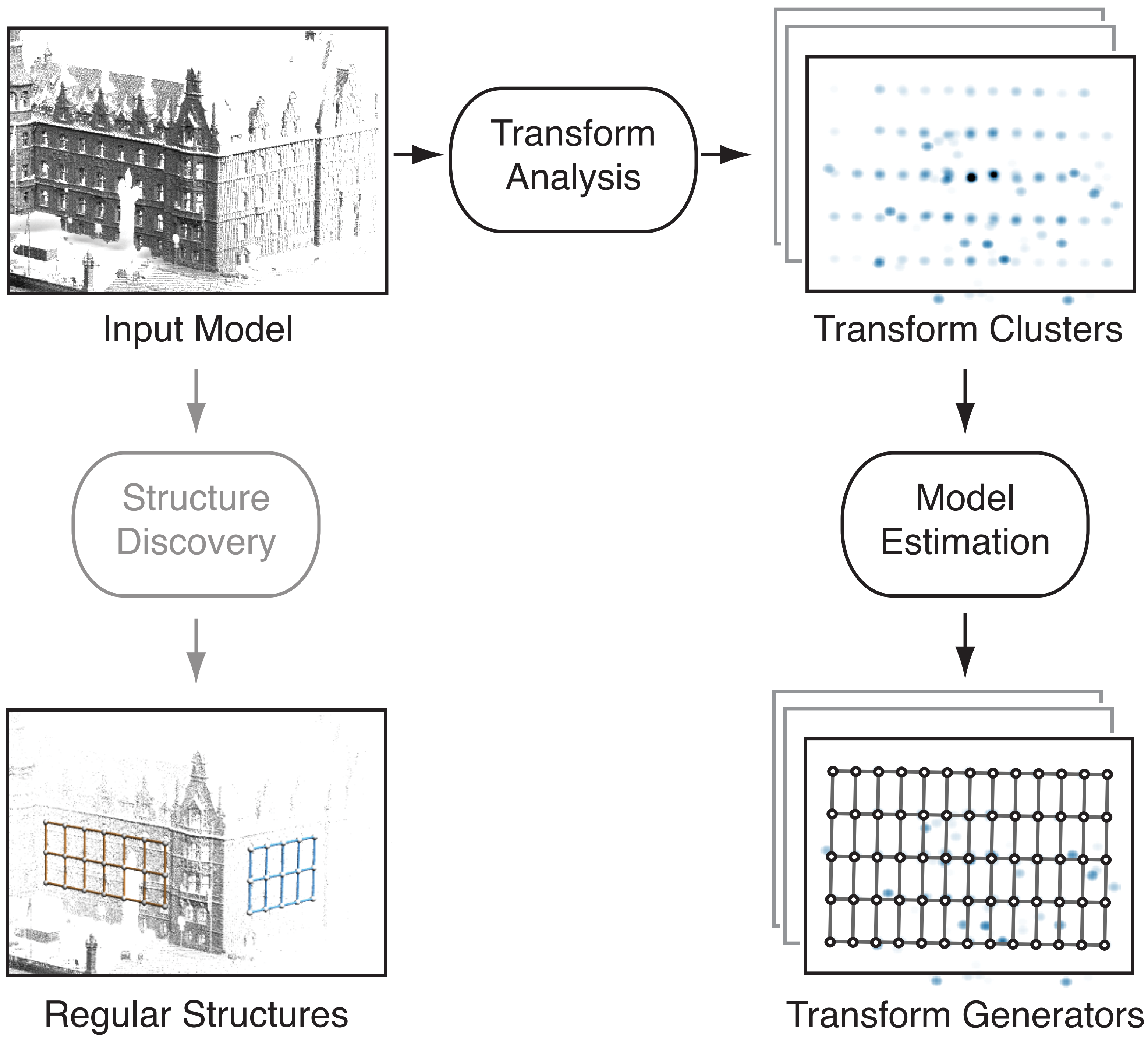
Structure Discovery



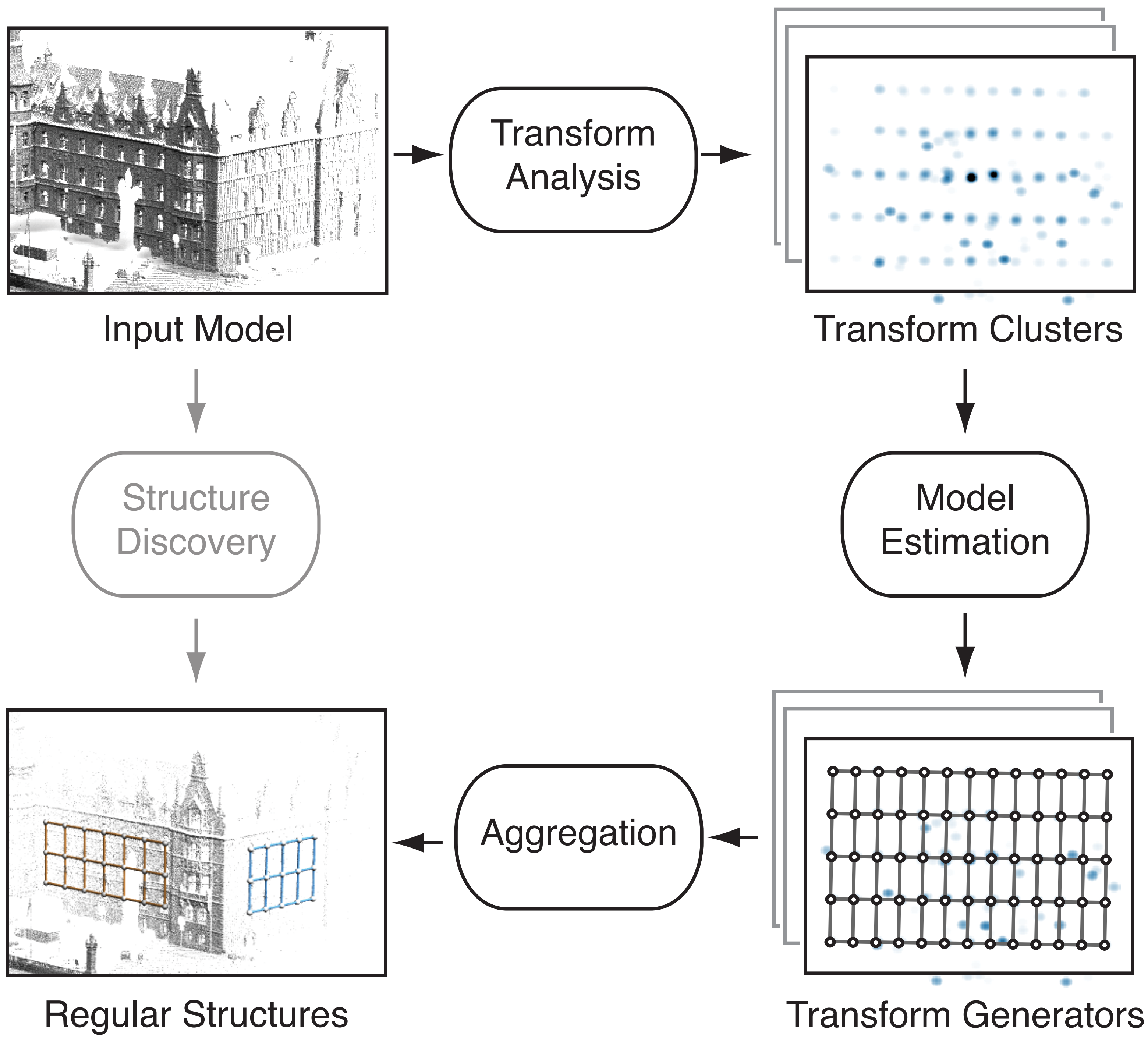
Structure Discovery



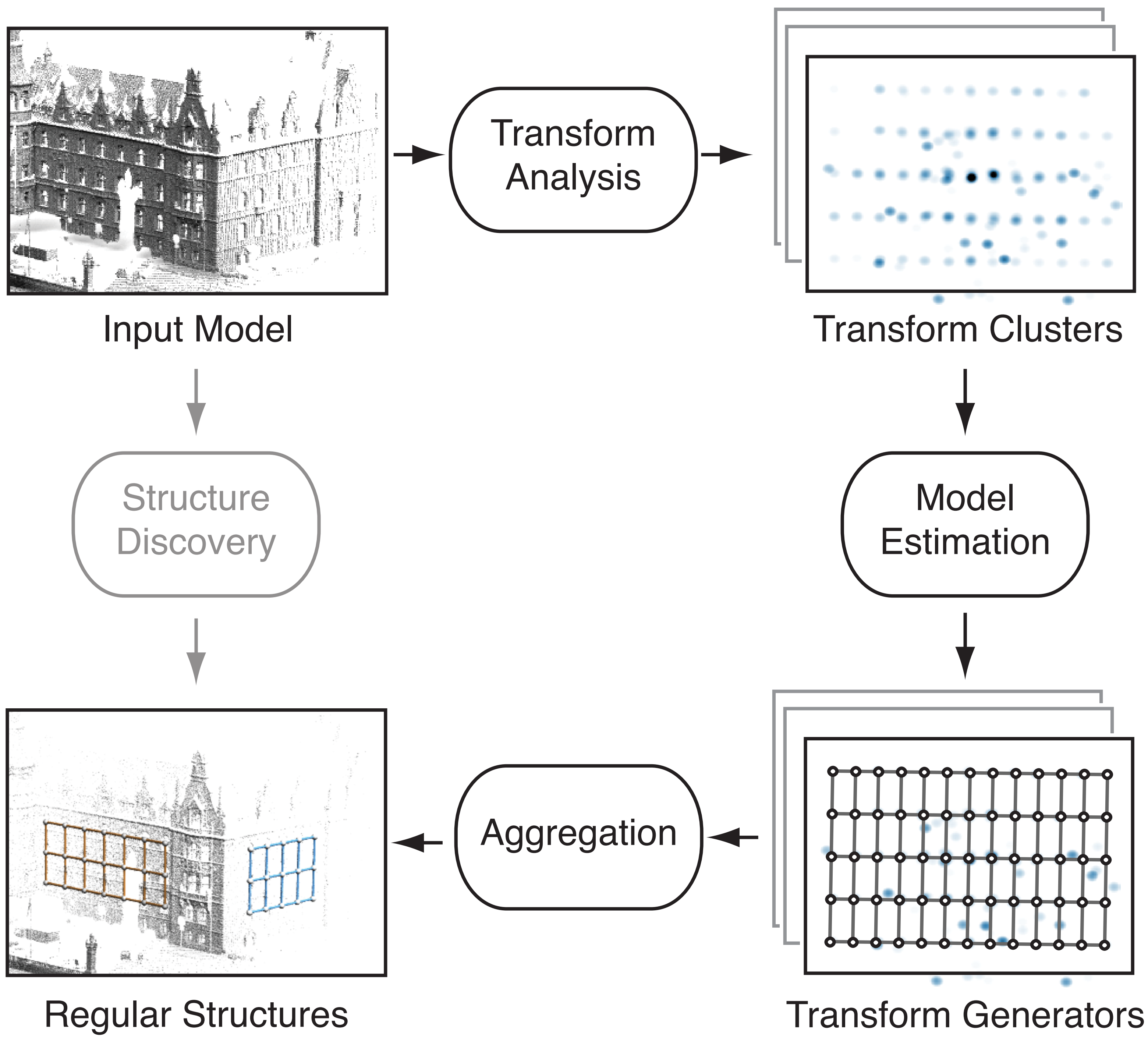
Structure Discovery



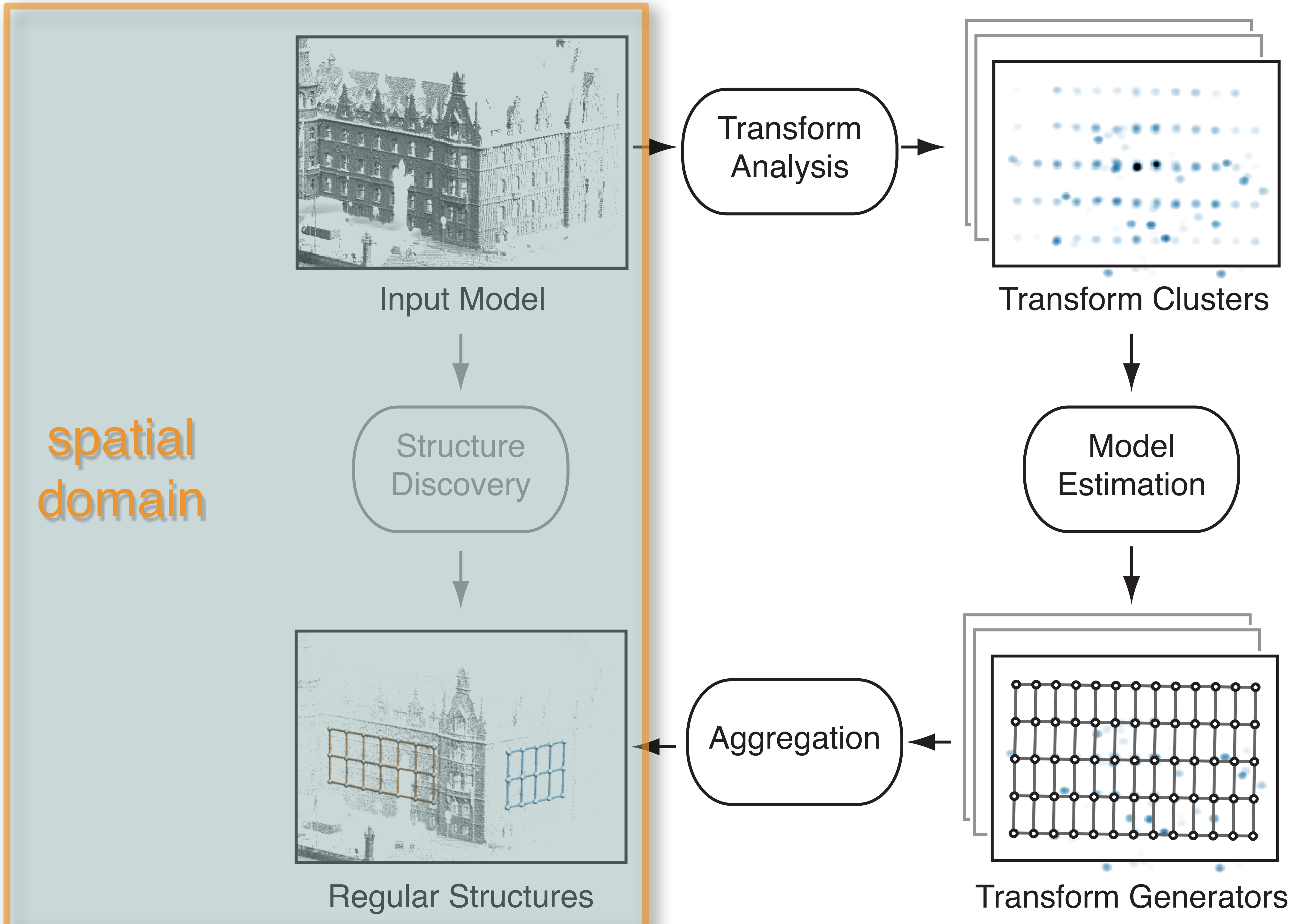
Structure Discovery



Structure Discovery



Structure Discovery



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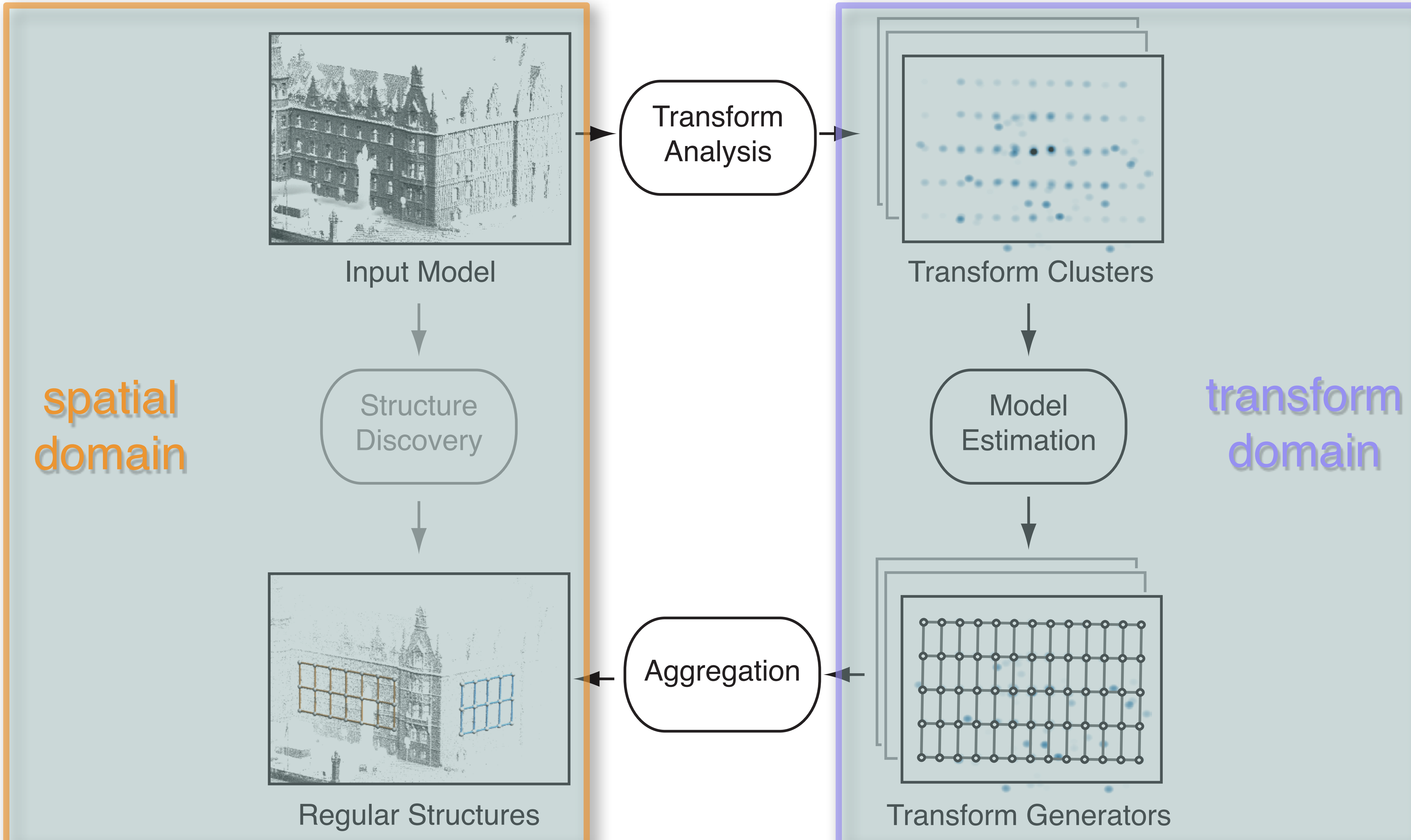

