Symmetry in Shapes – Theory and Practice

Niloy Mitra *Maksim Ovsjanikov* *Mark Pauly* *Michael Wand* *Duygu Ceylan*

UCL

ÉCOLE POLYTECHNIQUE ParisTech

EPFL ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

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Representations & Applications

Michael Wand

Saarland University / MPI Informatik

UNIVERSITÄT DES SAARLANDES

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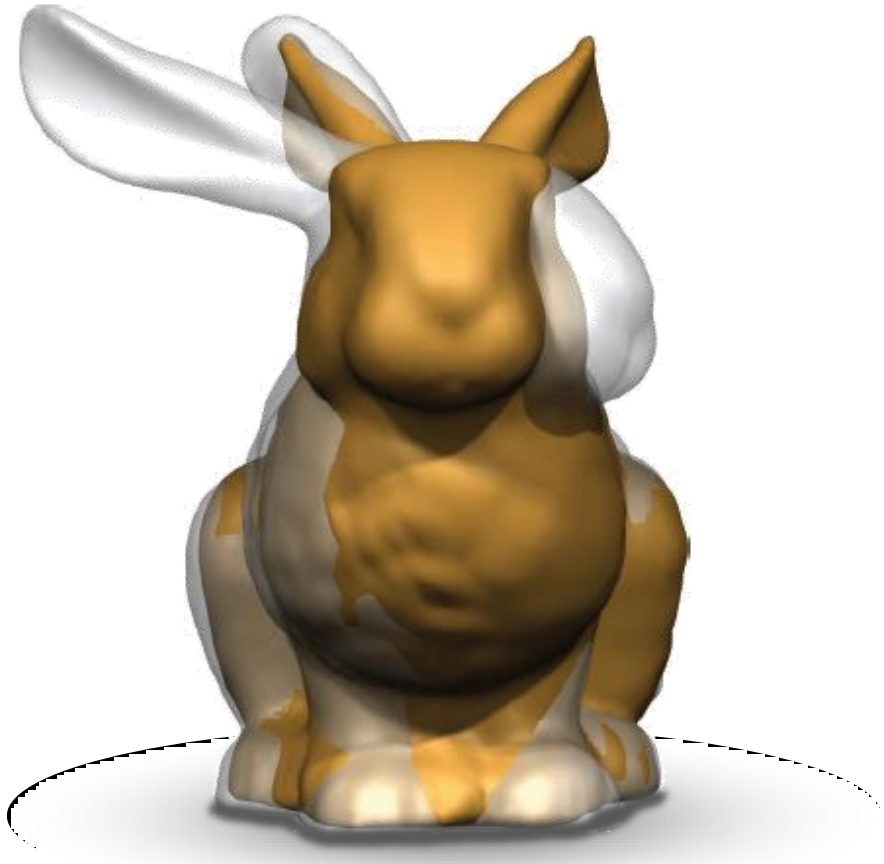


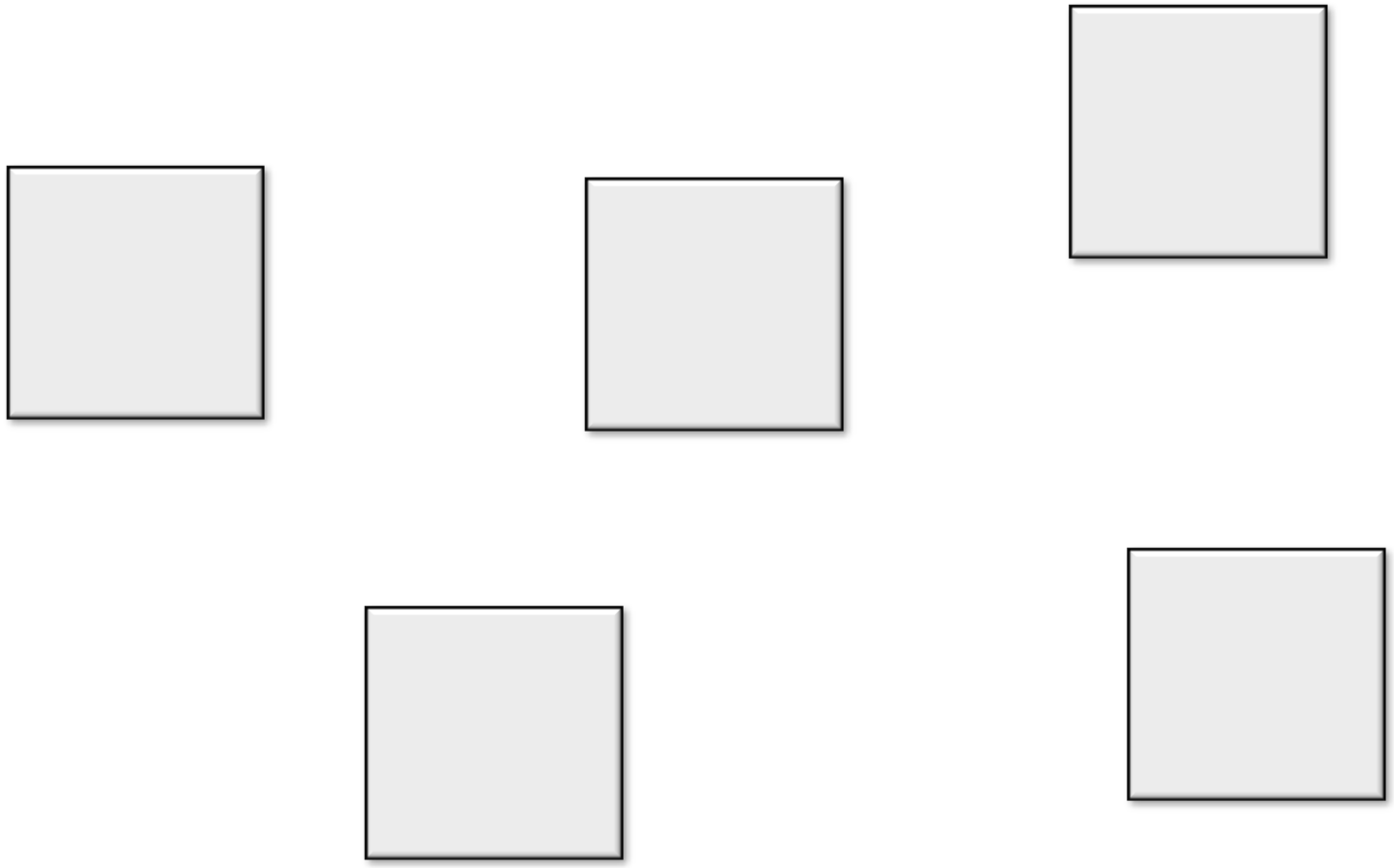
Representations

& Applications

Toy Example

How many building blocks are these?

Toy Example

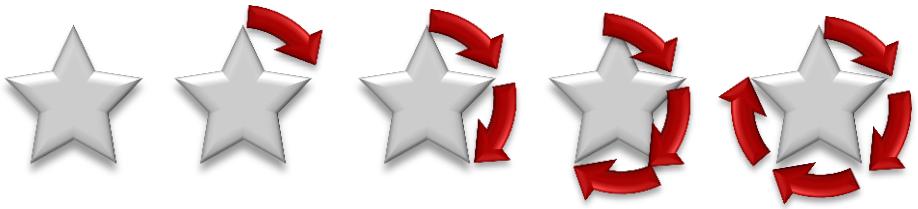


How many building blocks are these?

What is Symmetry?