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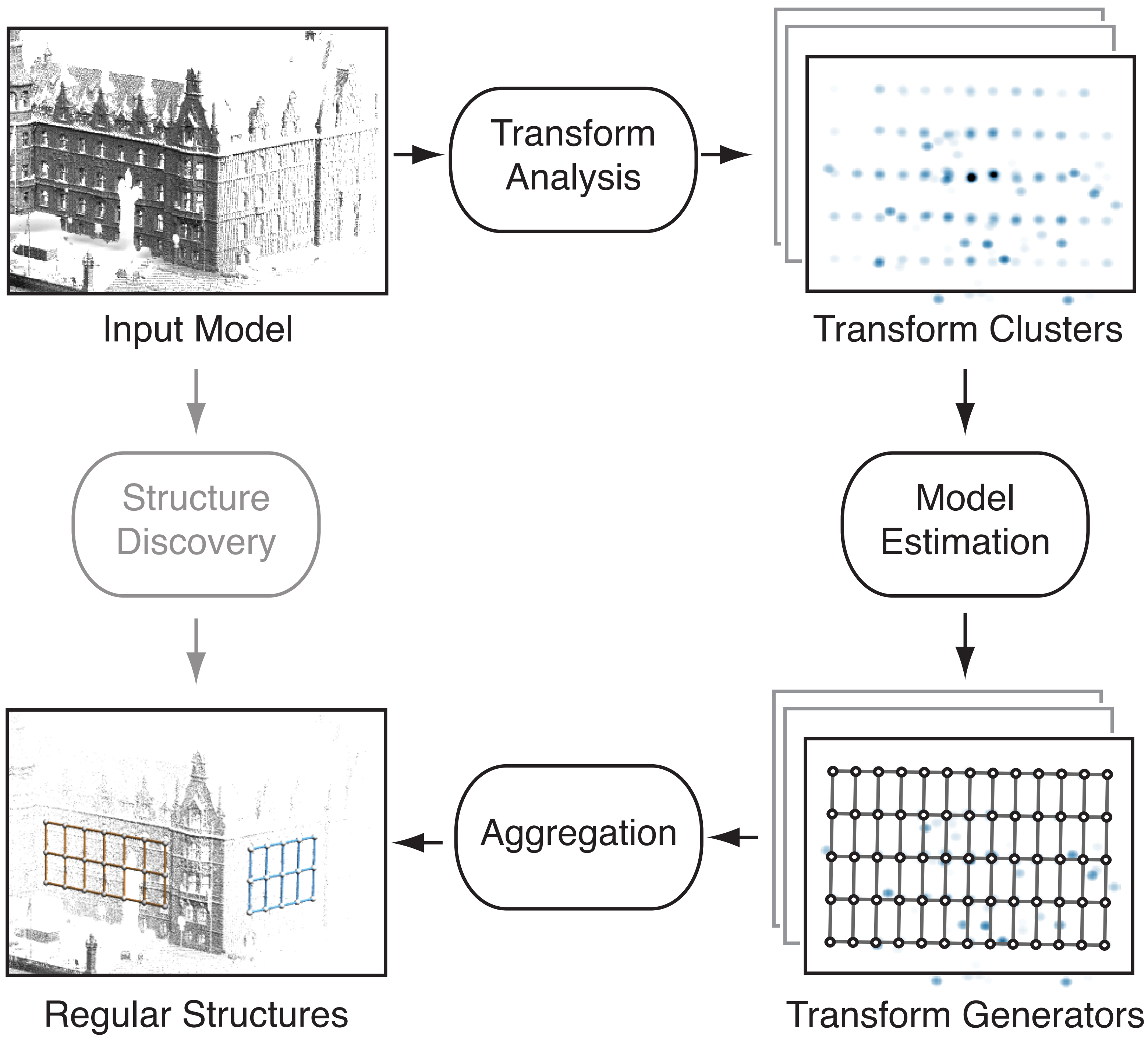

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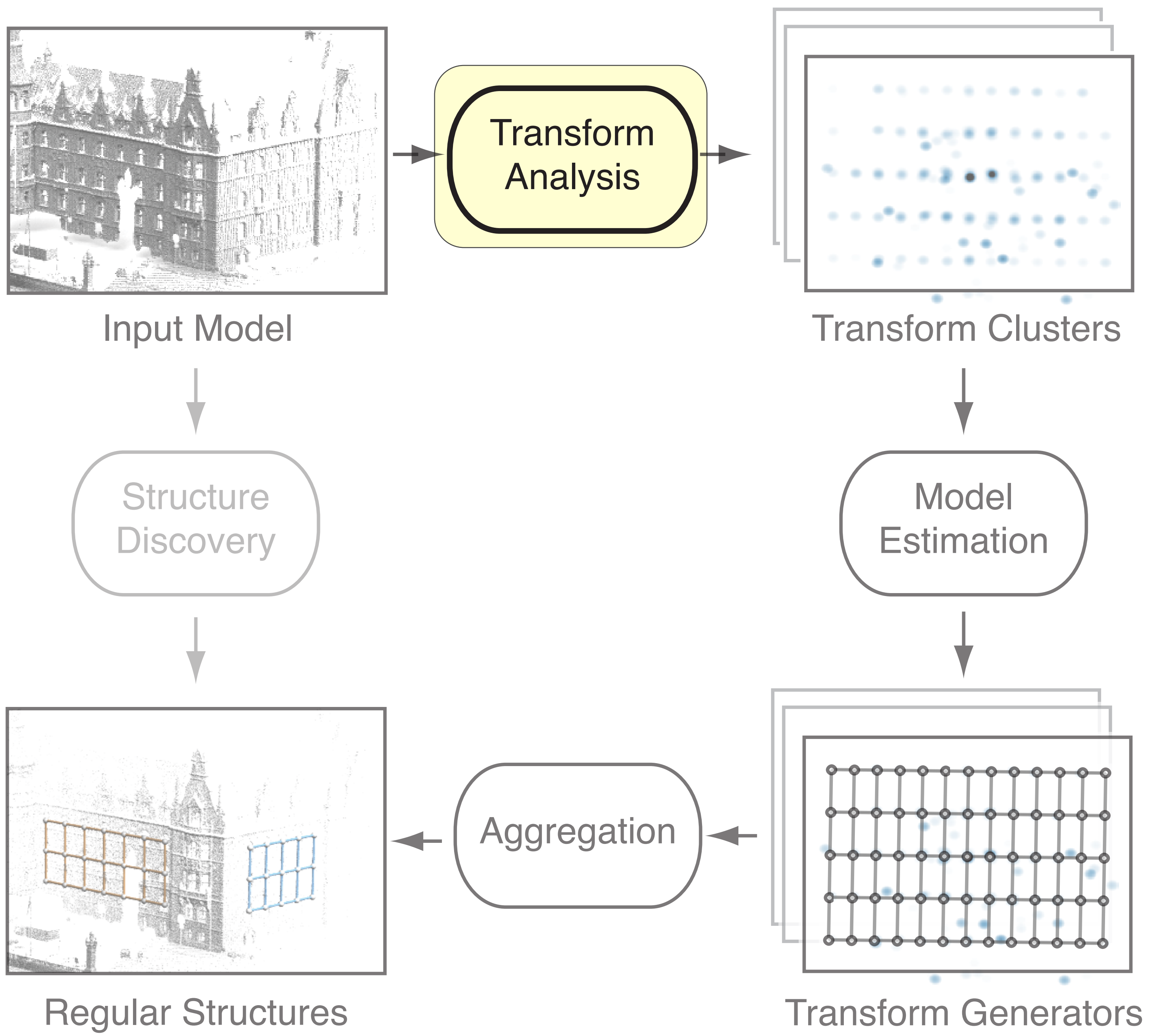
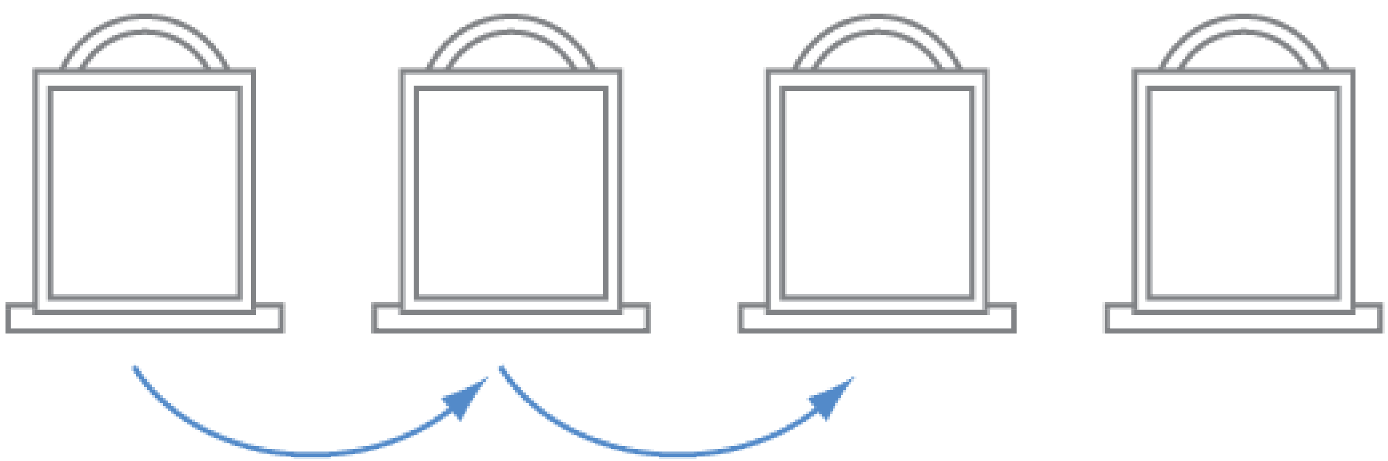


Transformations

Transformations

Transformations



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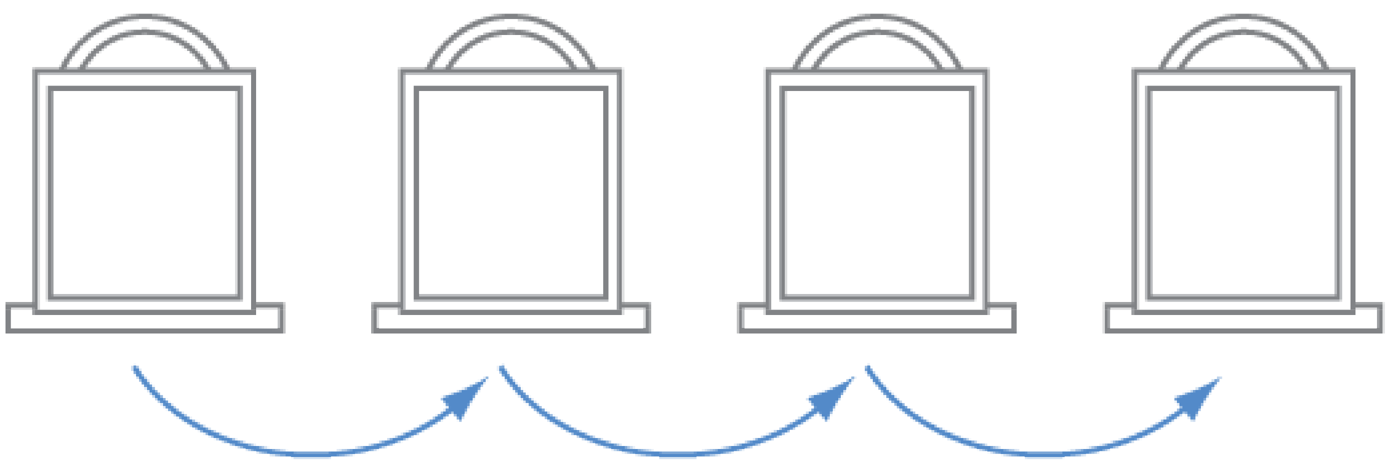

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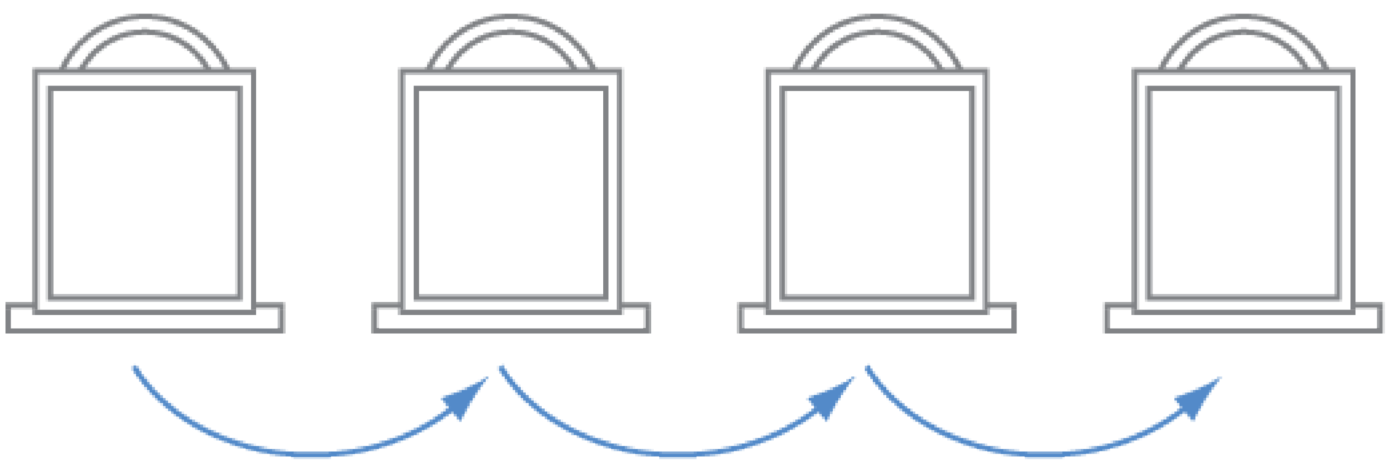

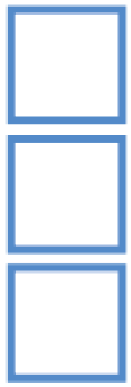

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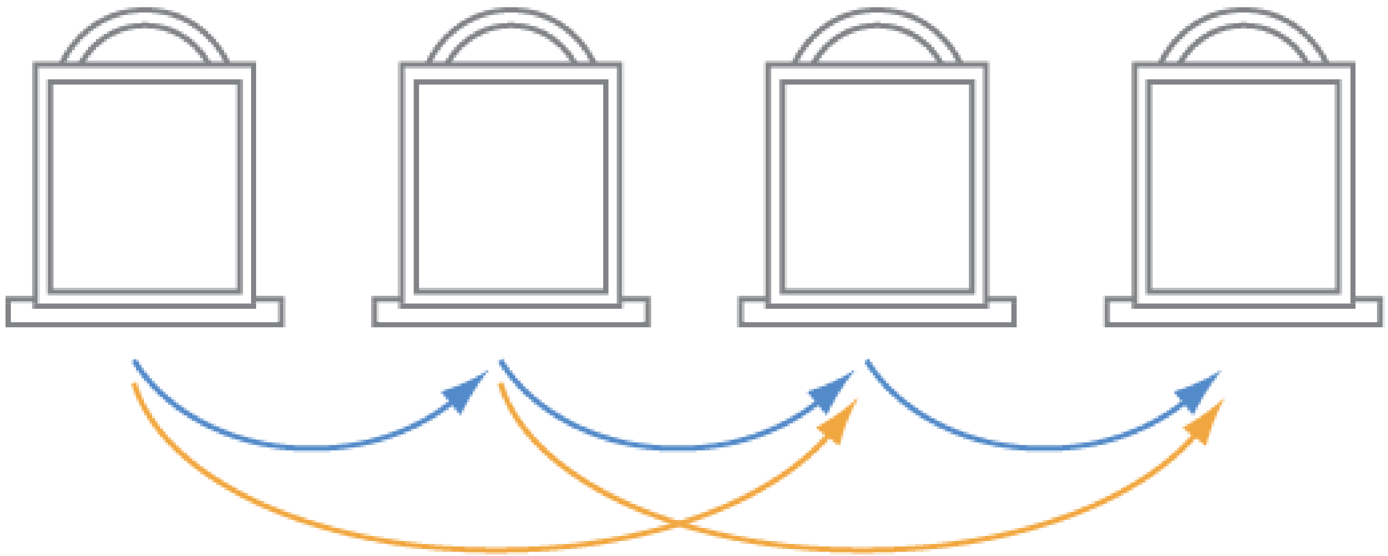

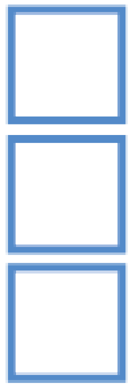

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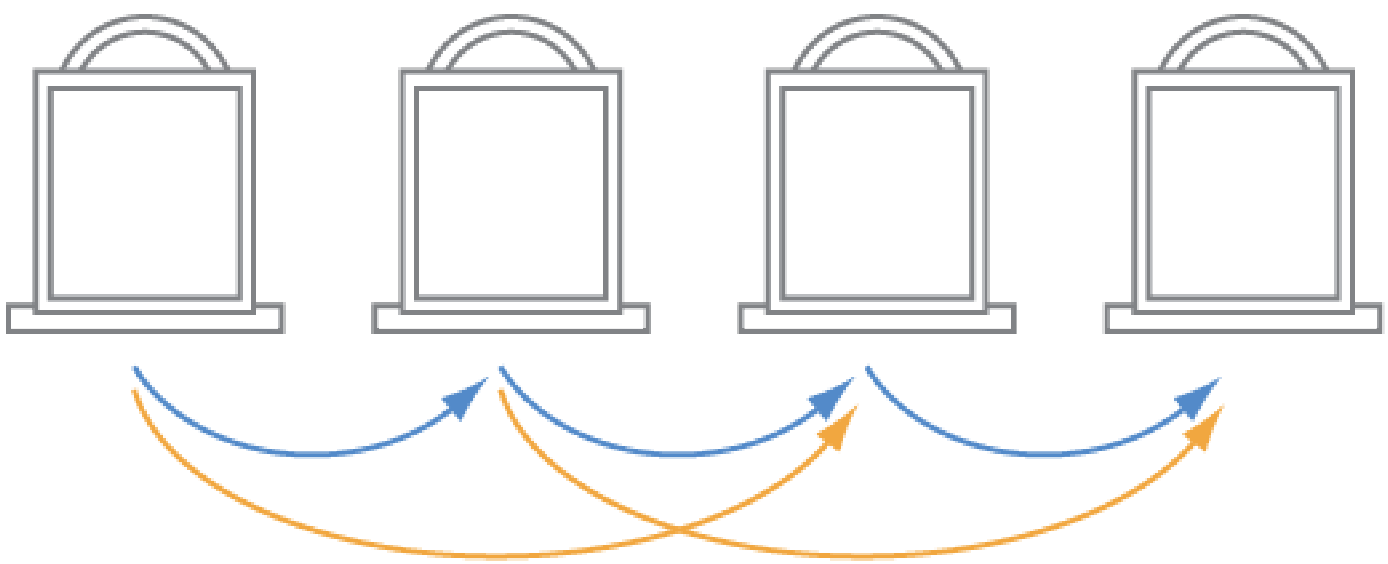