Set of operations f that leave object X intact

- $f(X) = X$

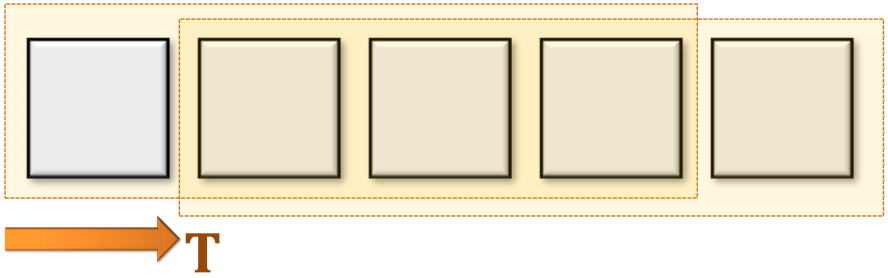


Operations $G = \{f | f(X) = X\}$ form a group

G encodes absent information

Derived Properties

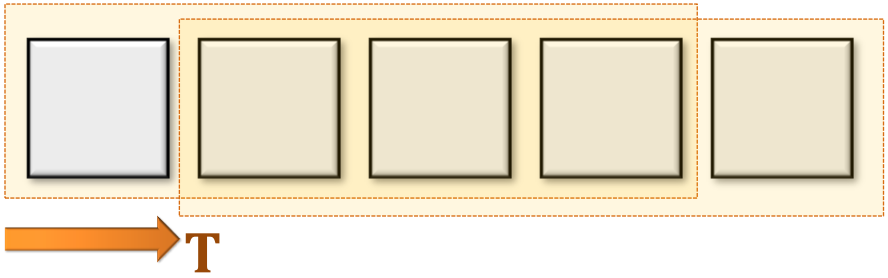
Pairwise Correspondences



Pairwise matches

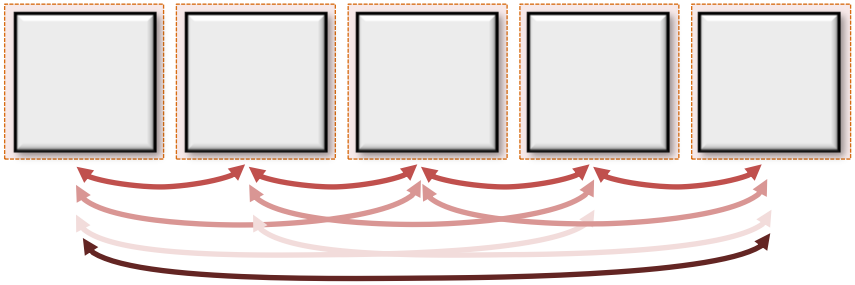
Derived Properties

Pairwise Correspondences



Pairwise matches

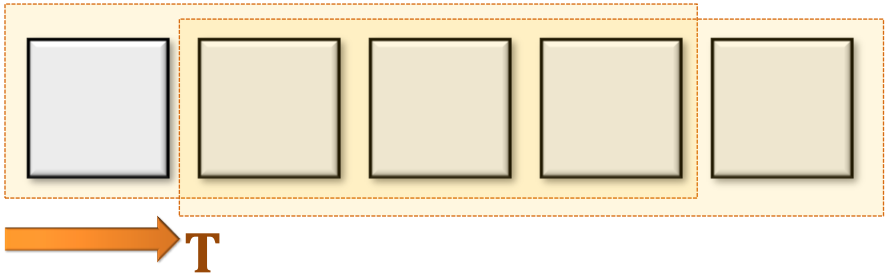
Permutation Groups



Exchangeable building blocks

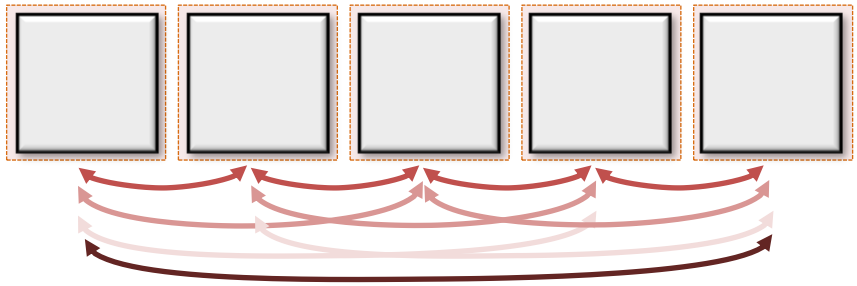
Derived Properties

Pairwise Correspondences



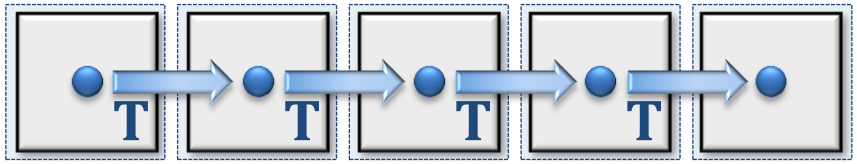
Pairwise matches

Permutation Groups



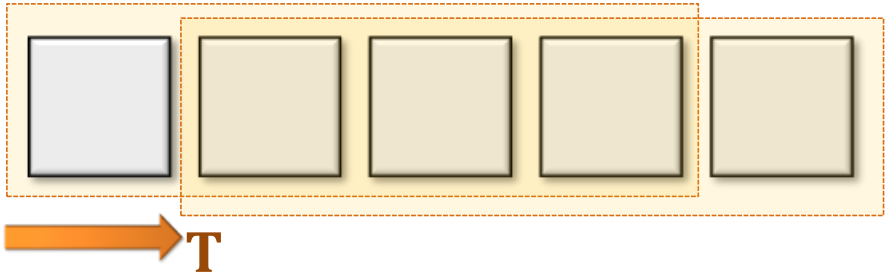
Exchangeable building blocks

Transformation Groups



Regular transformations
$\{\mathbf{T}^i | i \in \mathbb{Z}\}$

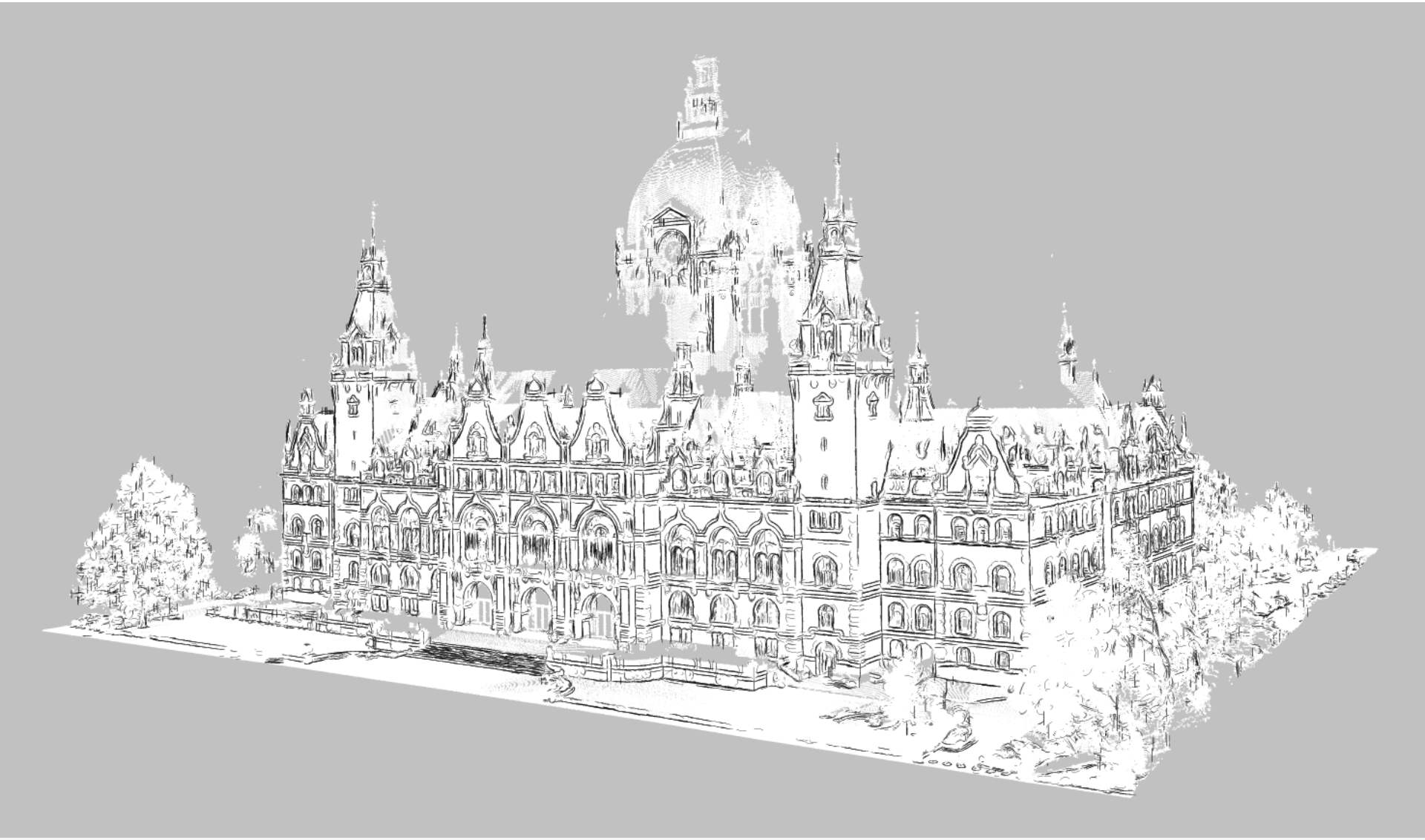
Pairwise Matches



Input Data (Point Cloud)

[data set: C. Brenner, IKG Univ. Hannover]

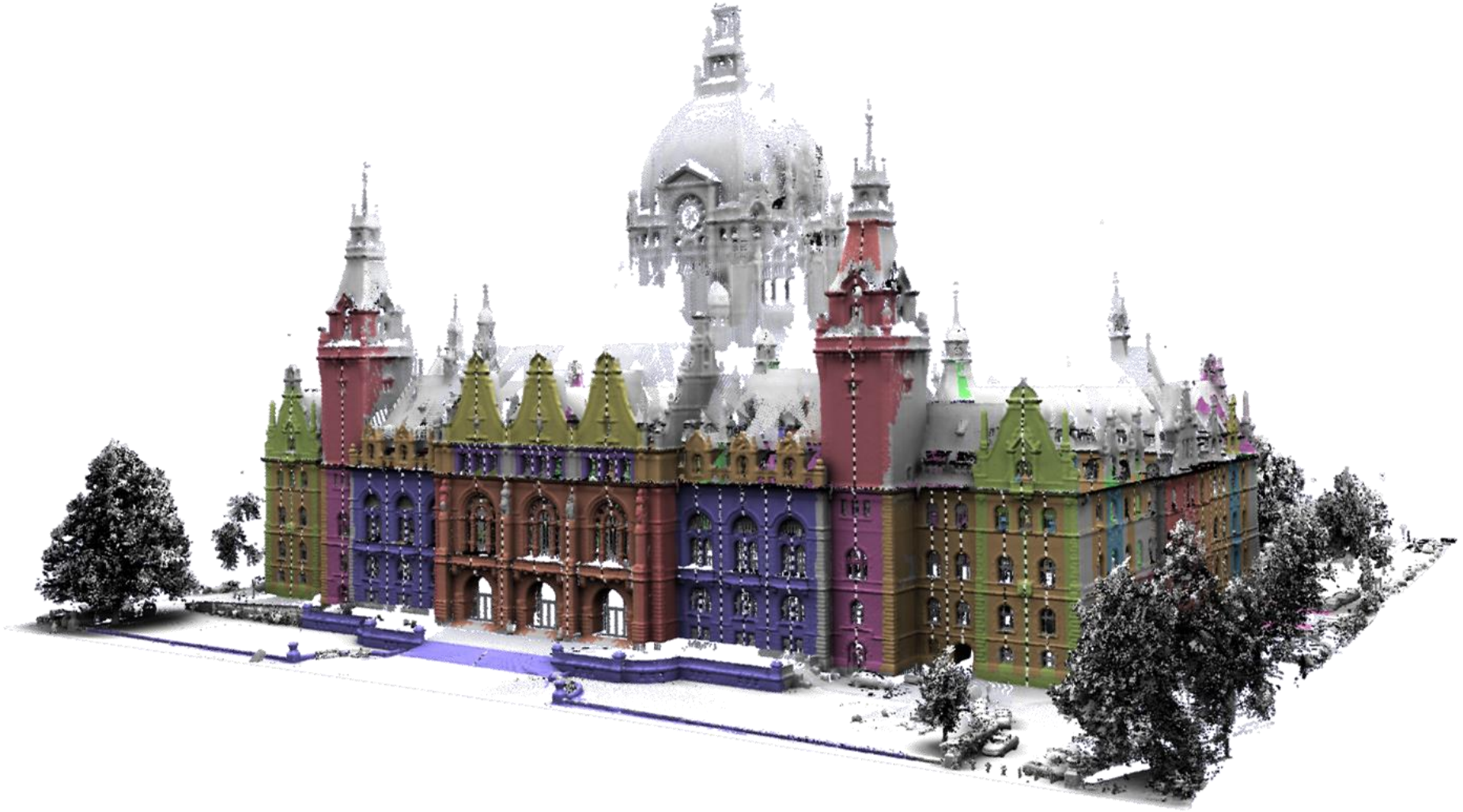
Feature Representation



[data set: C. Brenner, IKG Univ. Hannover]

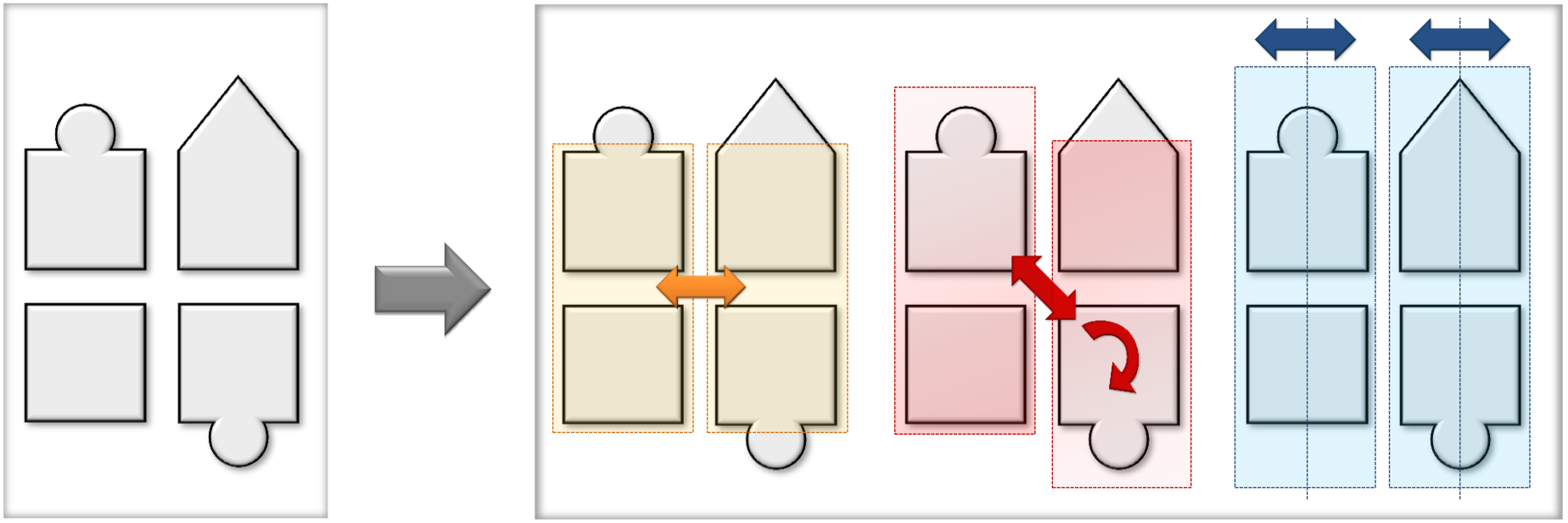
[data set: C. Brenner, IKG Univ. Hannover]

Result



[data set: C. Brenner, IKG Univ. Hannover]

Symmetry Detection



Partial Symmetry Detection

- Yields pairwise partial correspondences
- No symmetry groups (yet)

Applications

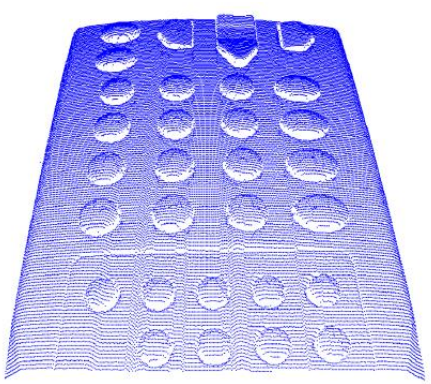
Pairwise correspondences

- Non-local denoising
- Symmetrization
- Constrained editing

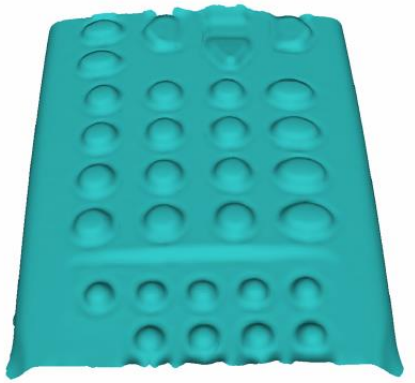
Techniques

- Correspondences transport information
- Simplification of pairwise relations
- Pairwise constraints as invariants