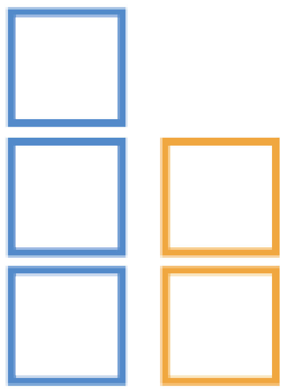

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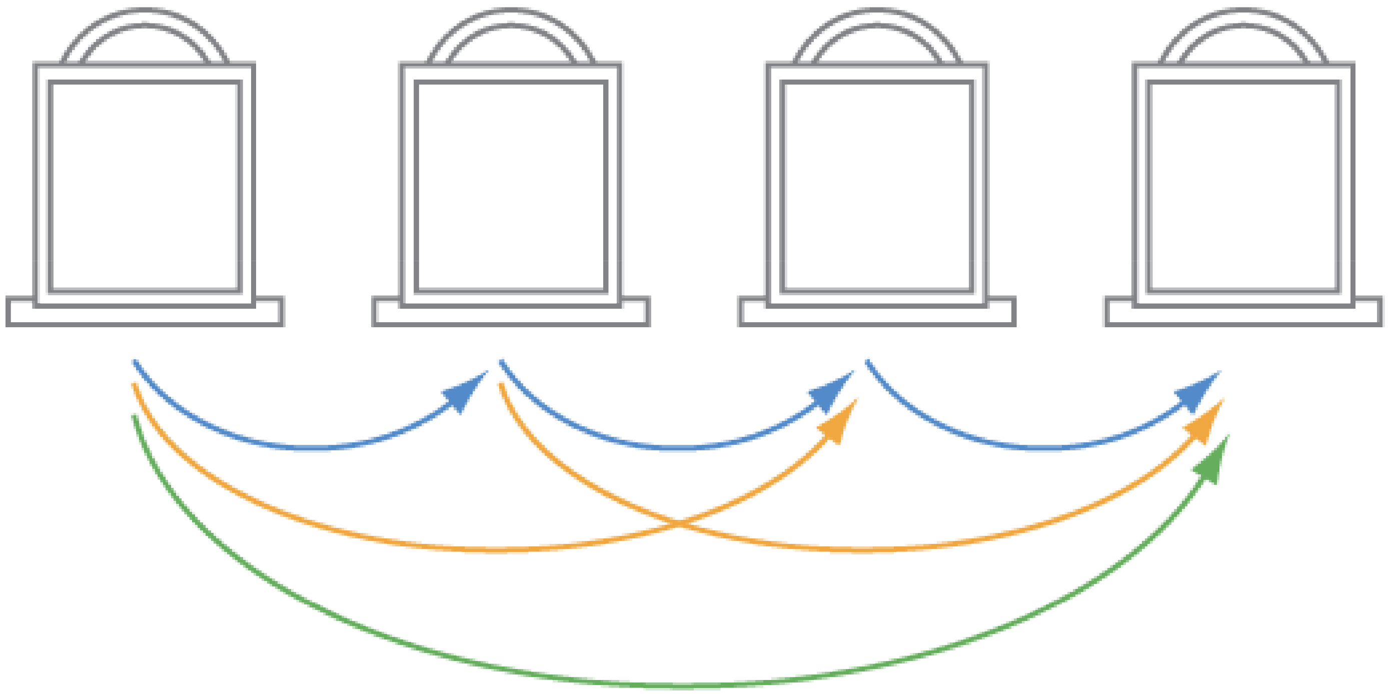

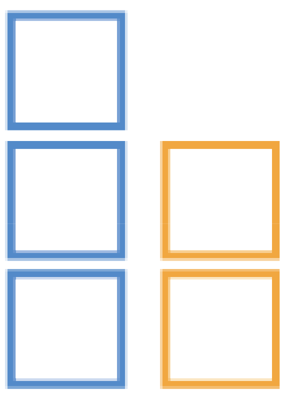

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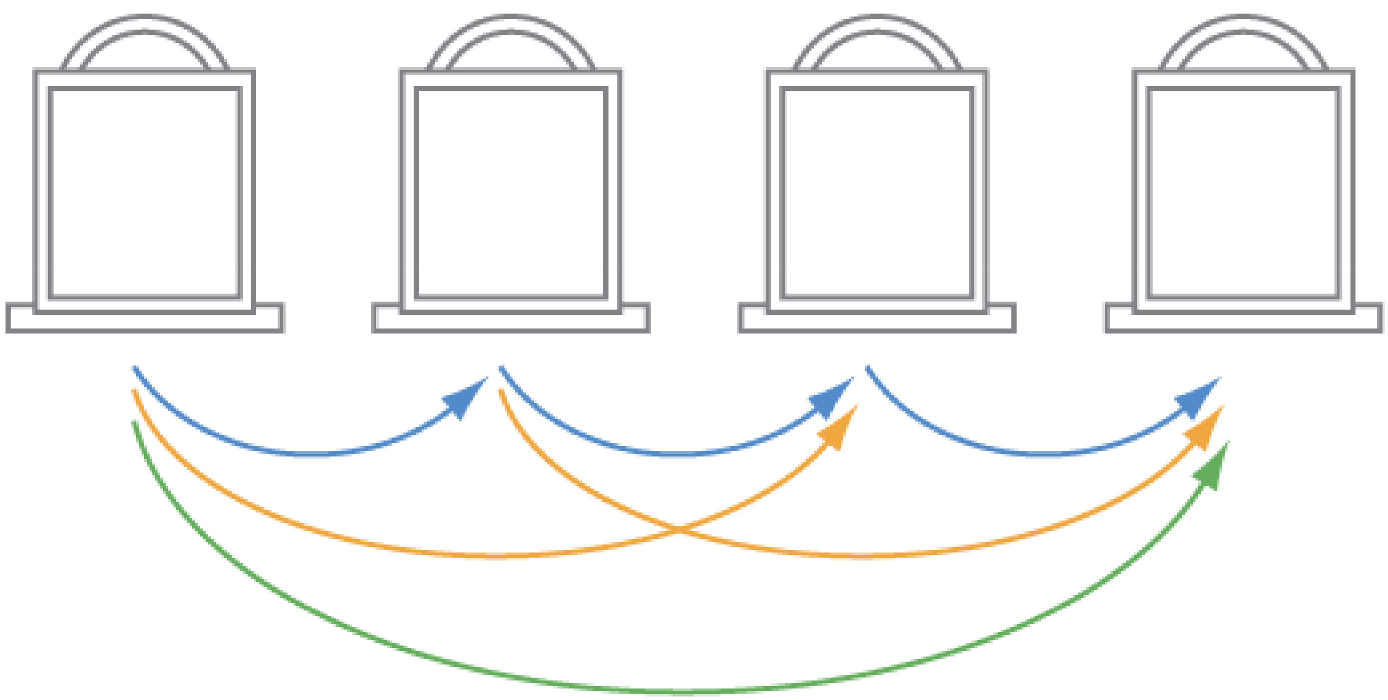

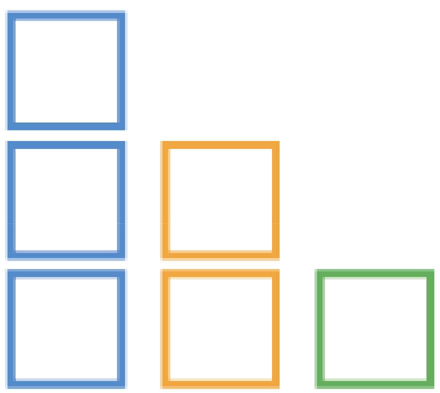

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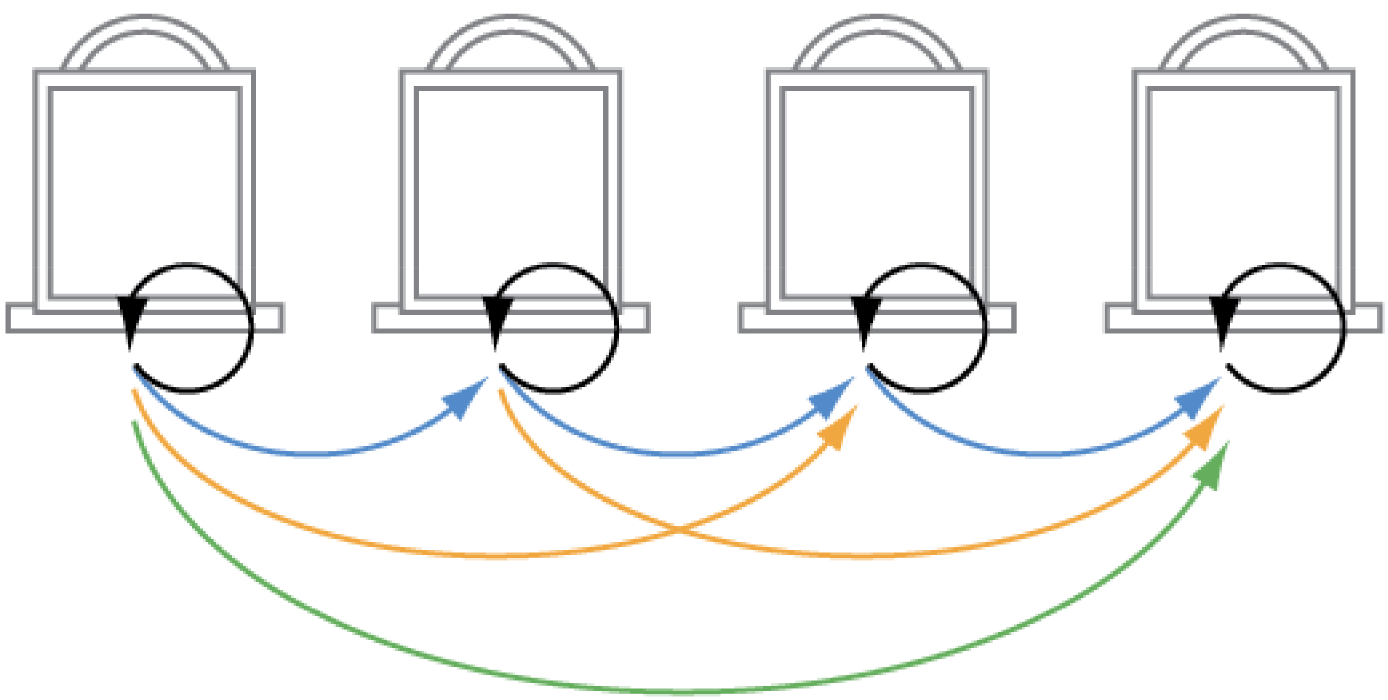

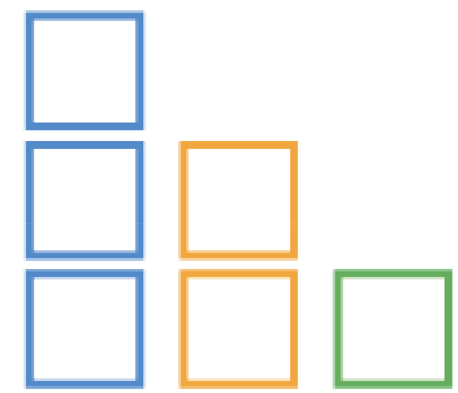

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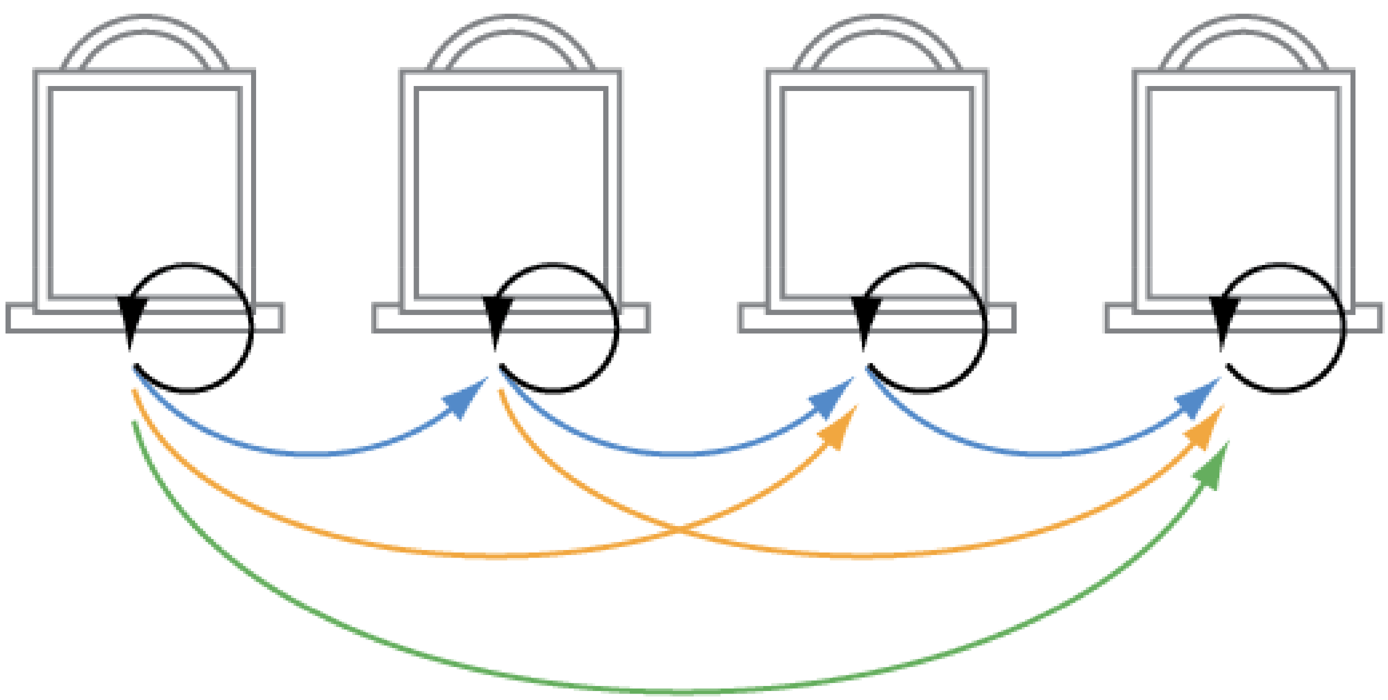

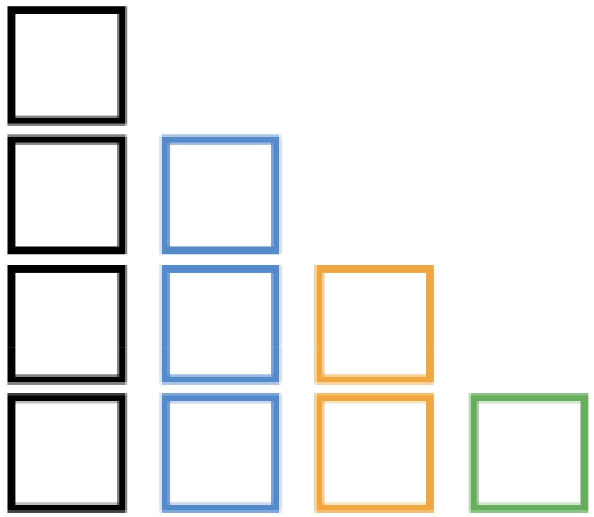

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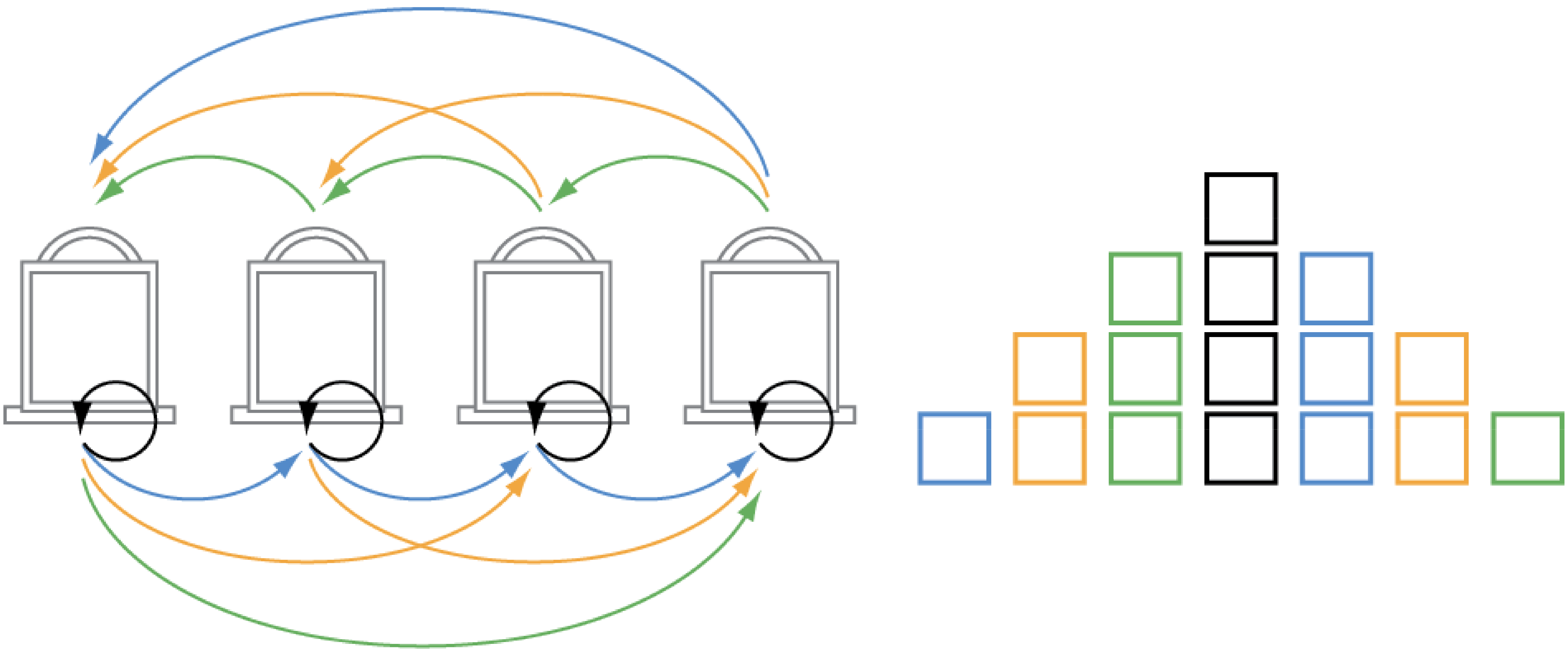