Non-Local Denoising



data



MLS

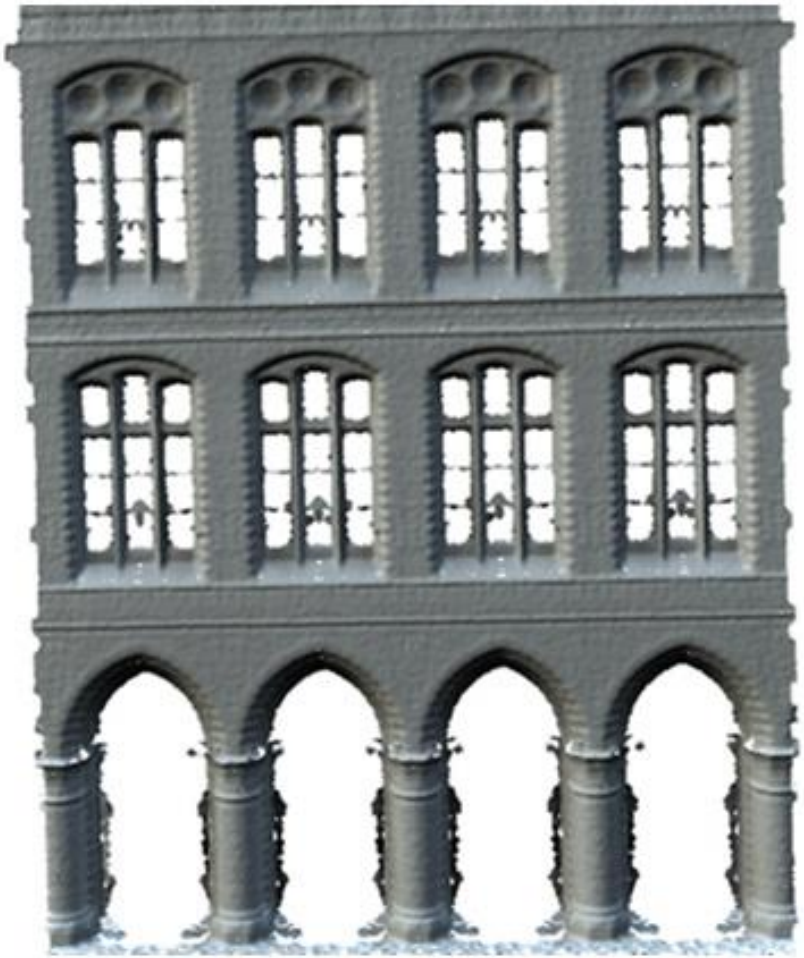
non-local

[Gal et al. 2007]

Non-Local Denoising

16 parts

MLS

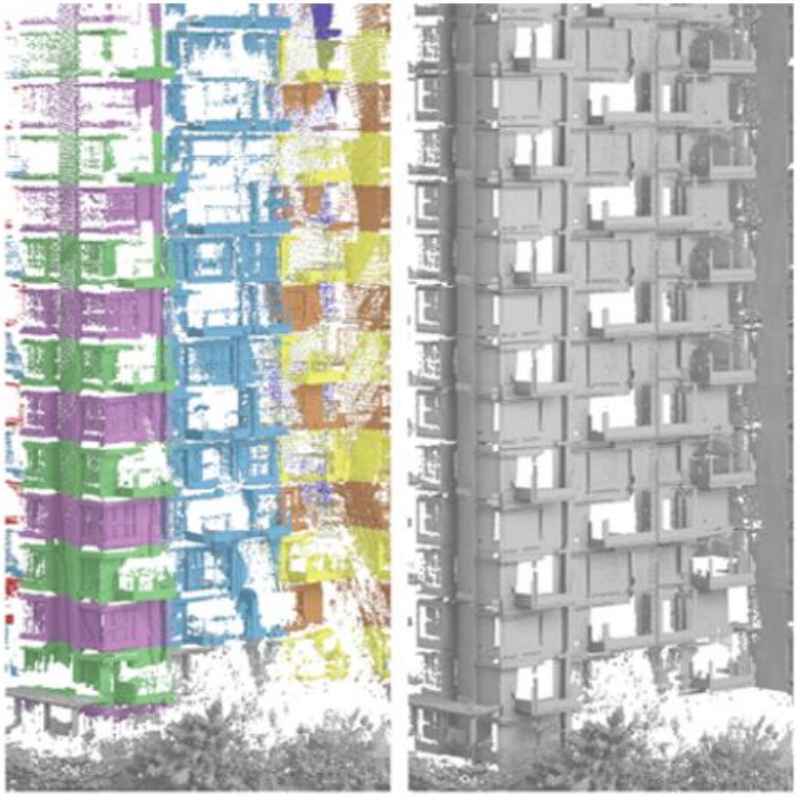


non-local

[data set: C. Brenner, University Hannover]

[Bokeloh et al. 2009]

Non-Local Denoising

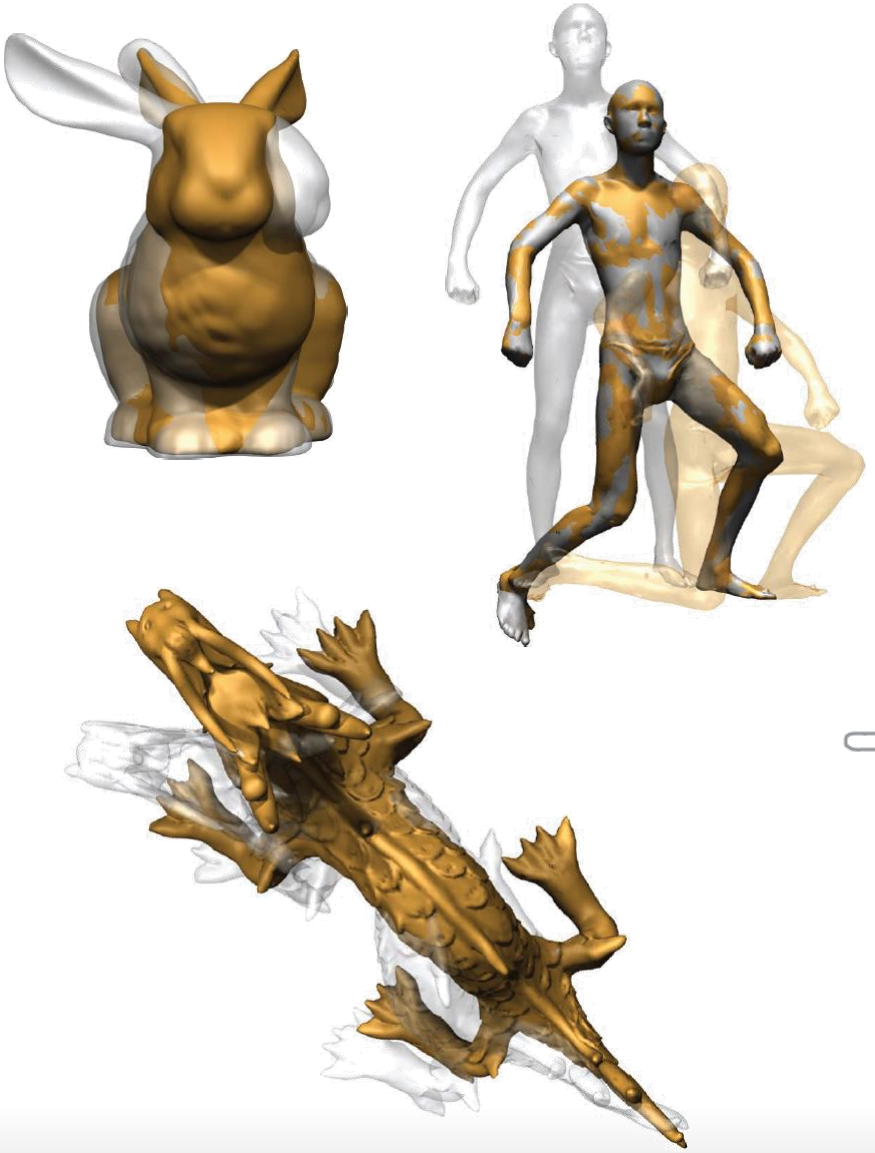


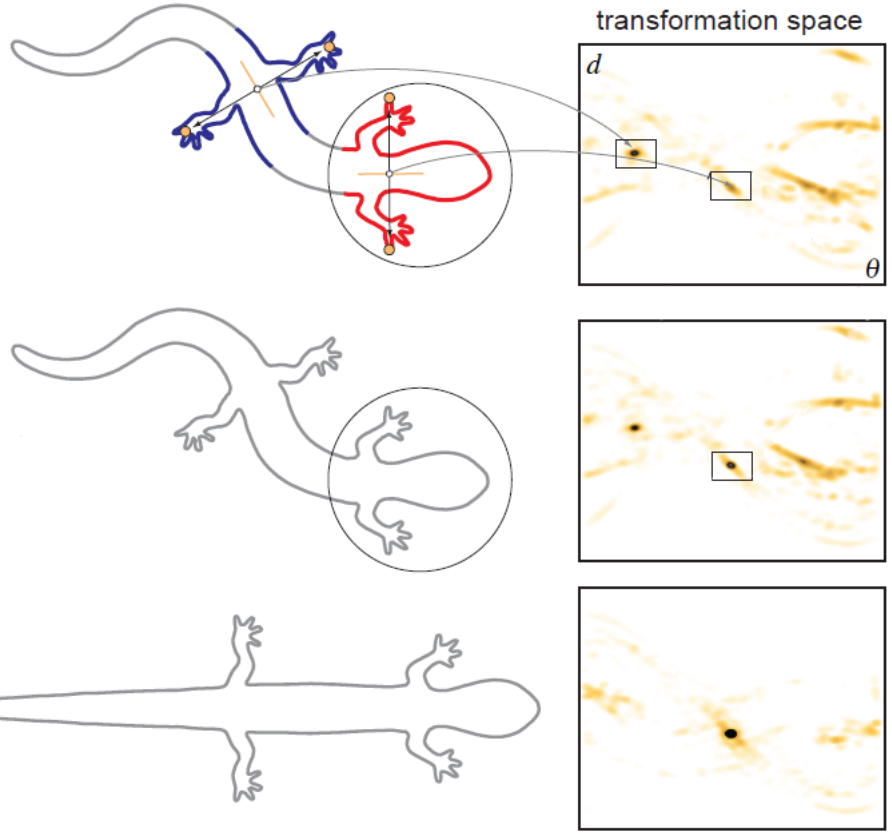
data

non-local denoising

[Zheng et al. 2010]

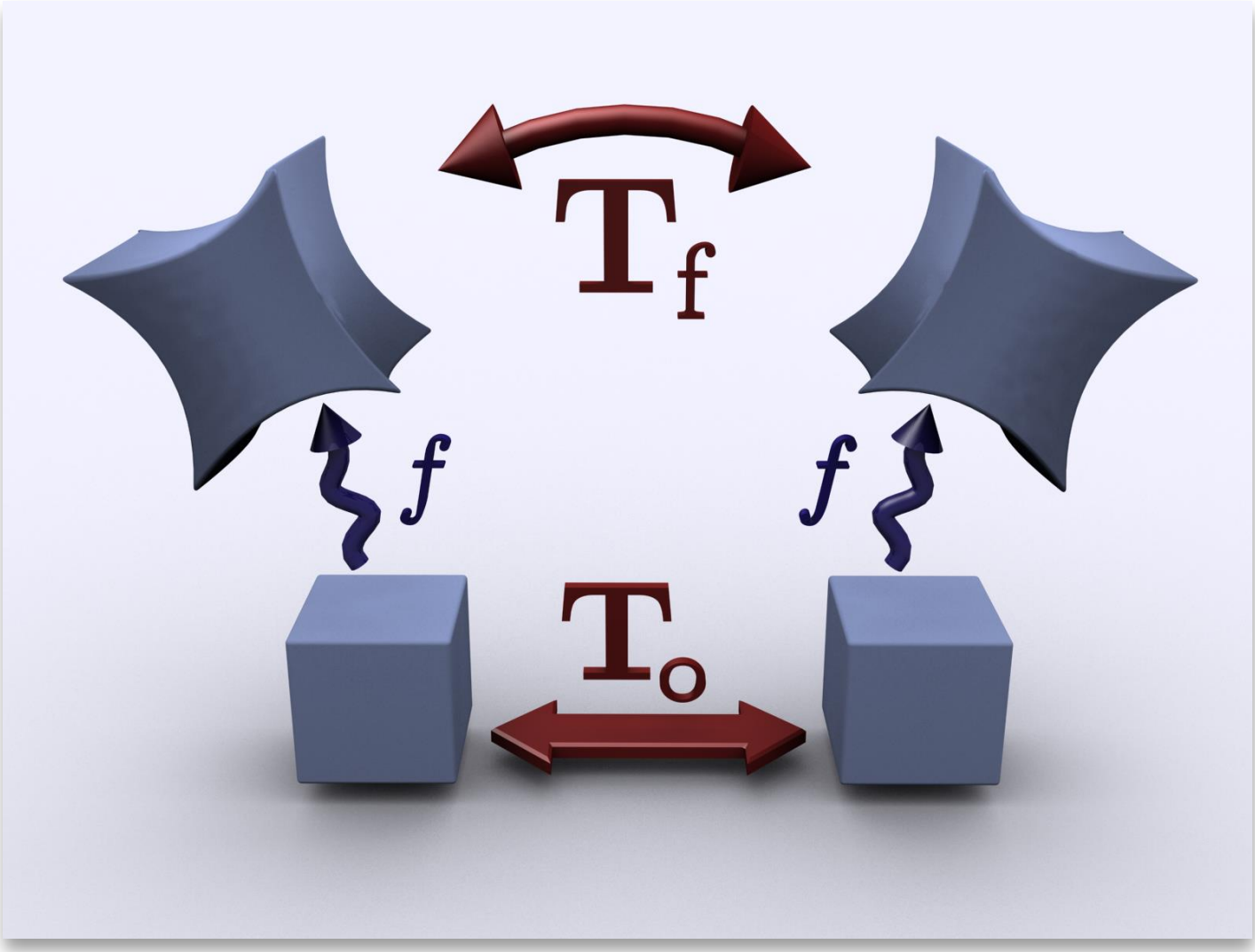
Symmetrization



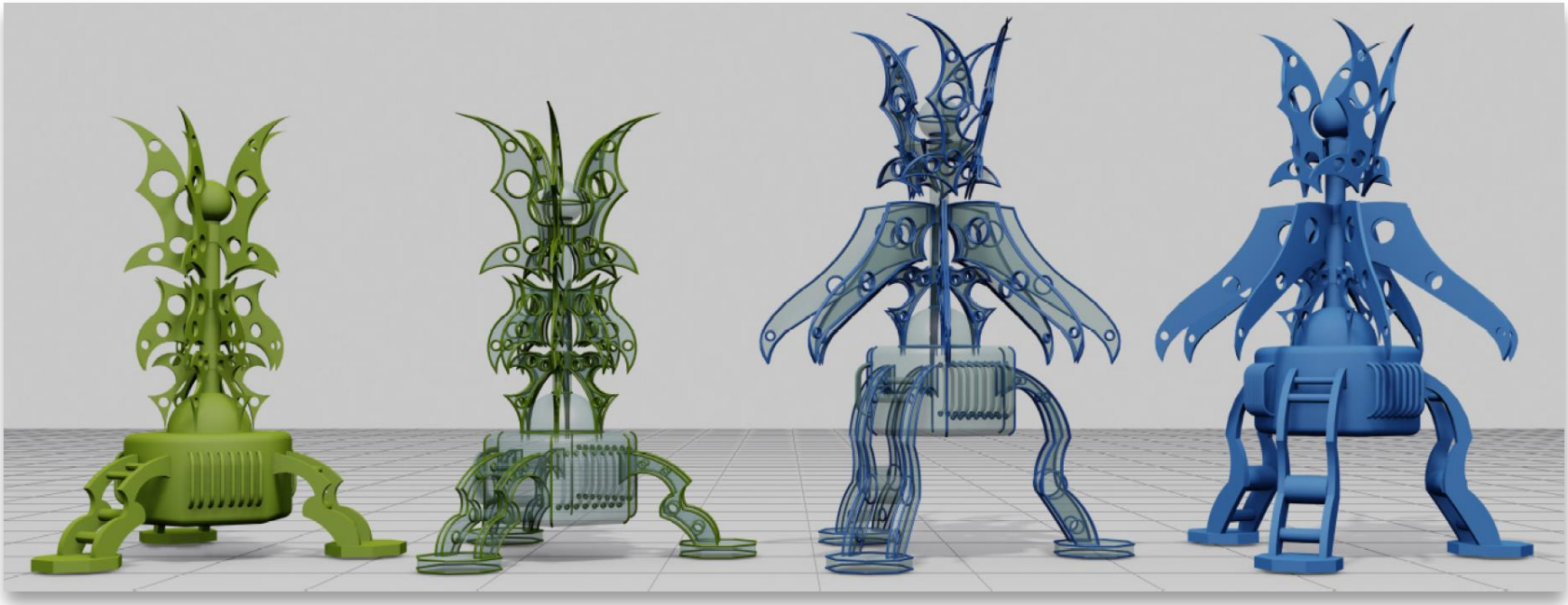


[Mitra et al. 2007]