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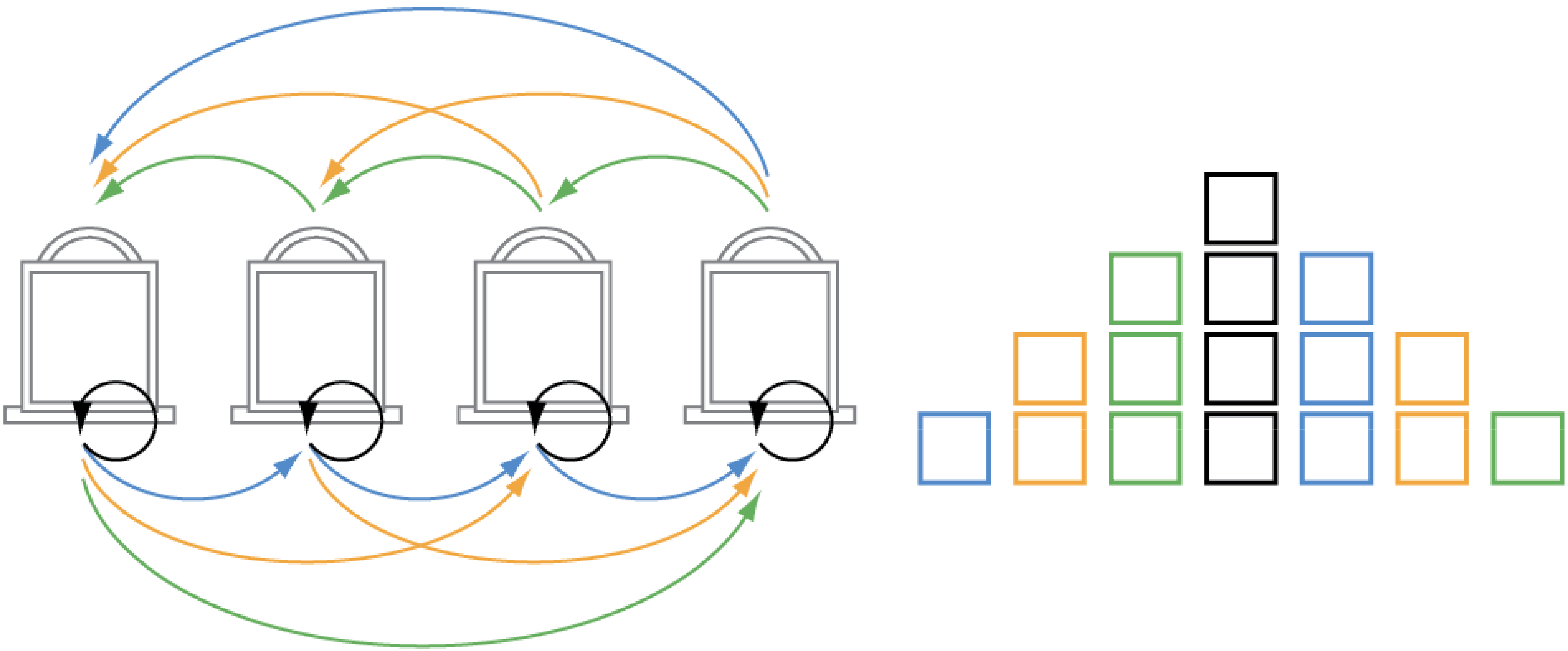
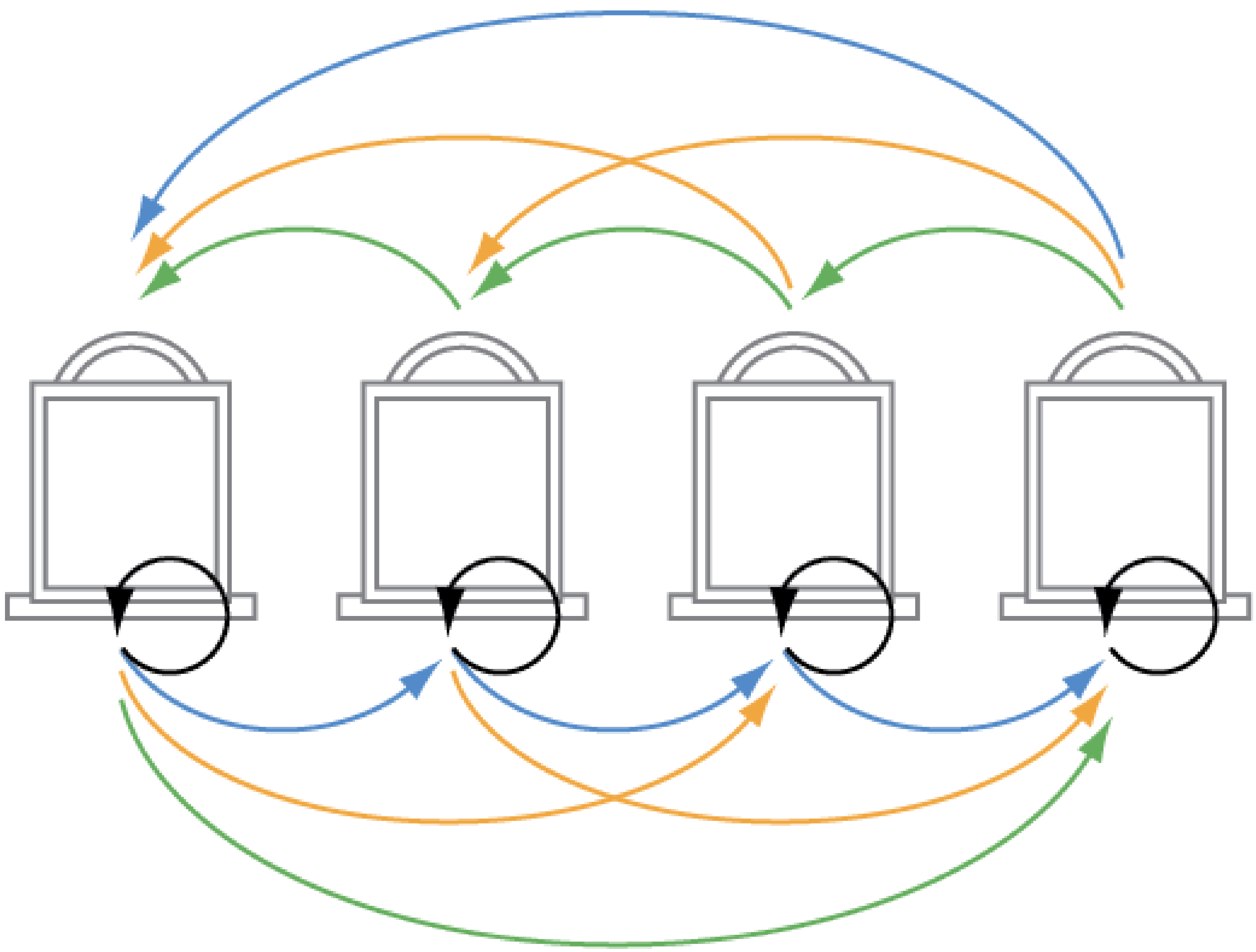
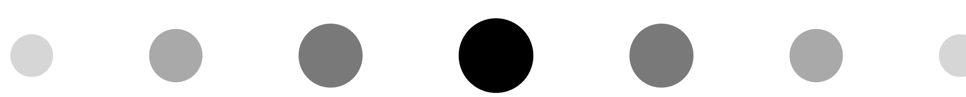


spatial domain

transformation space

Transformations

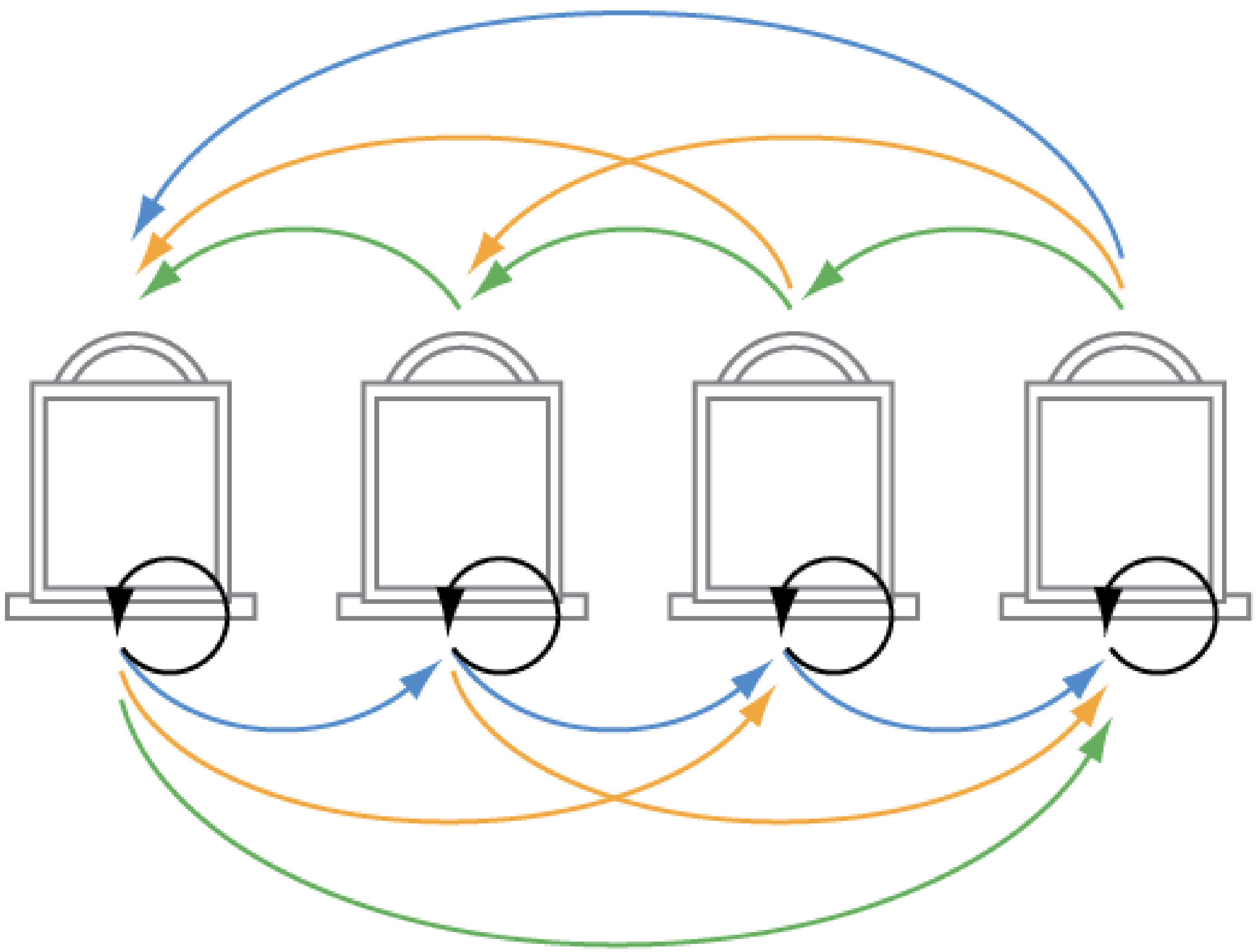




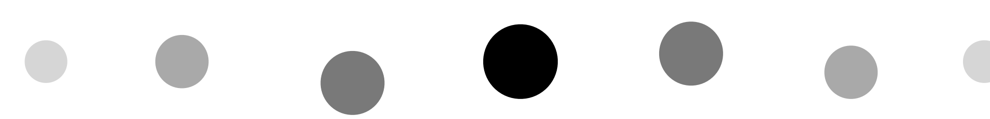
spatial domain

transformation space

Transformations

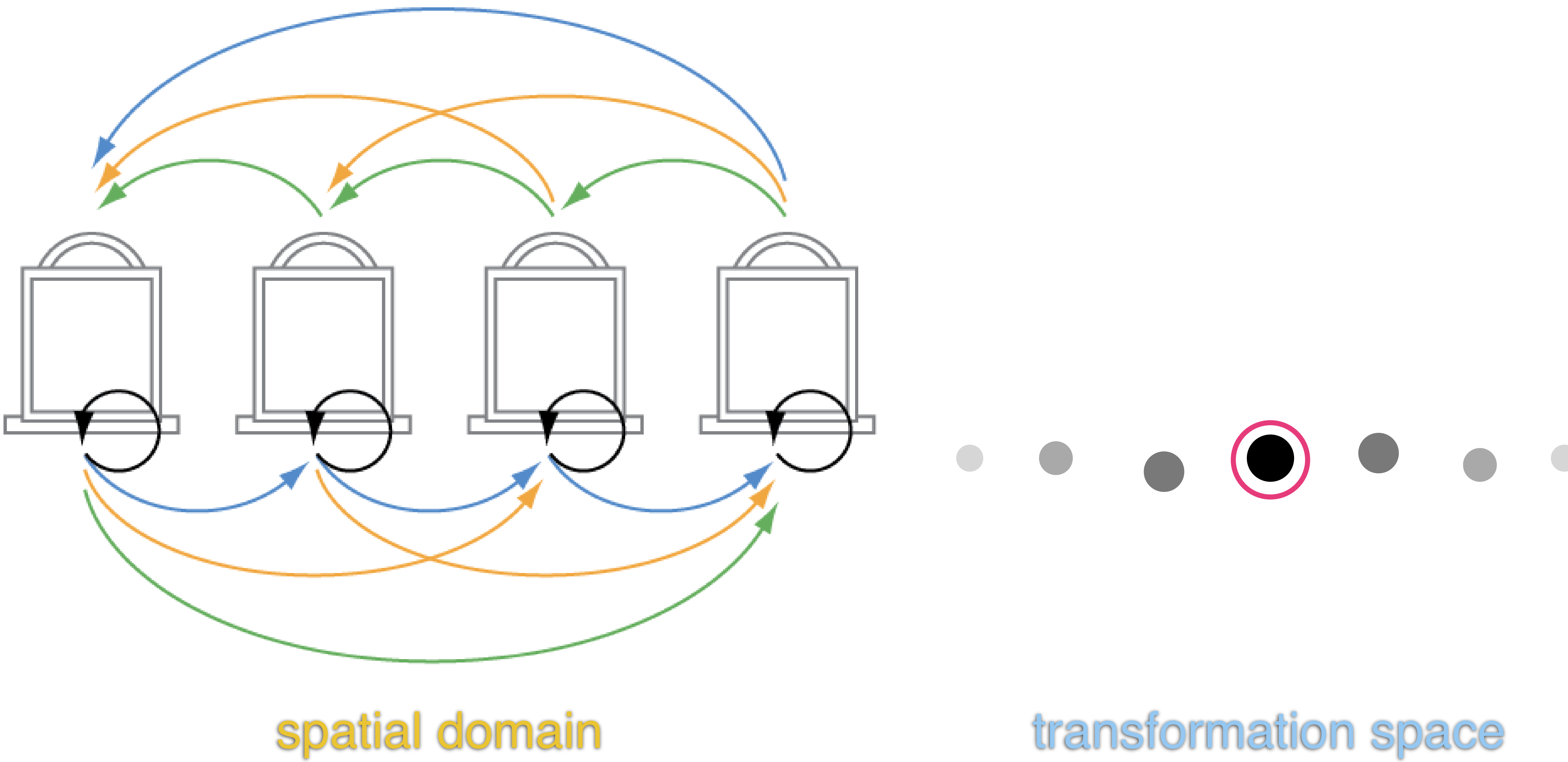


spatial domain

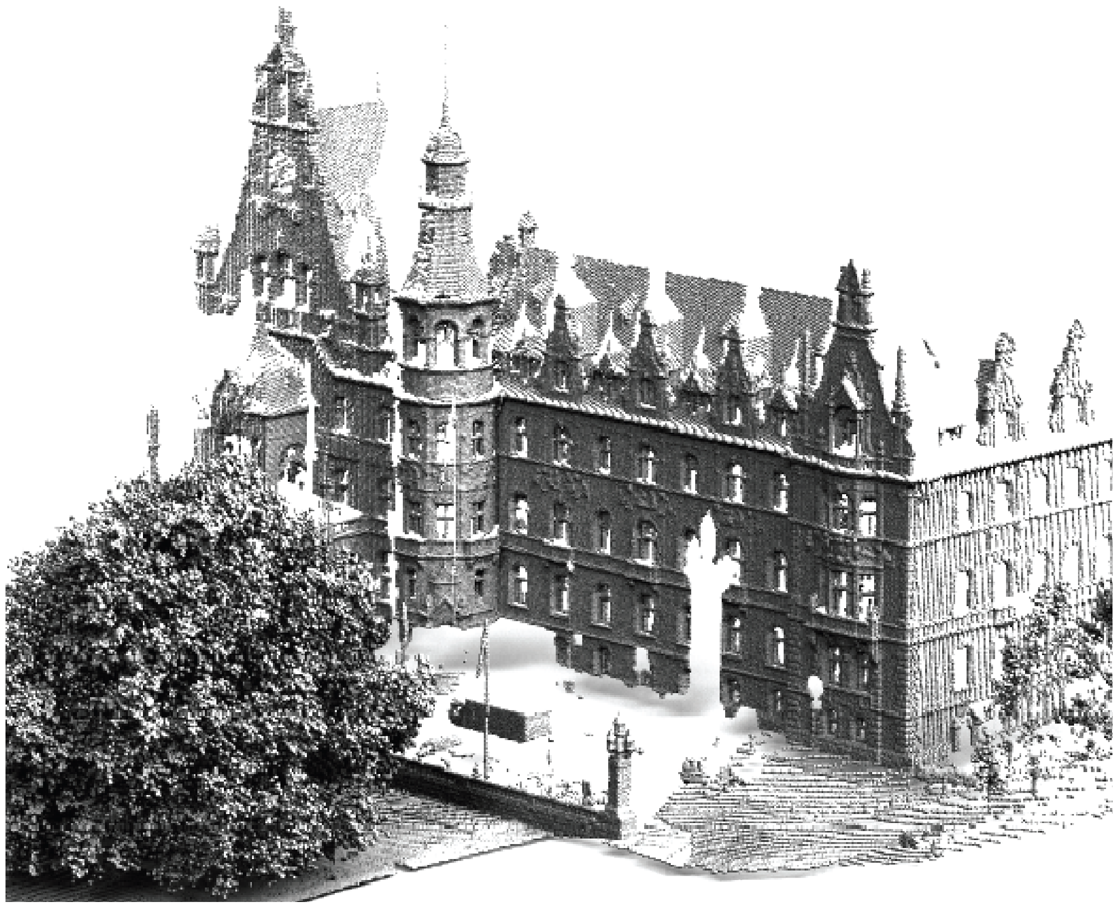


transformation space

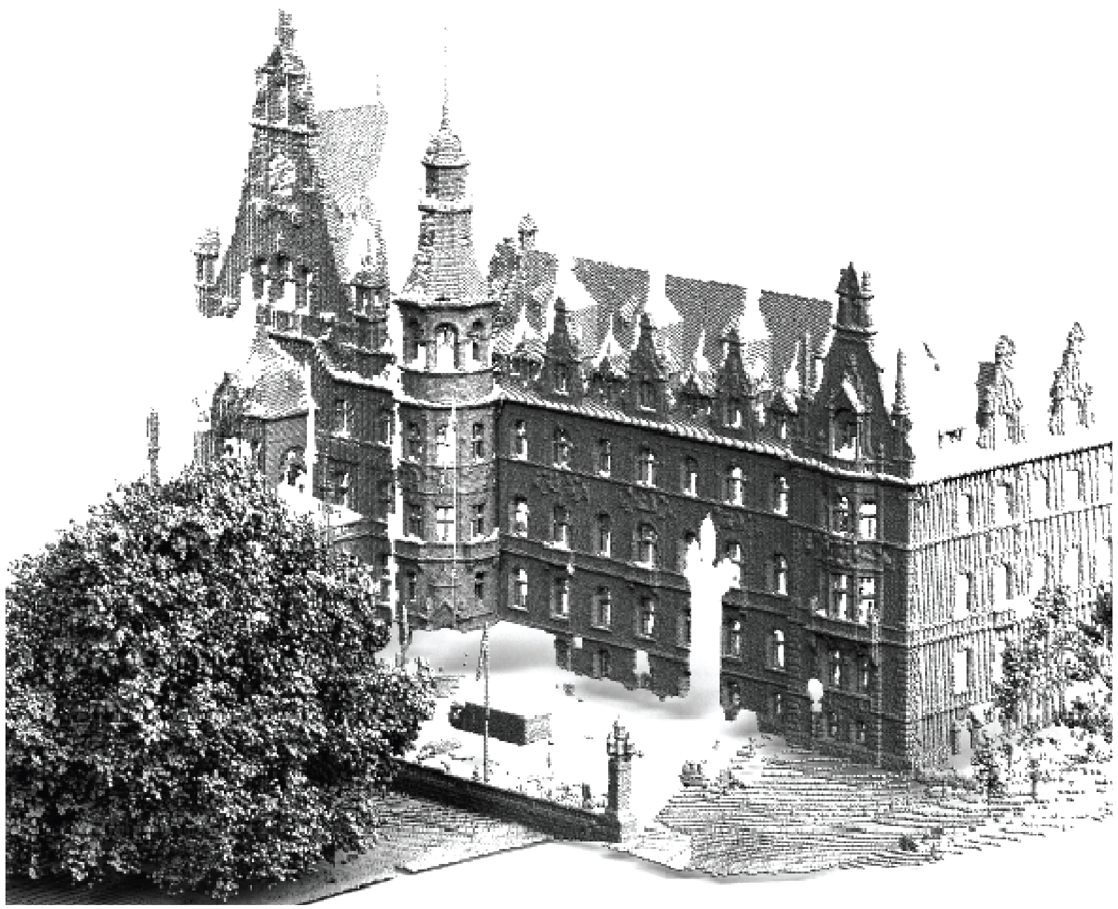
Transformations

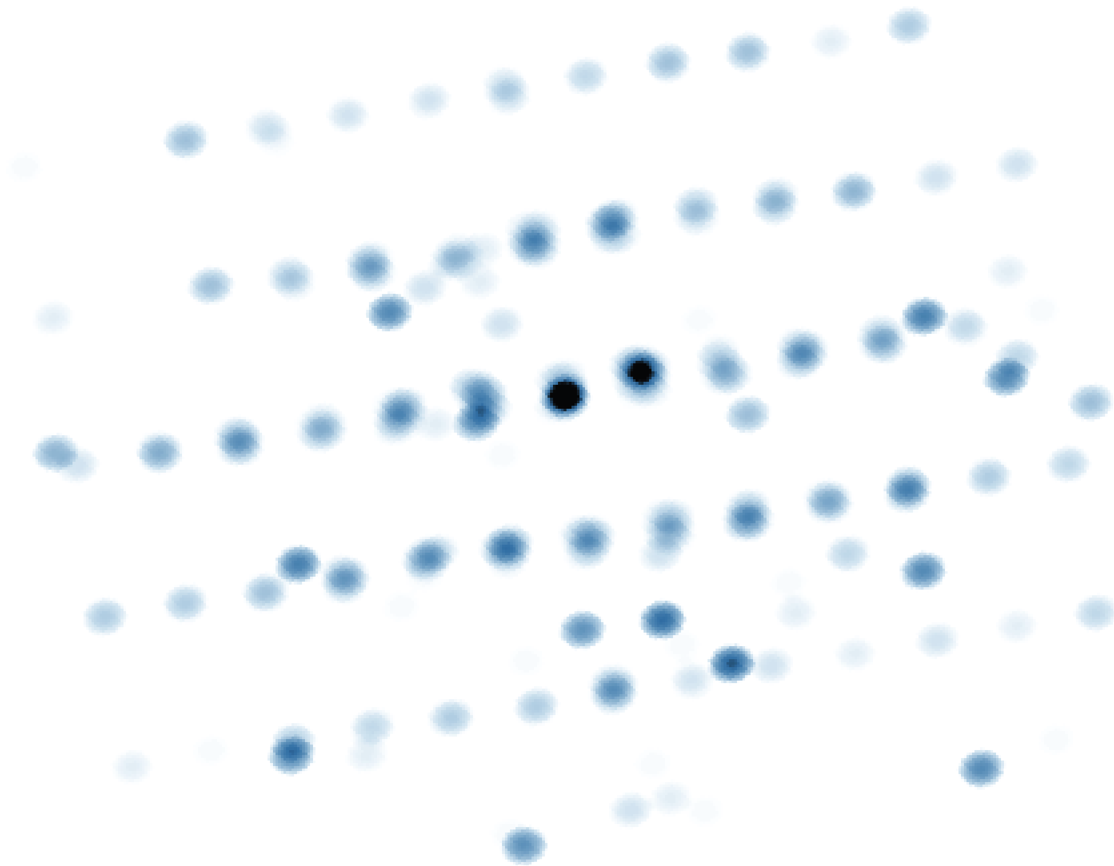


Model Estimation



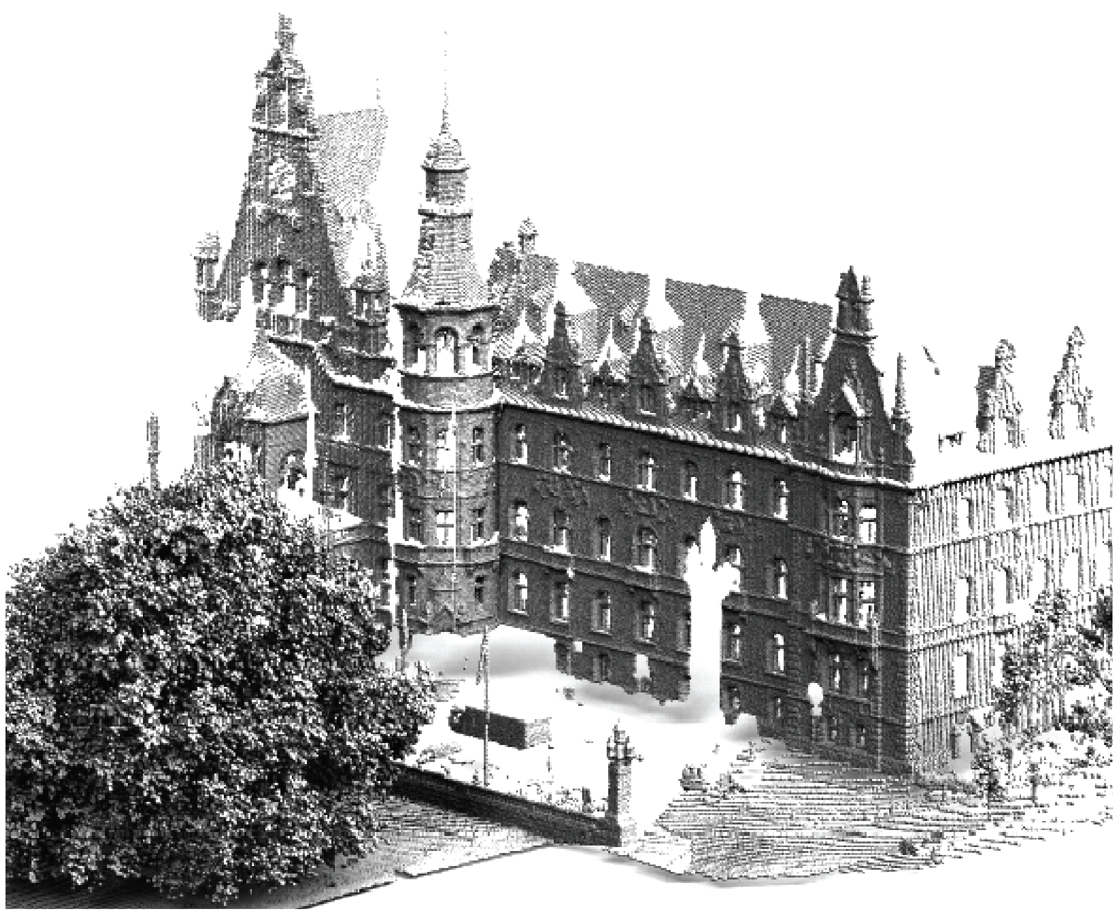
Model Estimation

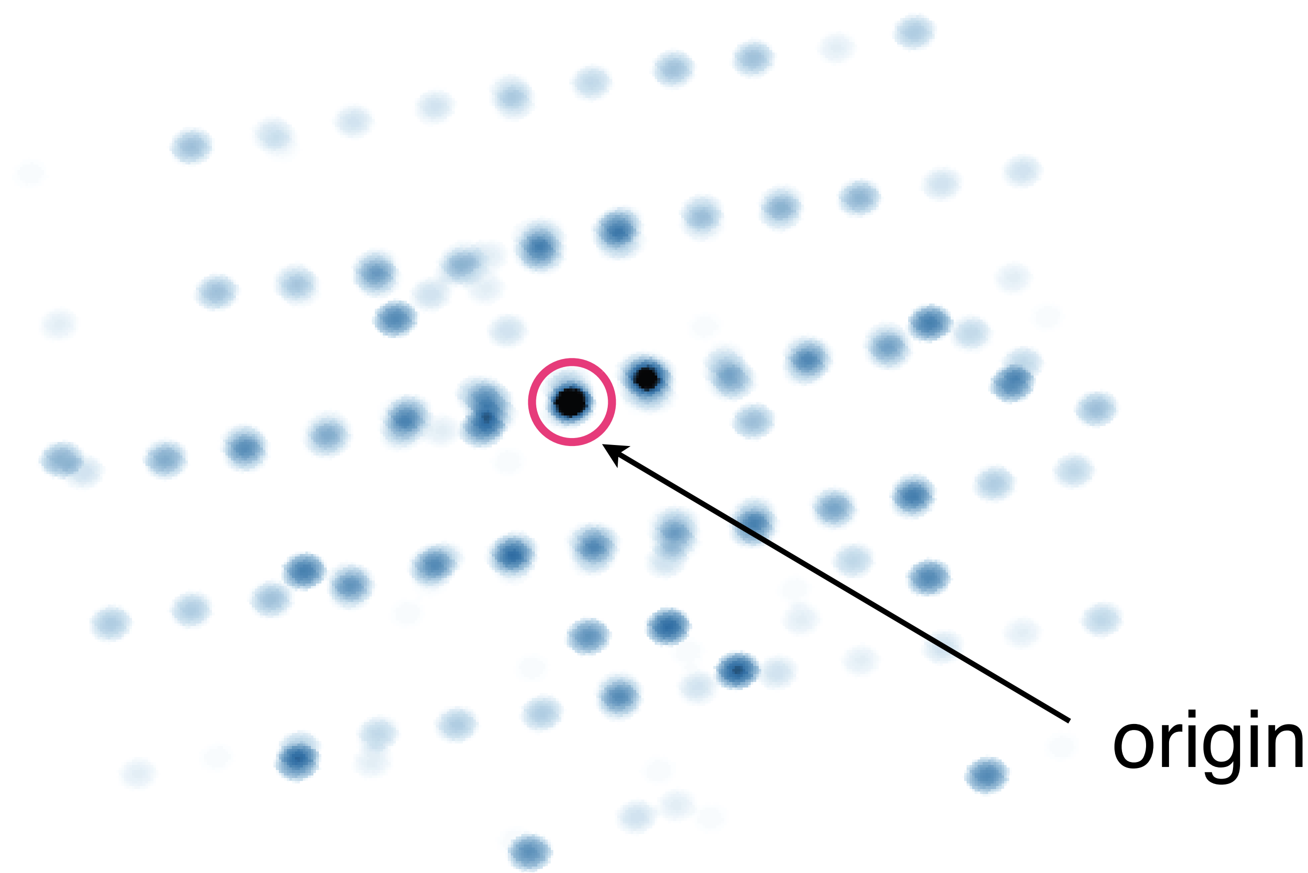




density plot of
pair-wise transformations

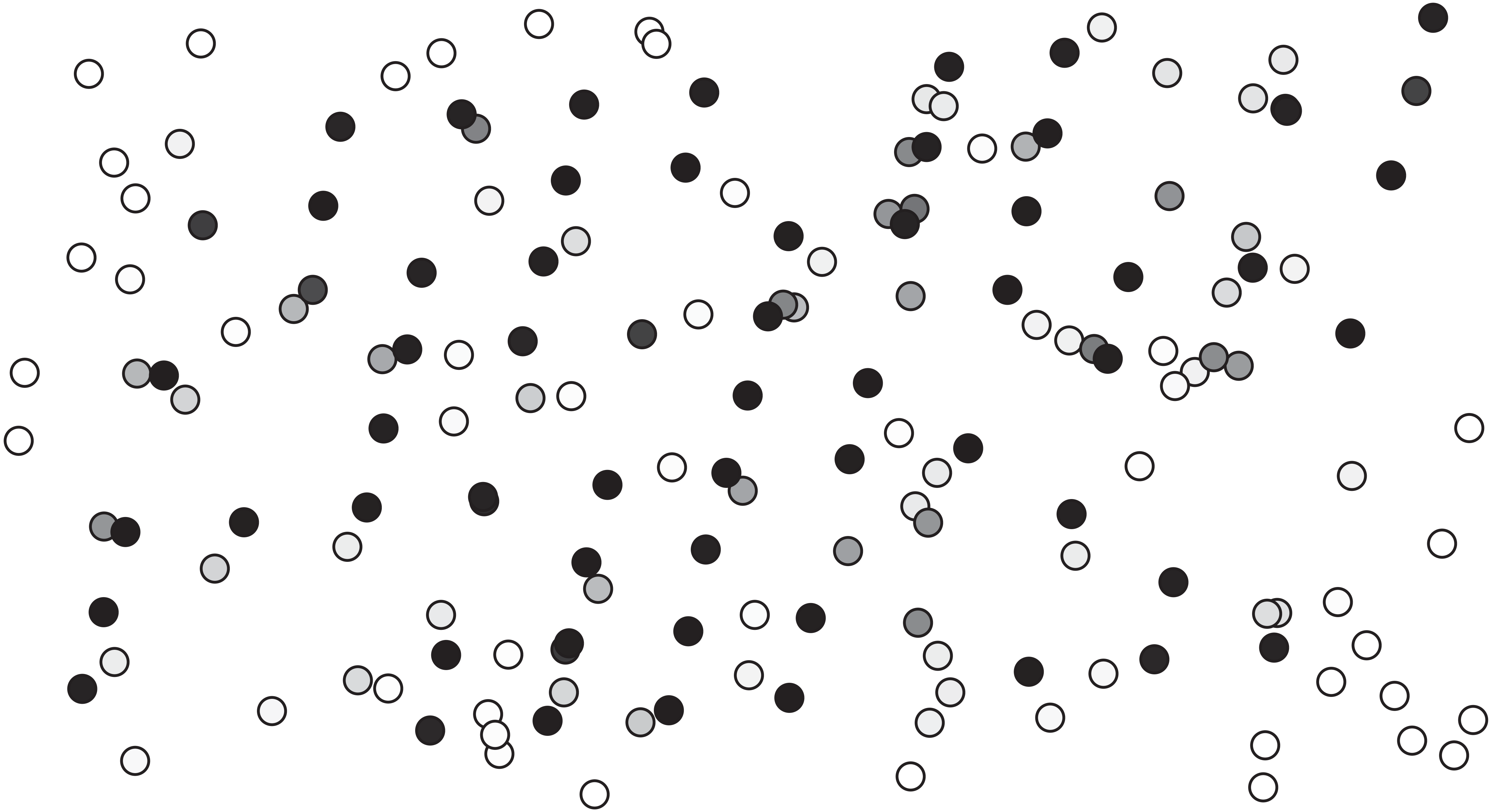