Symmetry Preserving Editing



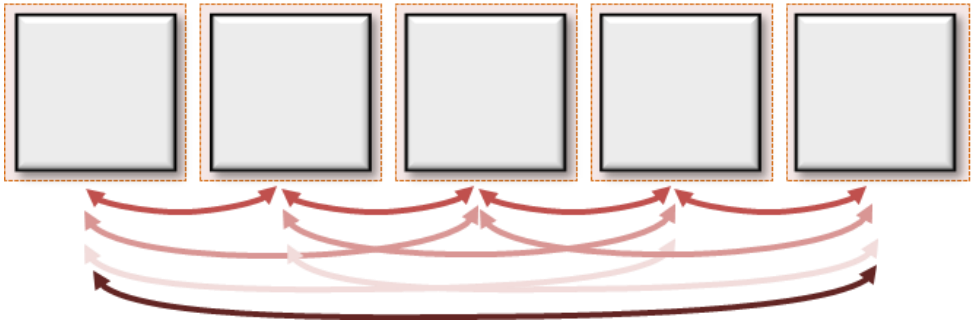
iWires



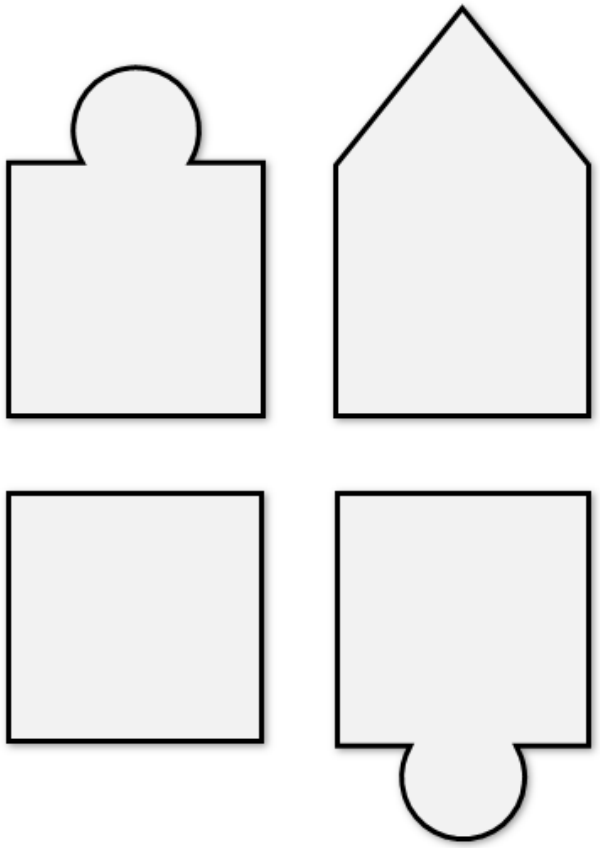
[Gal et al. 2009]

Symmetry-based propagation of edits: additional references
[Wang et al. 2011], [Zheng et al. 2011]