Model Estimation



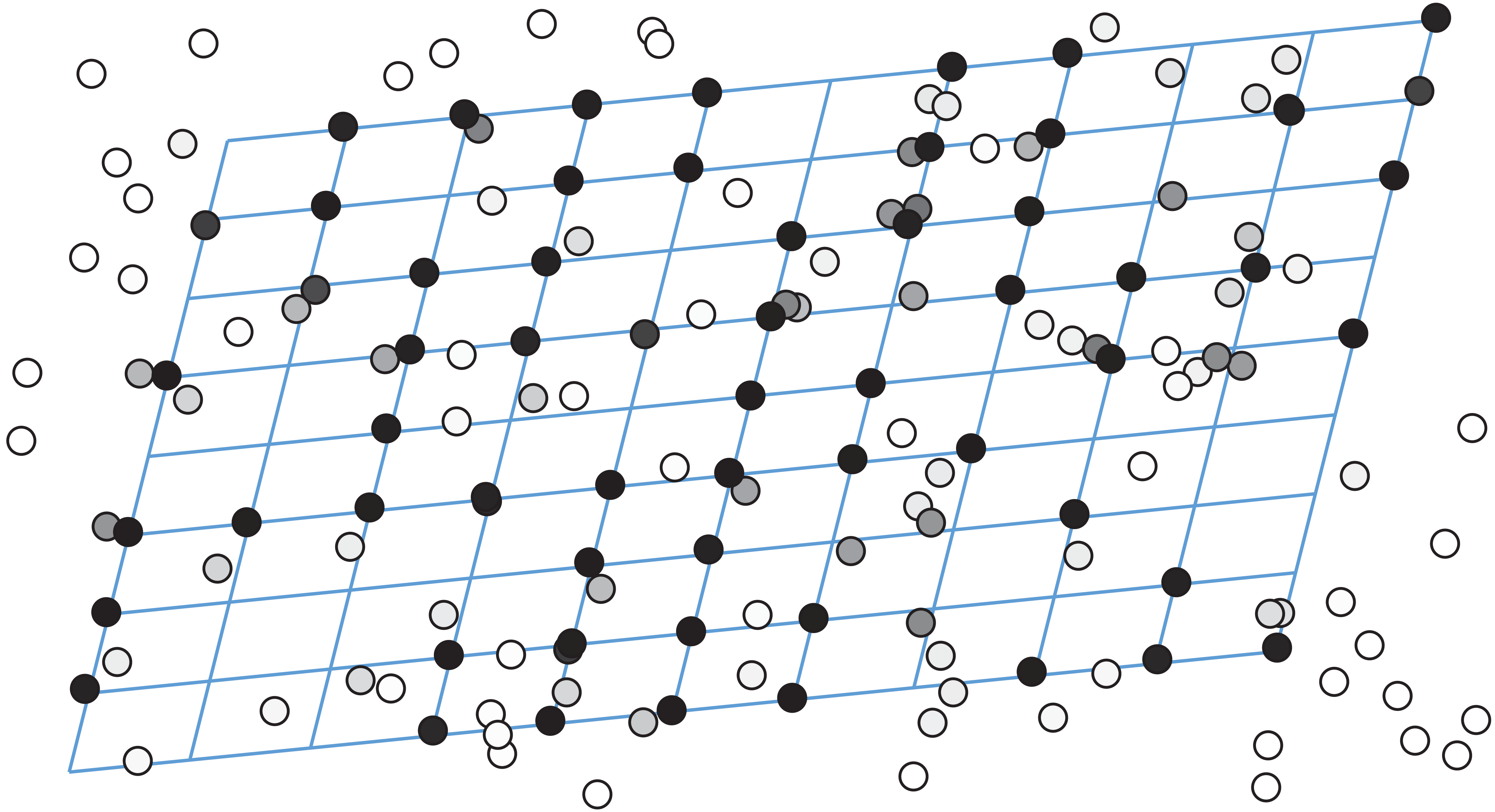


density plot of
pair-wise transformations

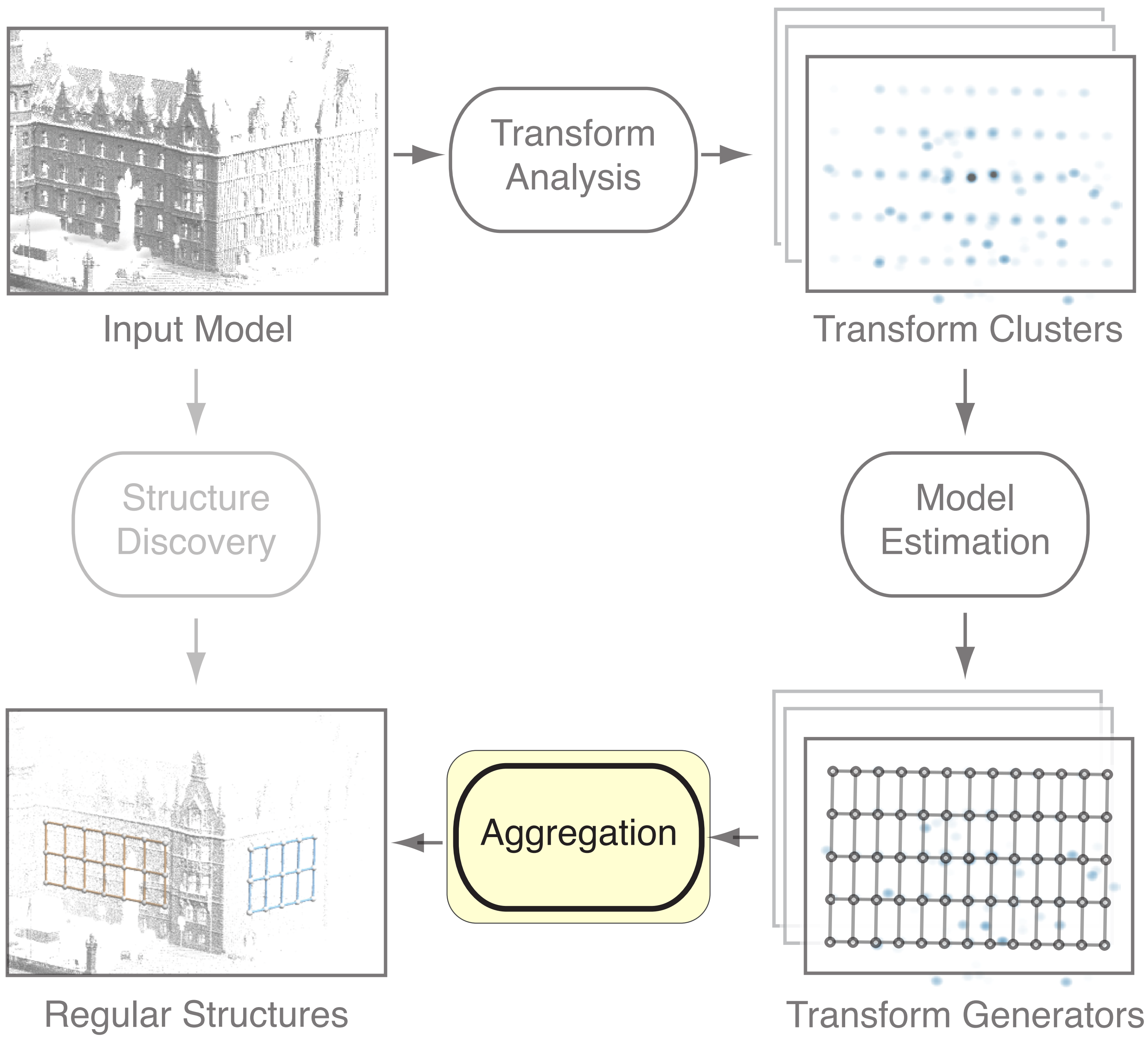
Optimization in Transform Domain



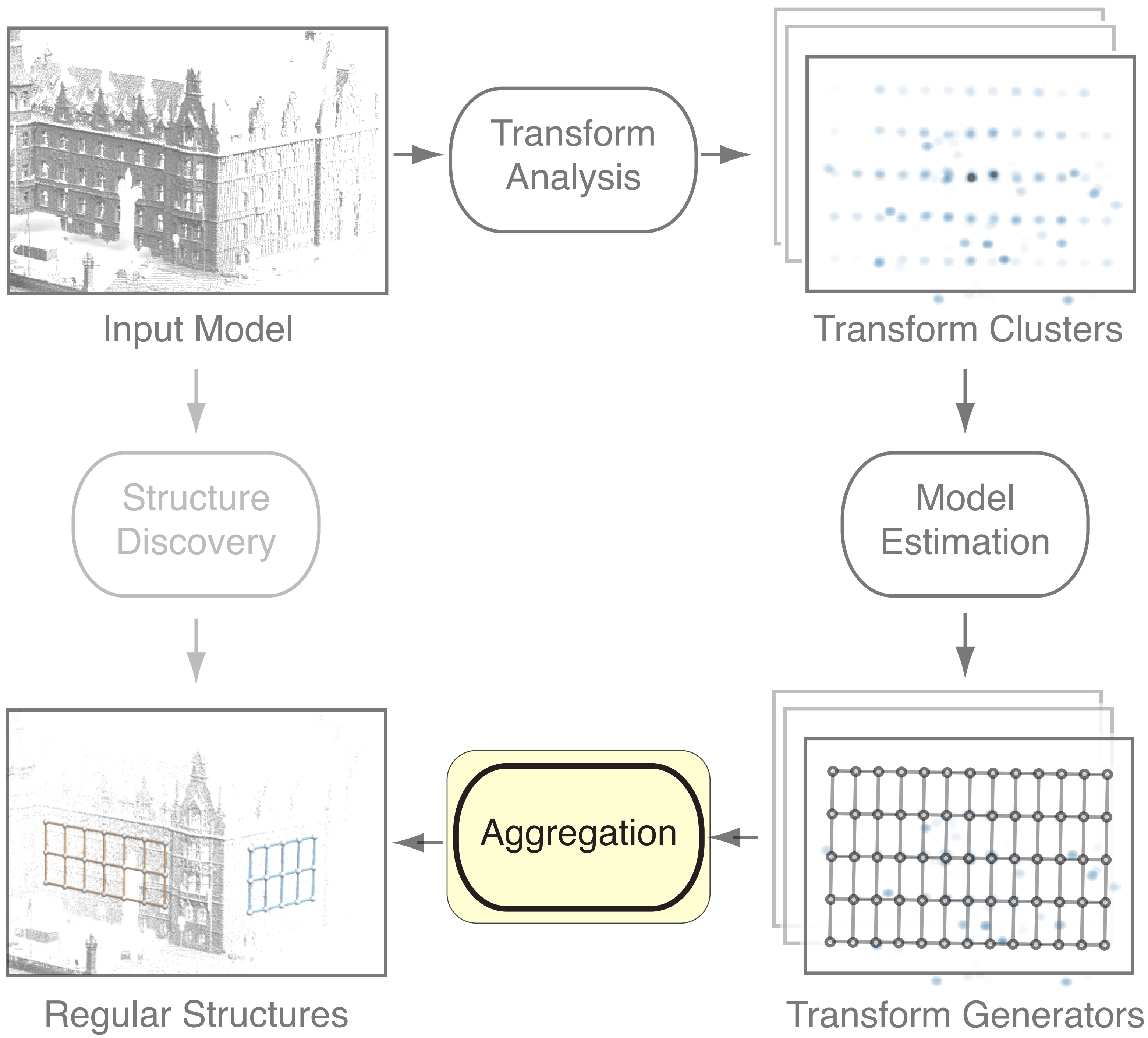
Optimization in Transform Domain



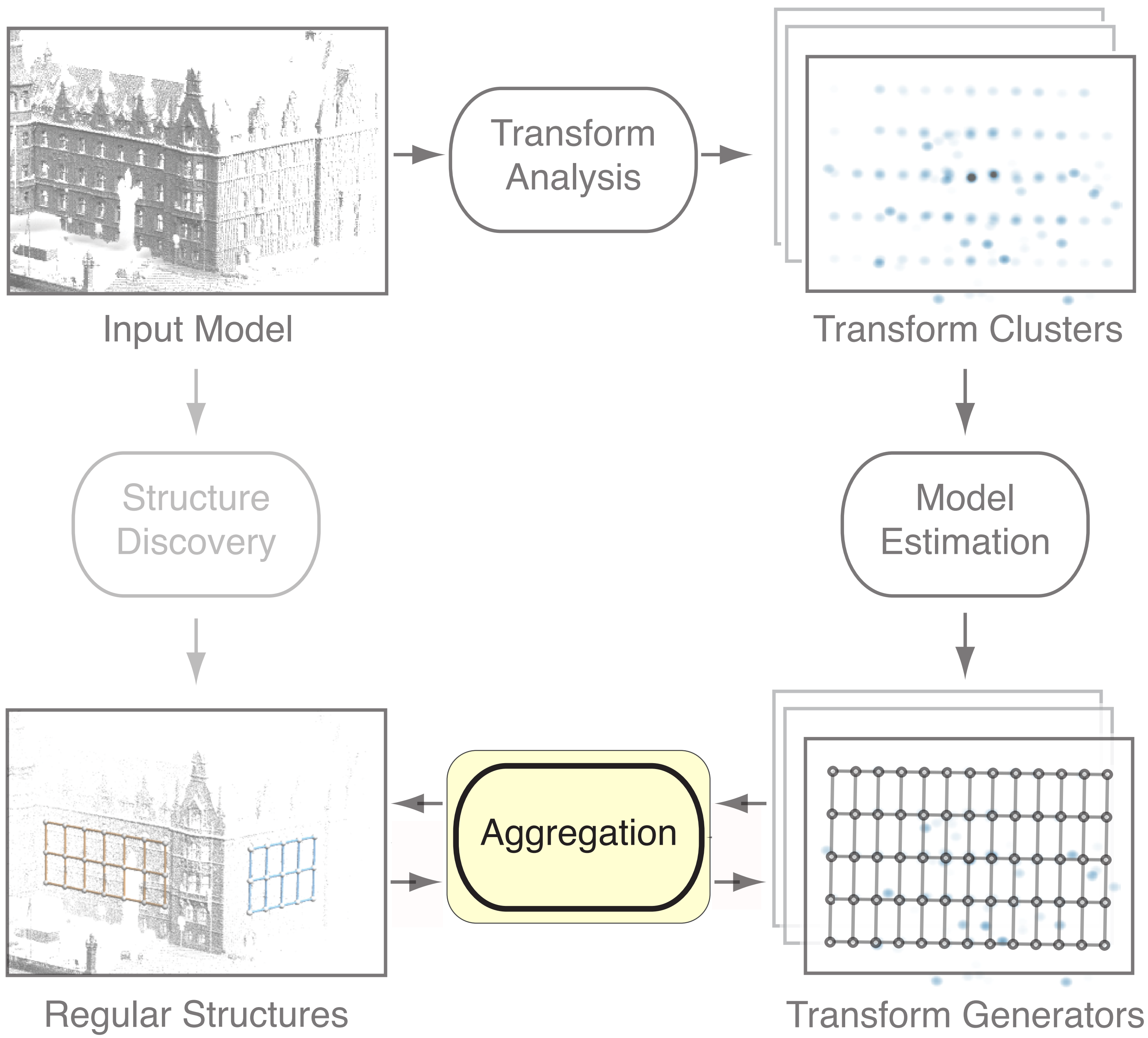
Structure Discovery



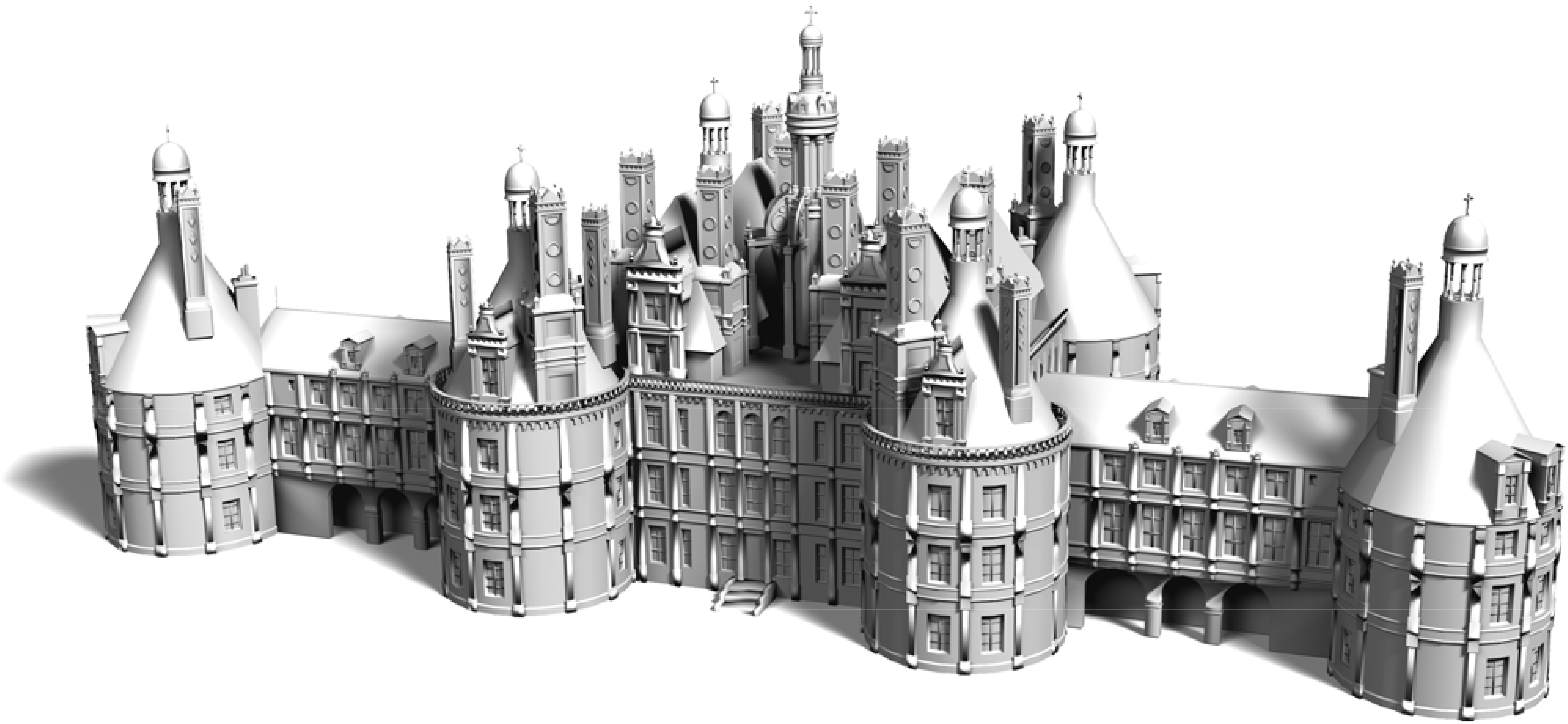
Structure Discovery



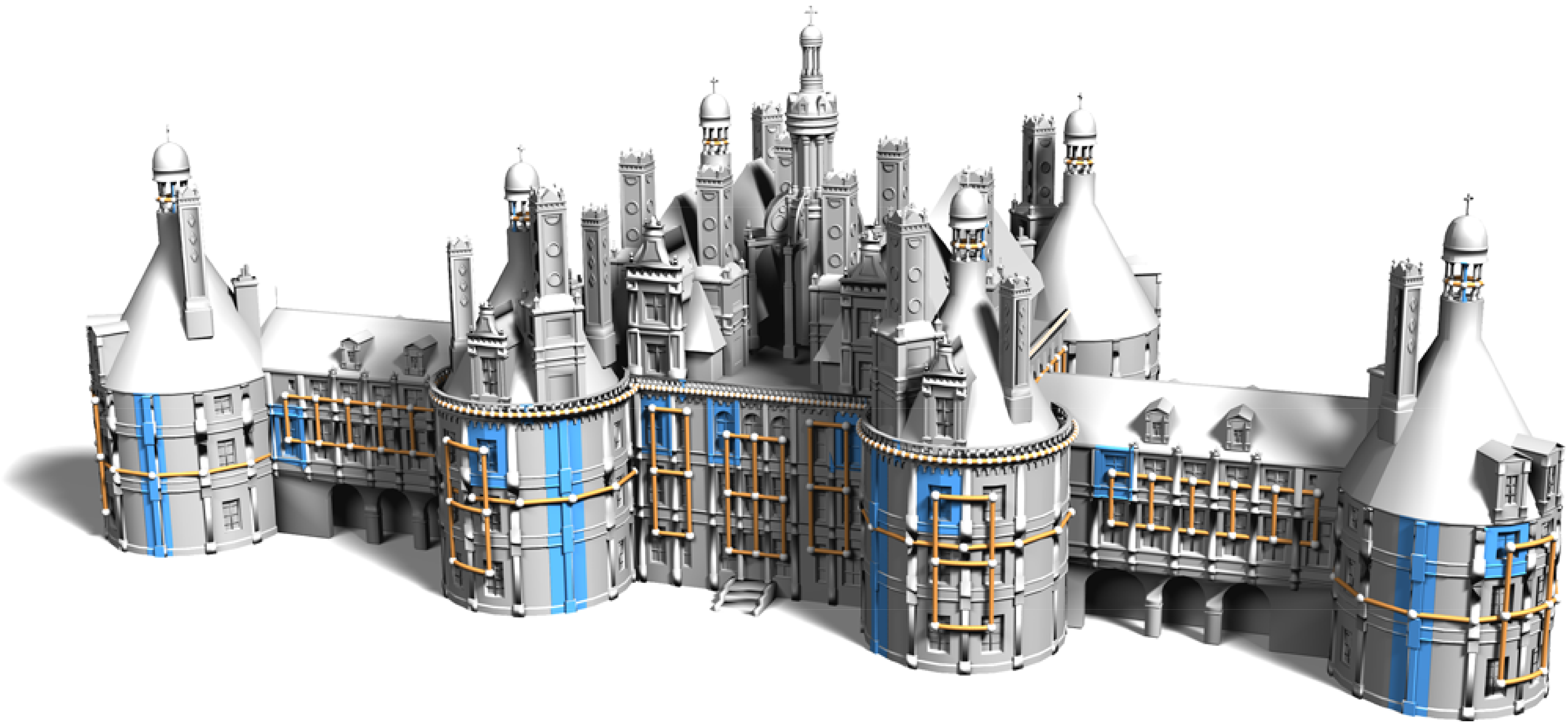
Structure Discovery



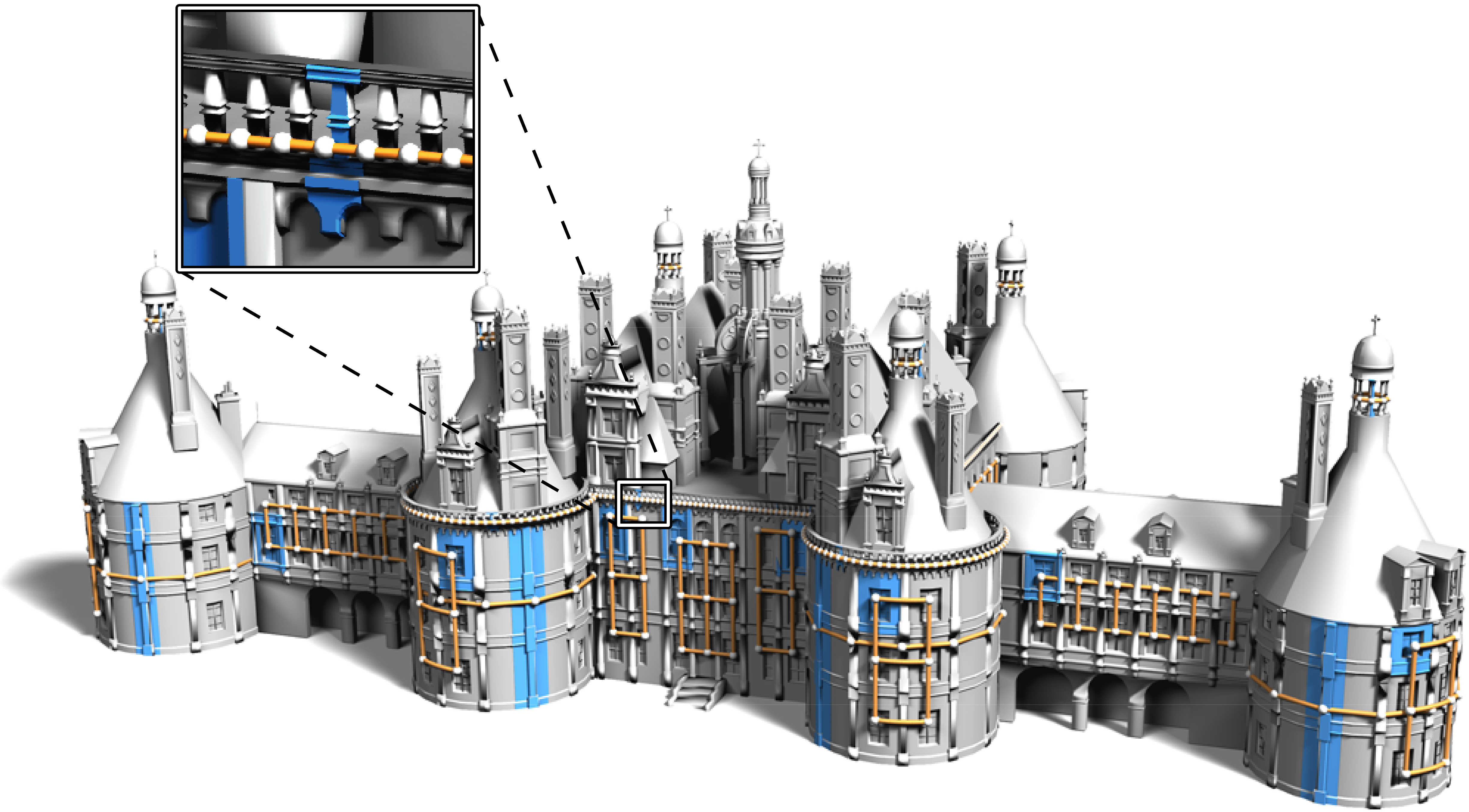
Chambord Castle



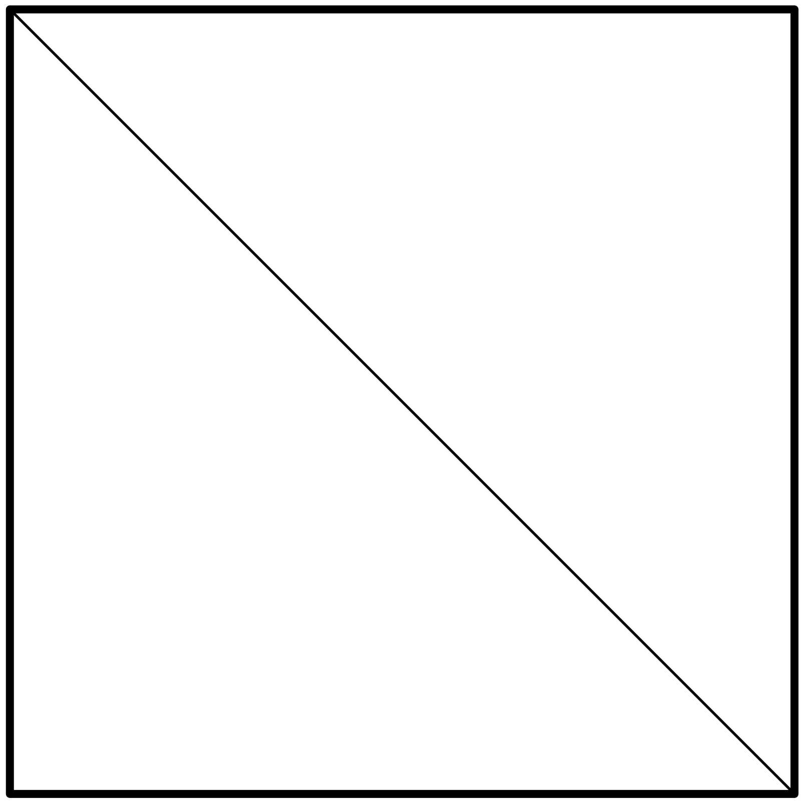
Chambord Castle



Chambord Castle

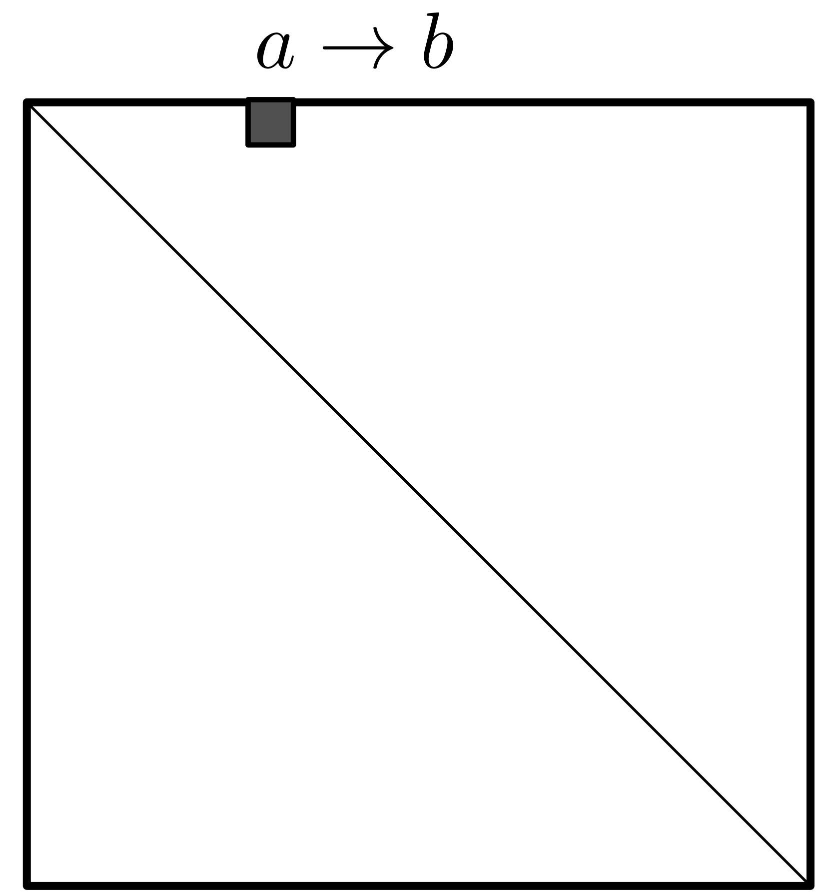


Symmetry in Correspondence Space



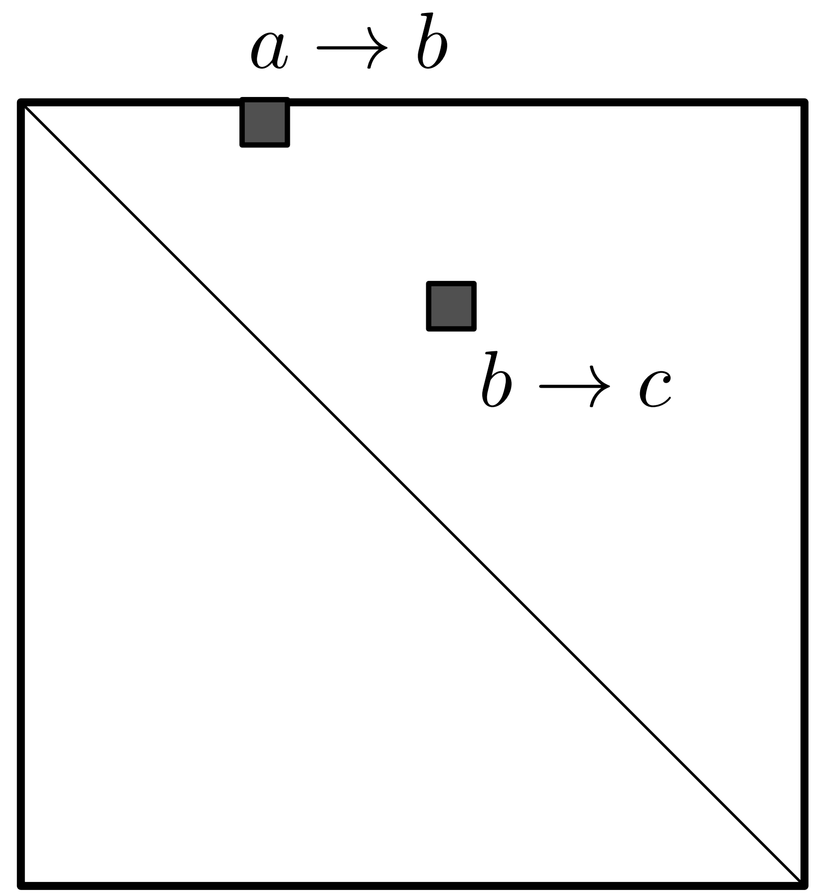
[Lipman et al. 2009]

Symmetry in Correspondence Space



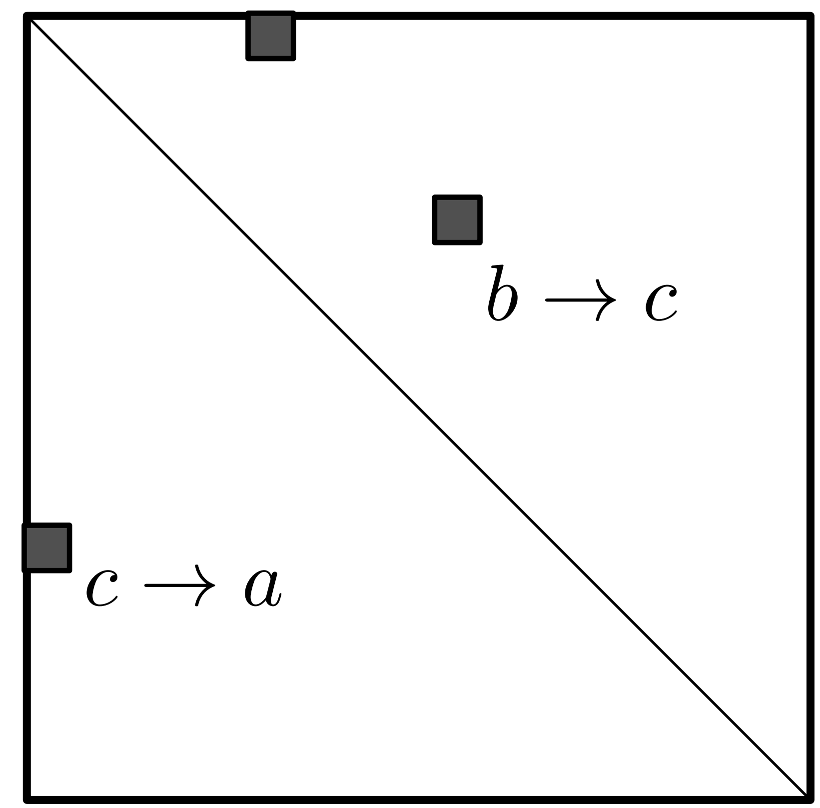
[Lipman et al. 2009]

Symmetry in Correspondence Space



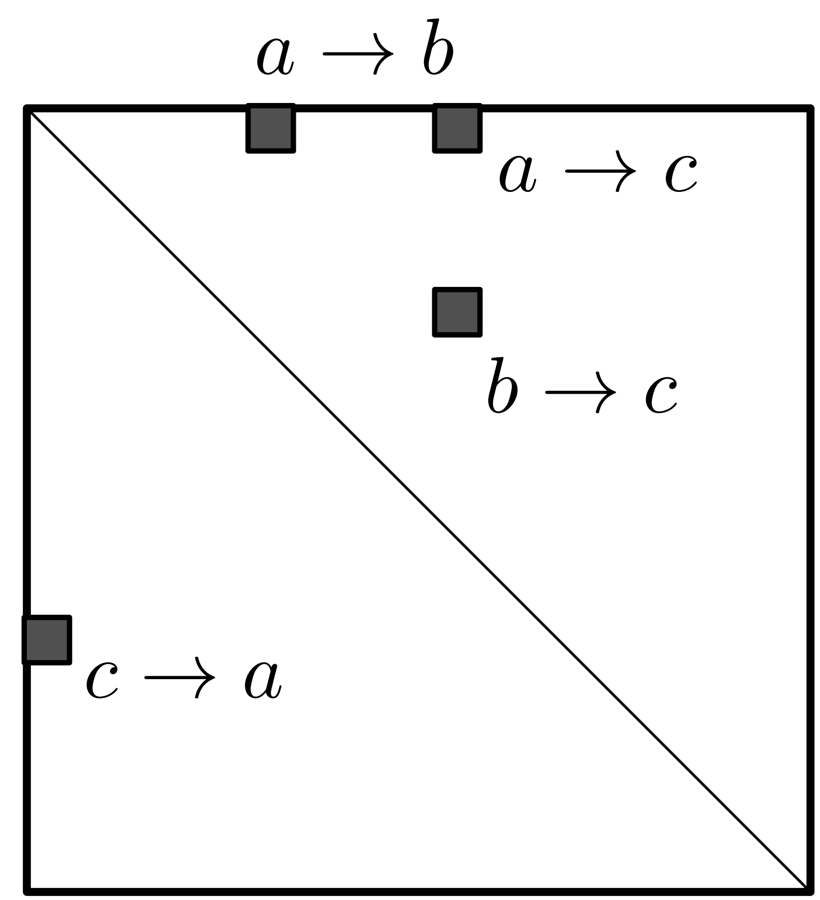
[Lipman et al. 2009]

Symmetry in Correspondence Space



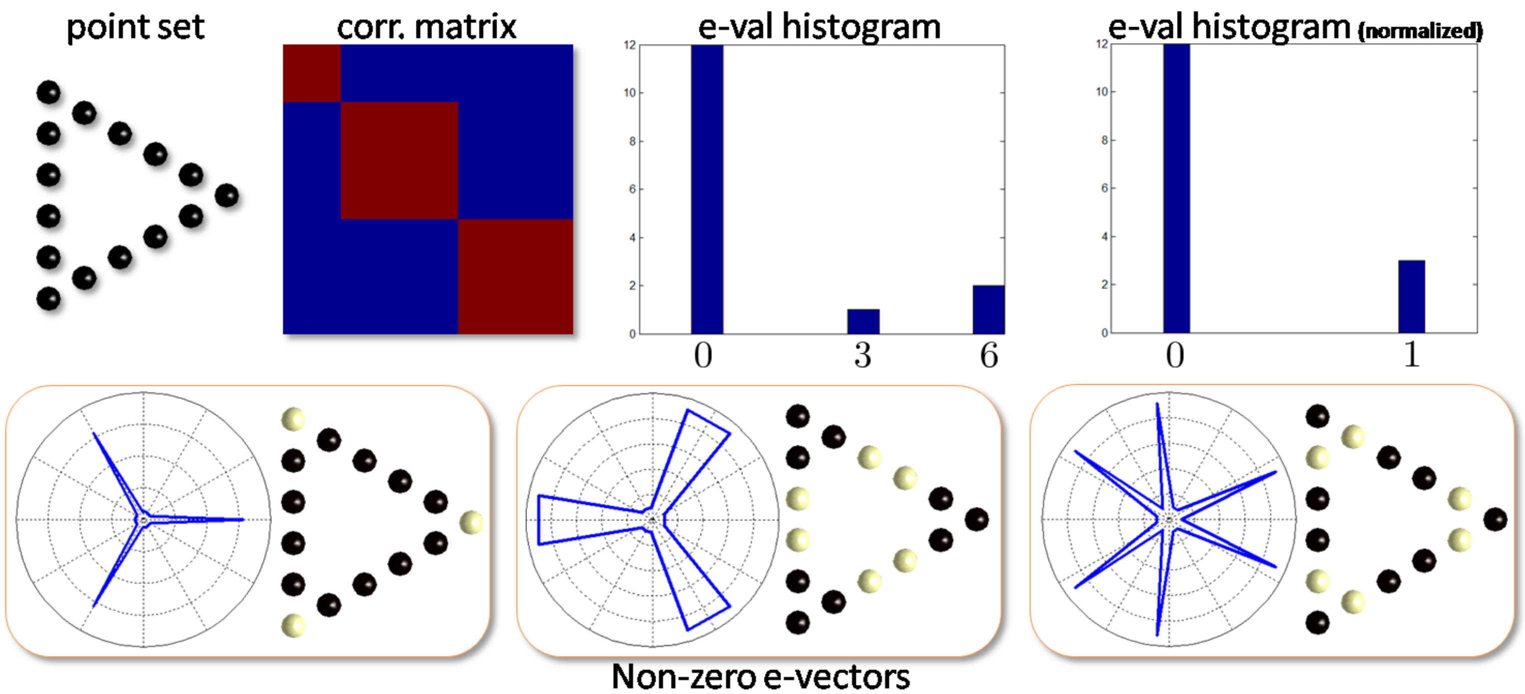
[Lipman et al. 2009]

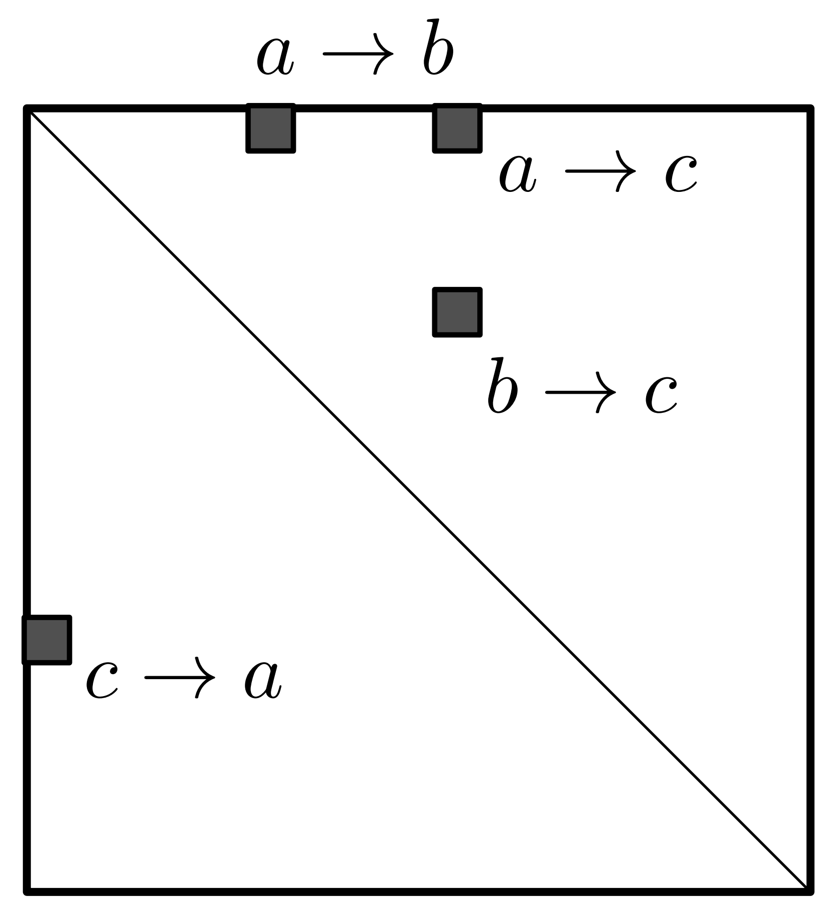
Symmetry in Correspondence Space



[Lipman et al. 2009]

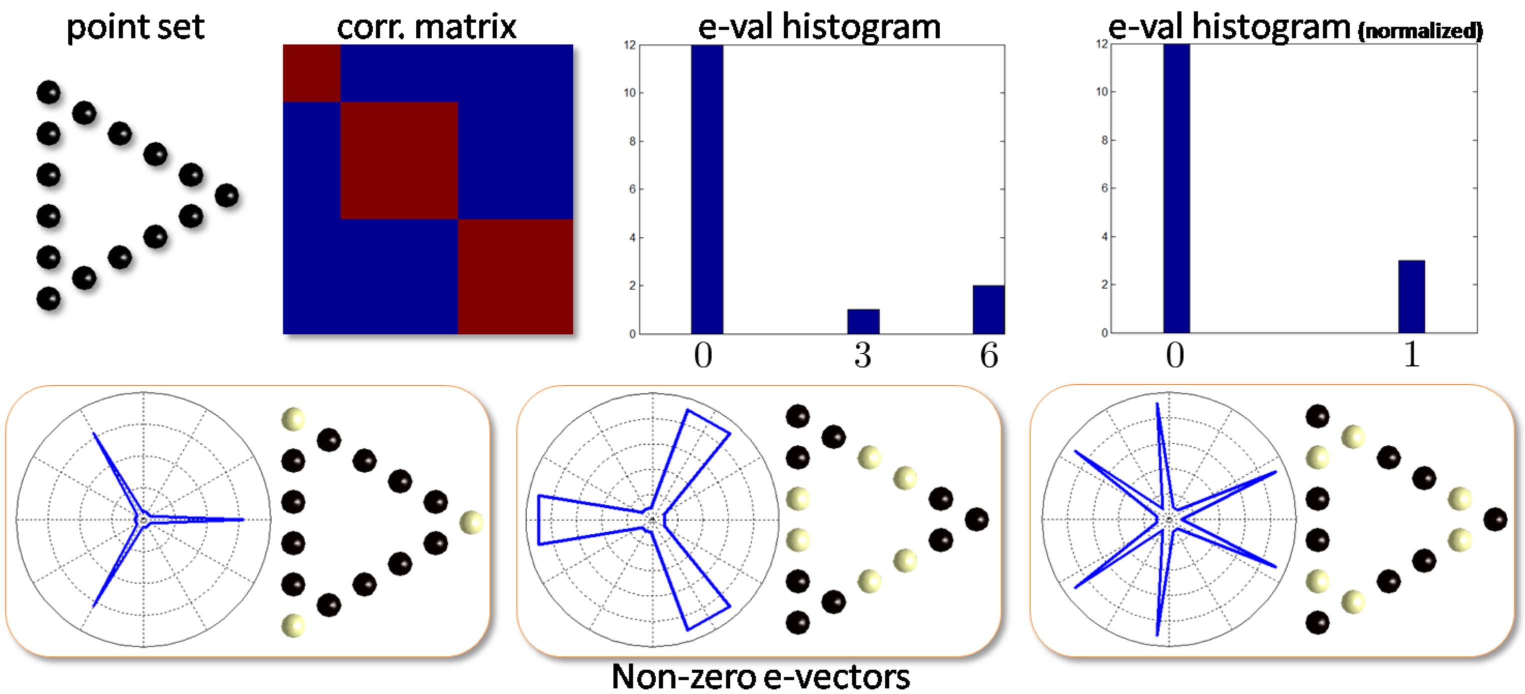
Symmetry in Correspondence Space

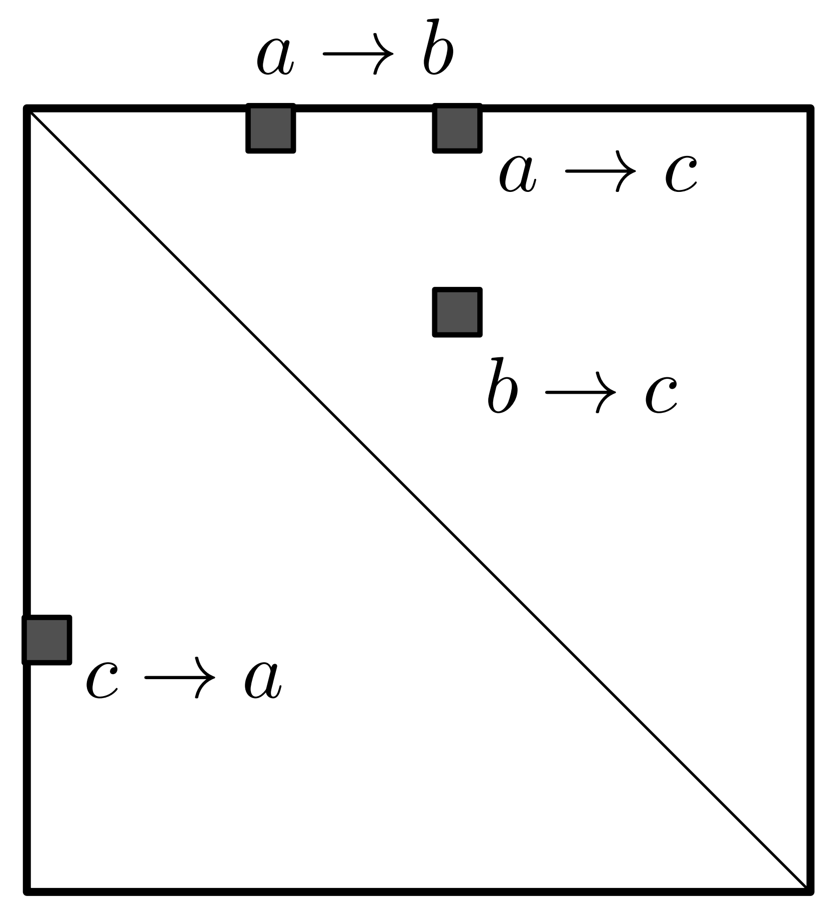




[Lipman et al. 2009]