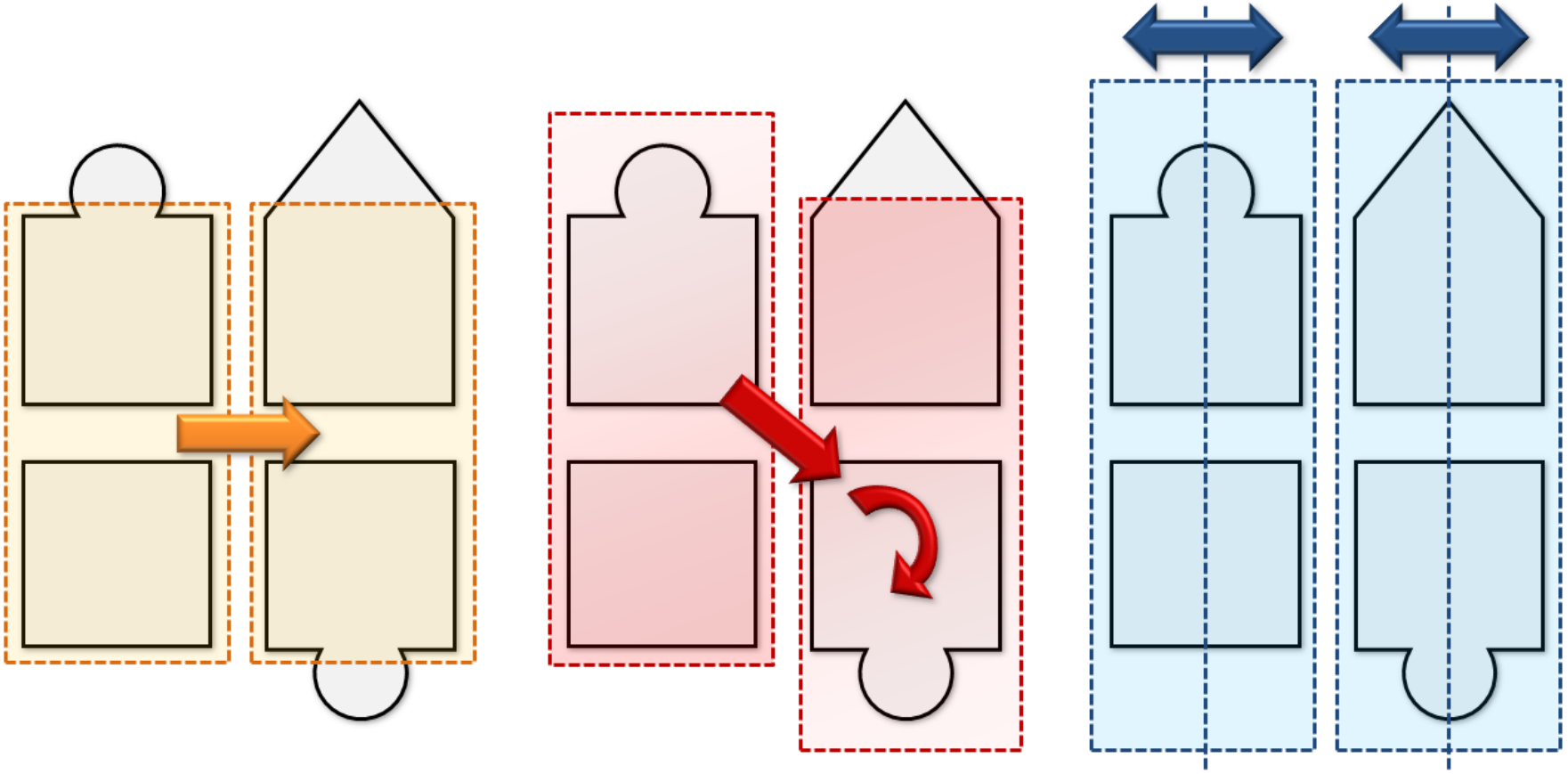
Permutation & Building Blocks



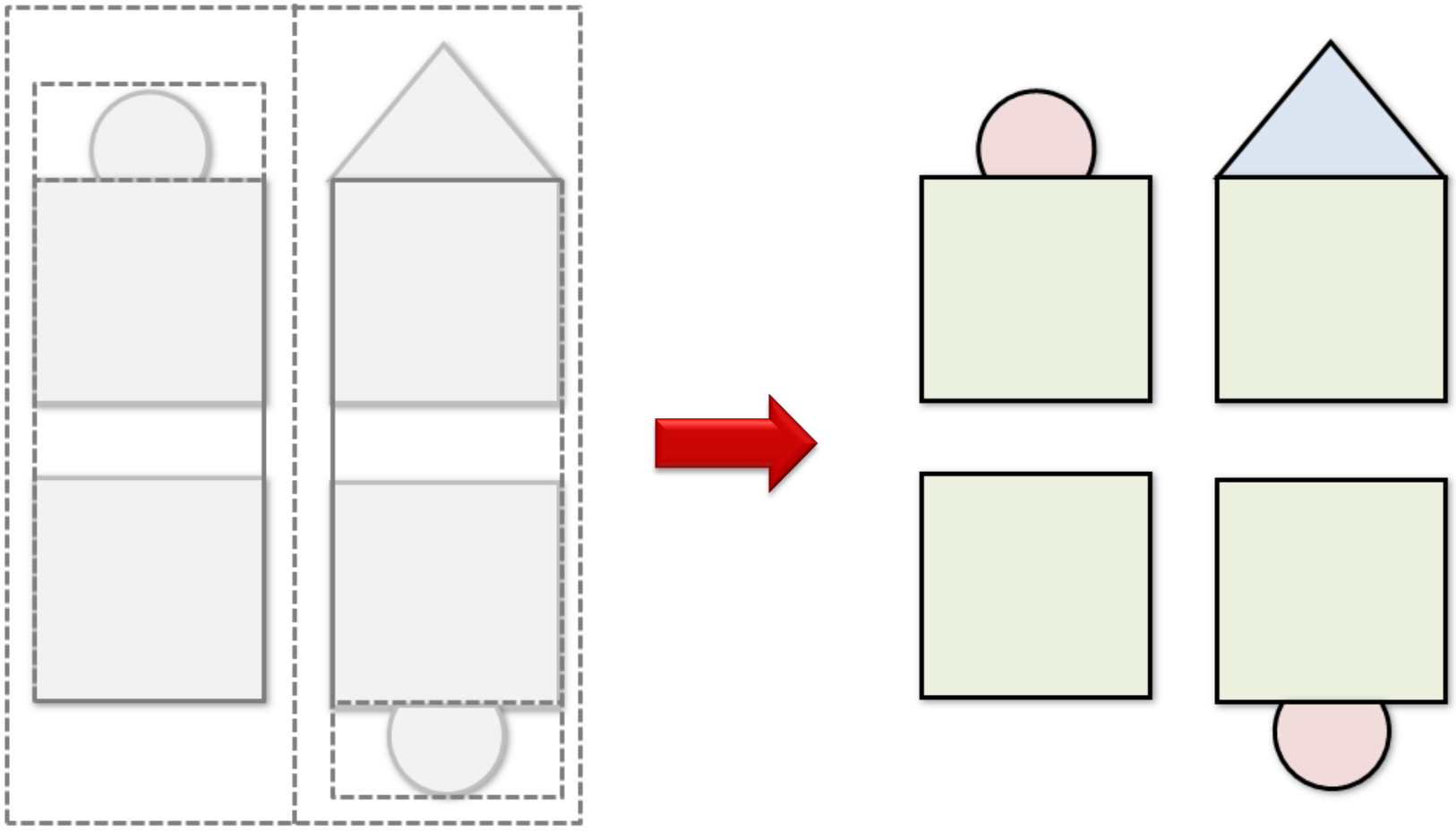
Example Scene



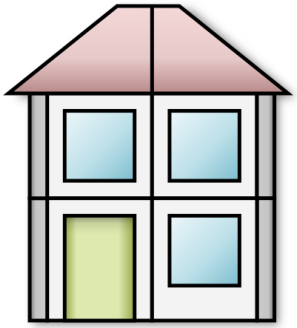
Pairwise Correspondences

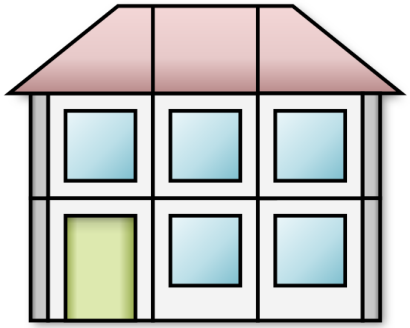


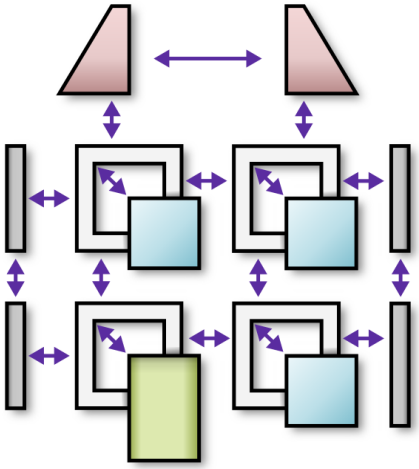
Cutting at the Boundaries

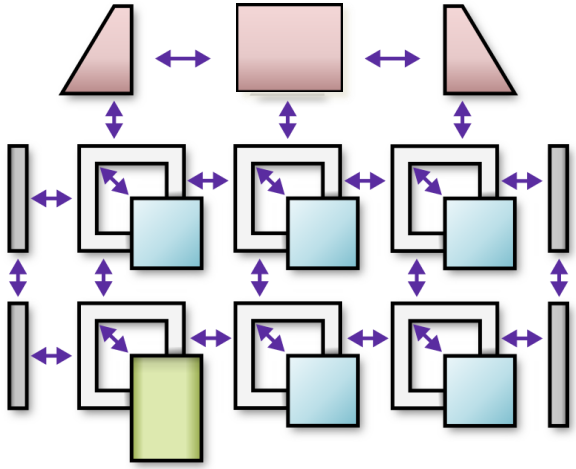


Microtiles

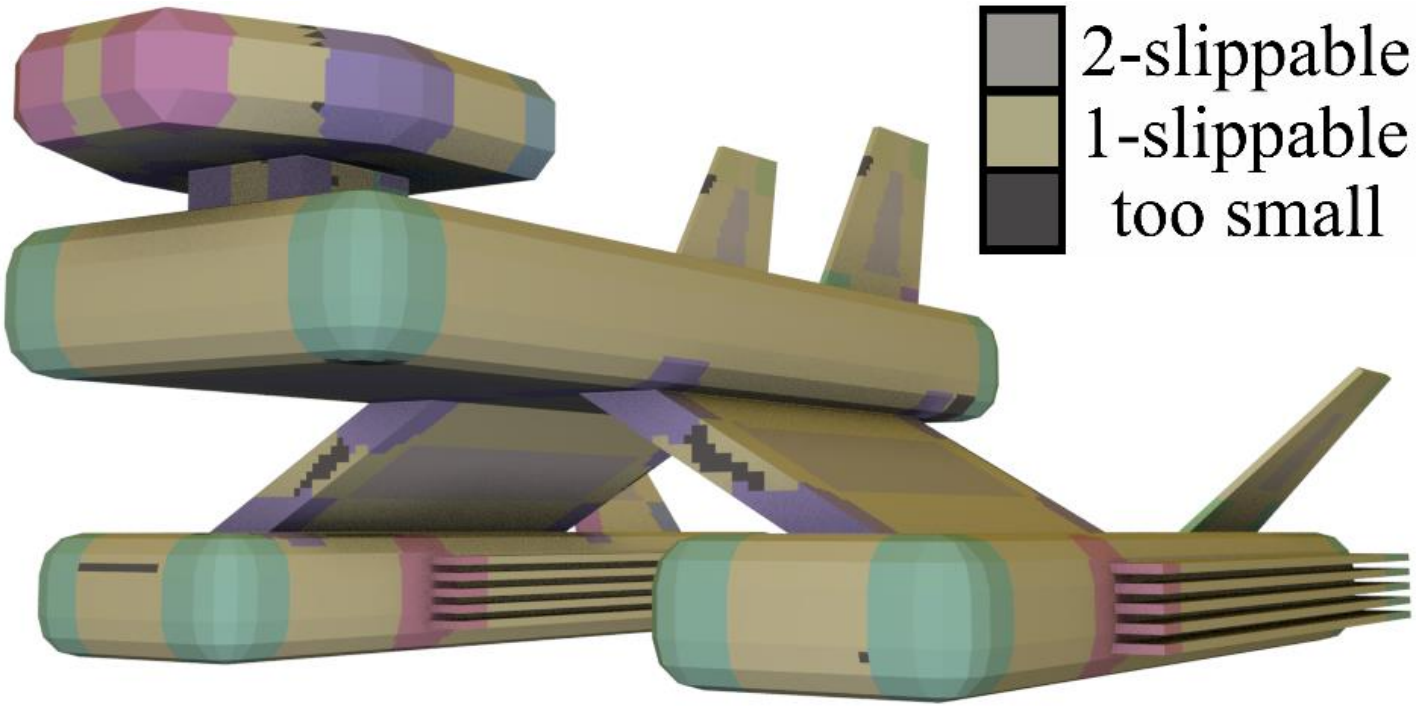








3D Result



Properties

General framework

- Need point-wise equivalent relations

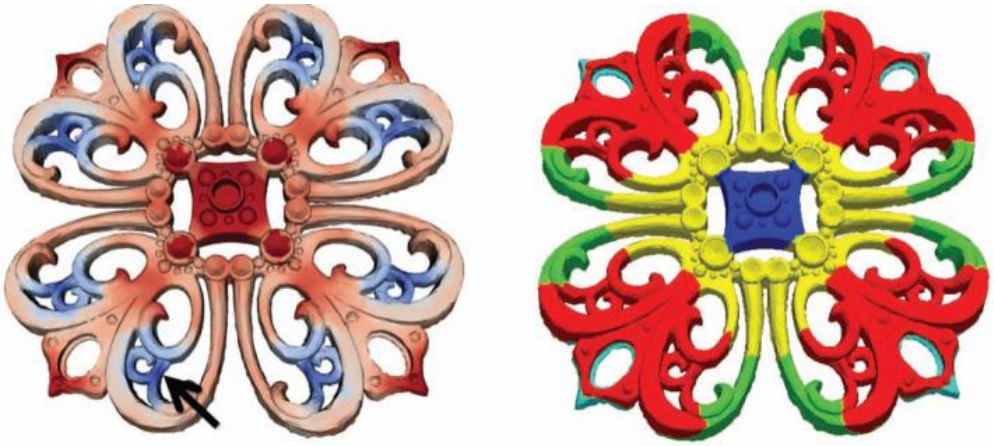