Symmetry in Correspondence Space

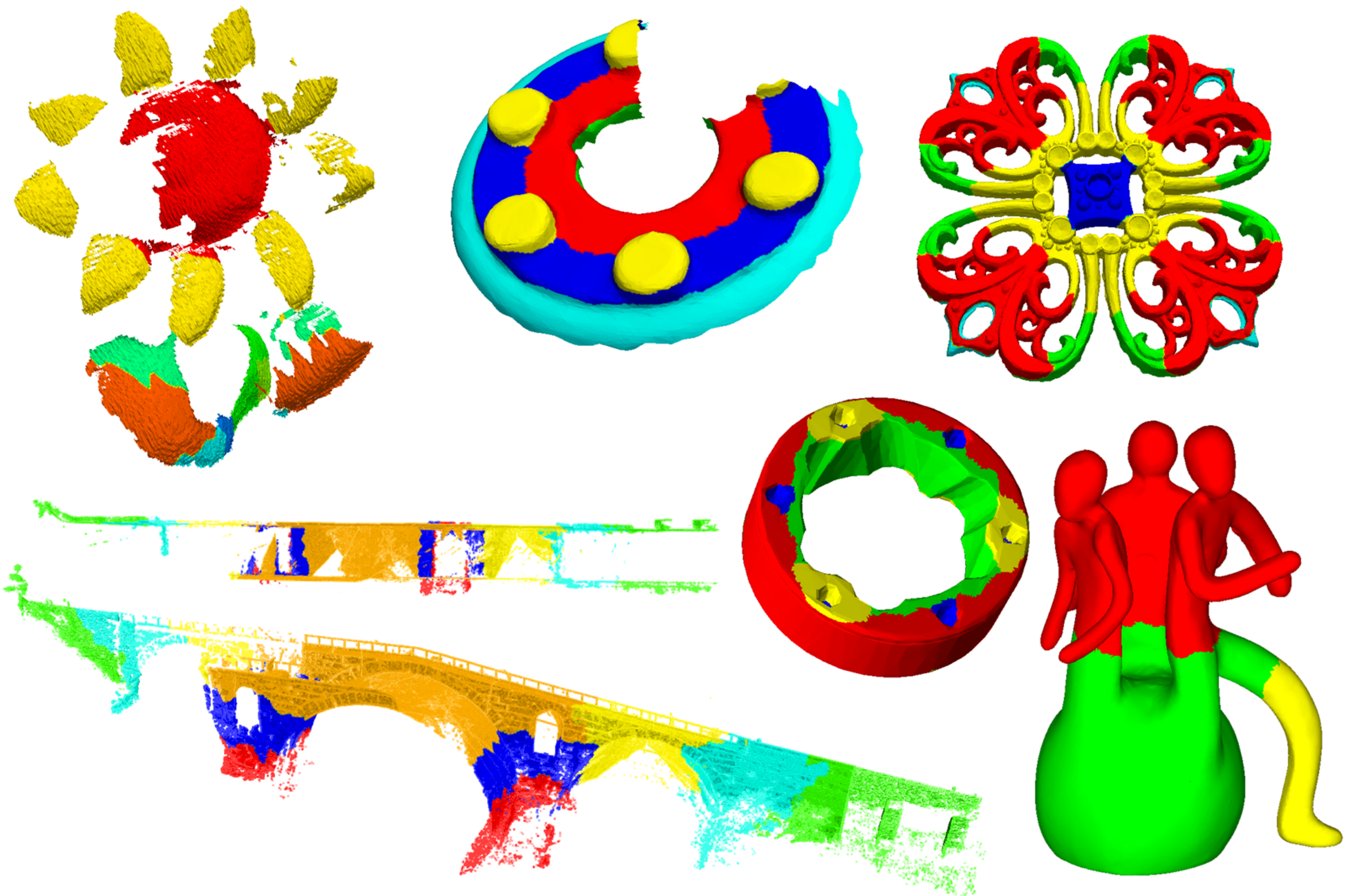




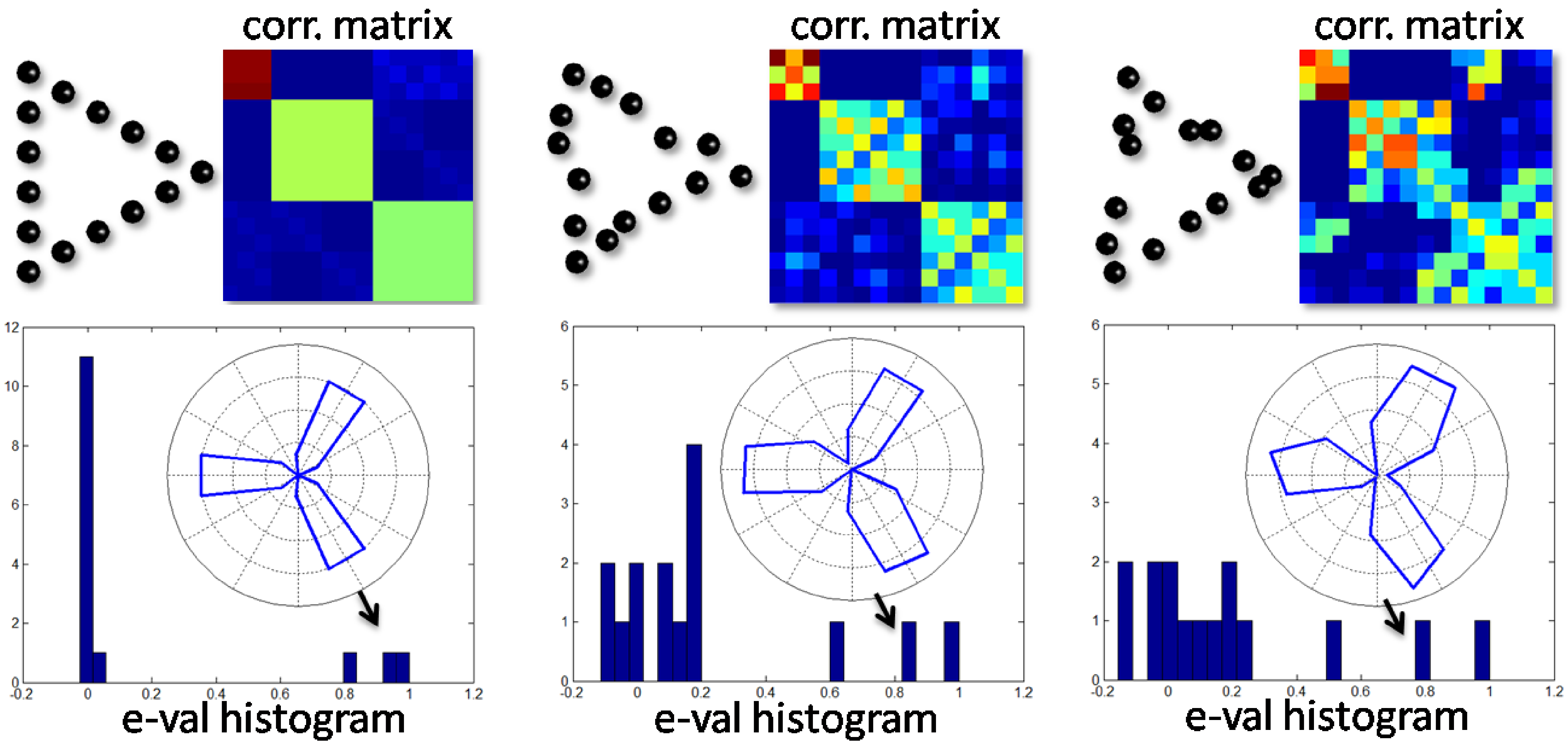
finding cliques *amounts to* spectral analysis on similarity matrix

[Lipman et al. 2009]

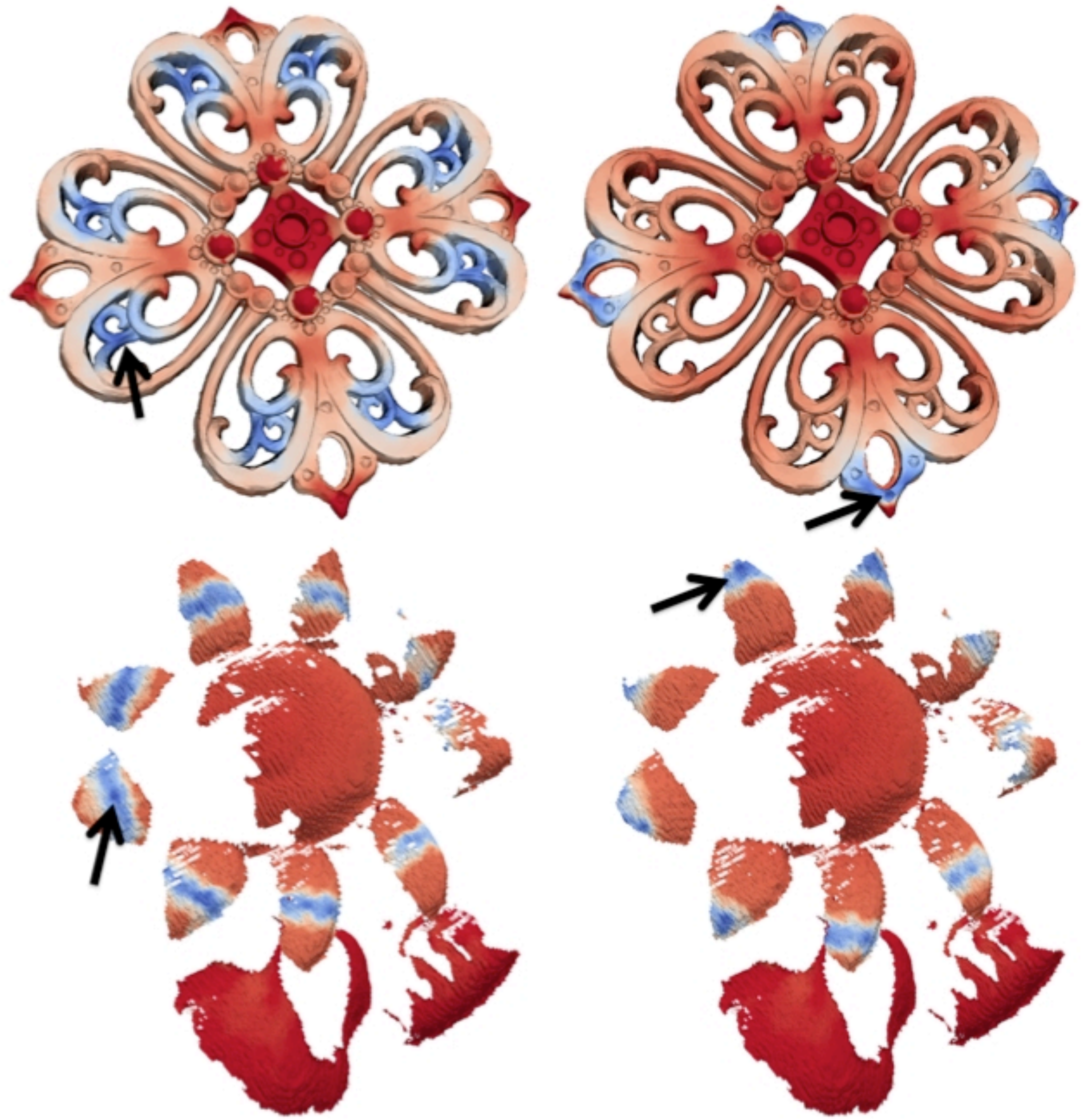
Symmetry-factored Segmentation



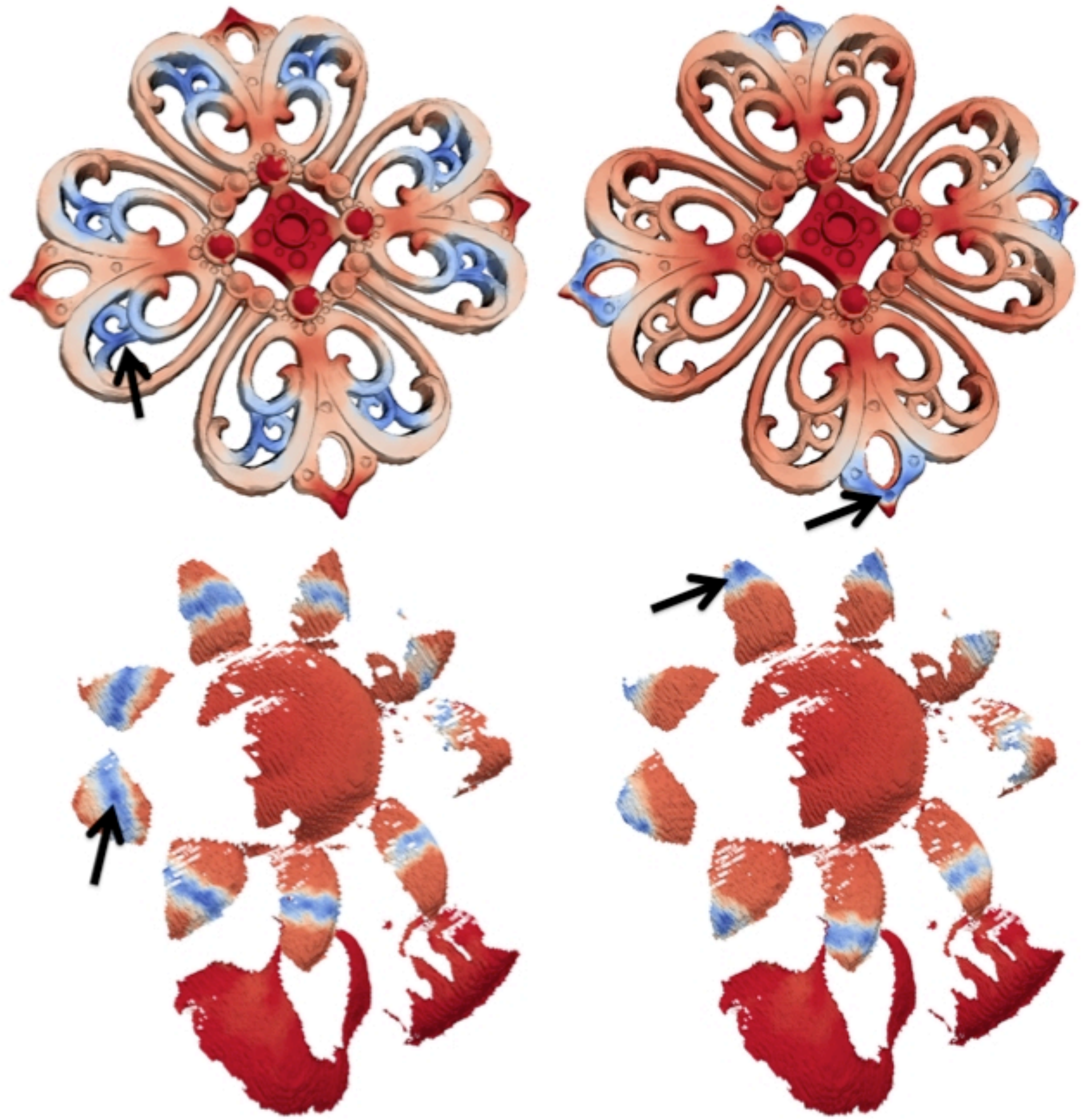
Symmetry in Correspondence Space



Symmetry in Correspondence Space

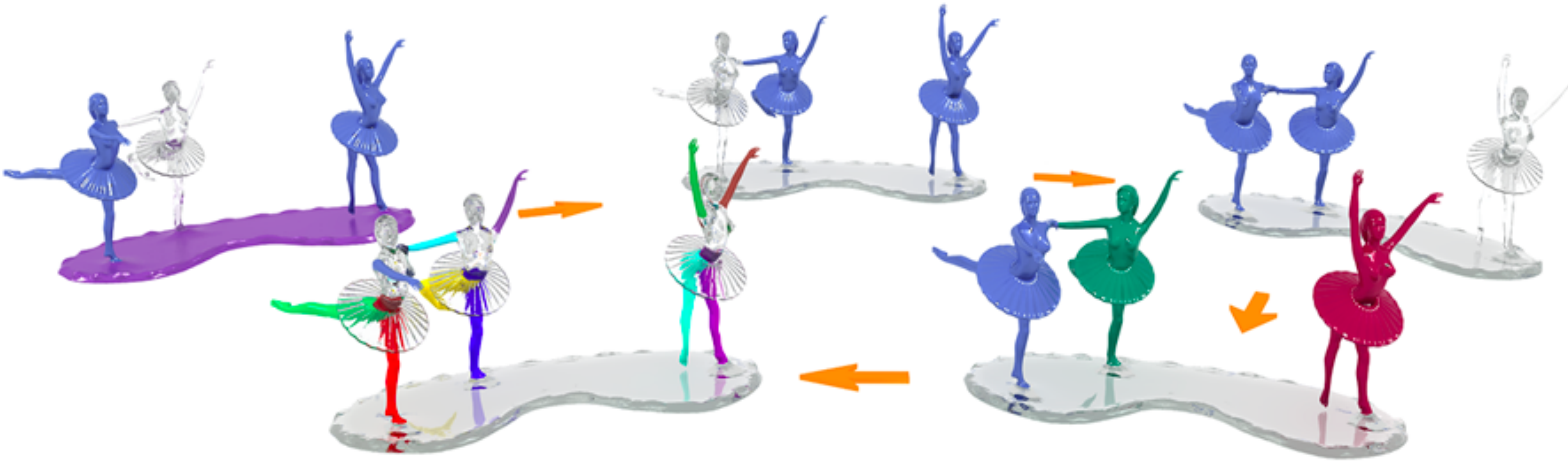


Symmetry in Correspondence Space



finding cliques *amounts to* spectral analysis on similarity matrix

Multi-scale Symmetry Detection



[Xu et al. 2012]