Canonical, unique decomposition

Every point of every piece is unique

- Microtiles cannot have partial correspondences

Microtiles reveal permutation groups

Symmetry Factored Embedding



[Lipman et al. 2010]

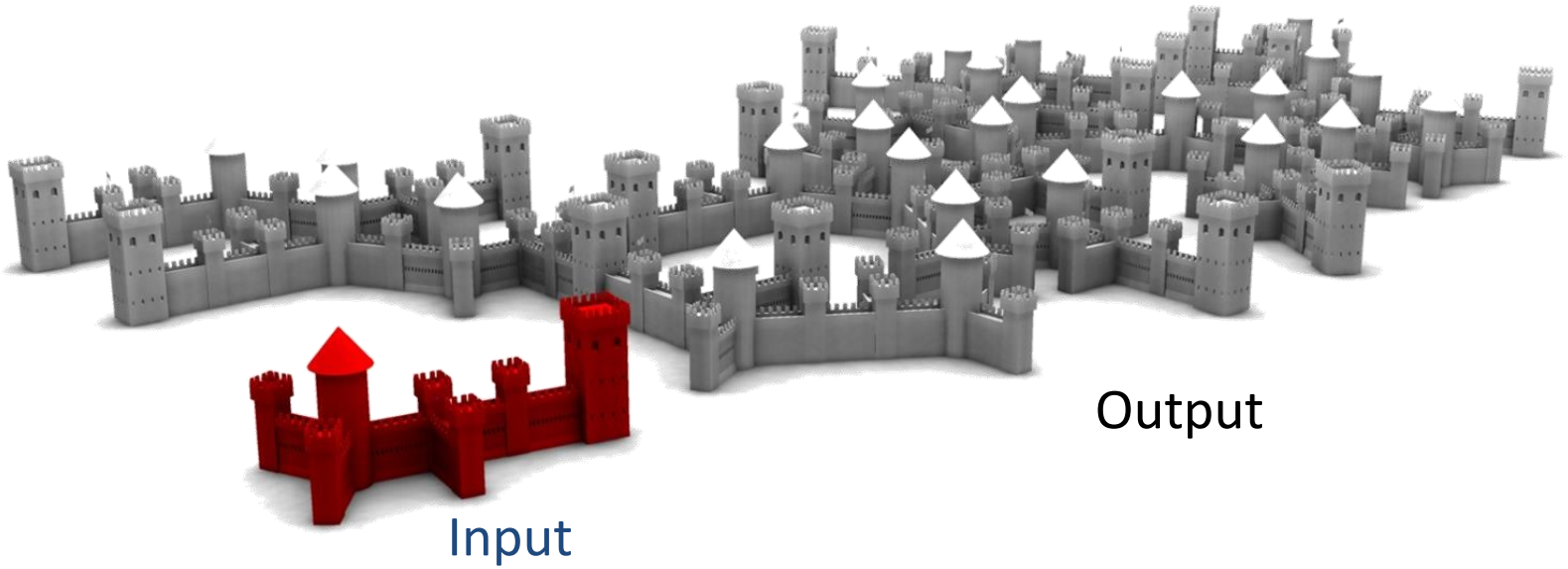
Related Concept

- Points that map together in once piece
- Consistent orbits
- Ignores transformation, point-wise orbits

Inverse Procedural Modeling

Rules from example geometry

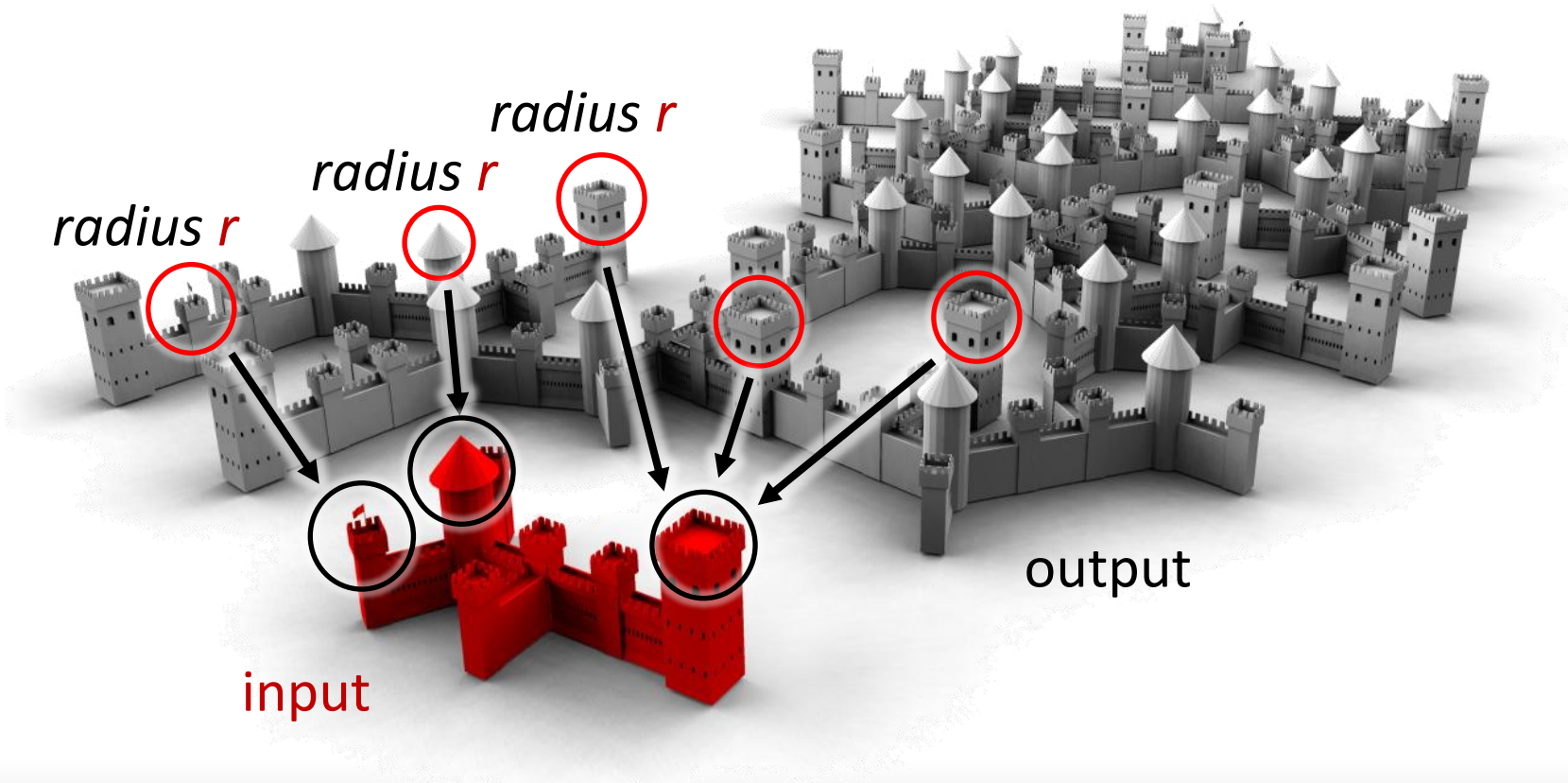
- Example model
- Compute rules describing a class of similar models



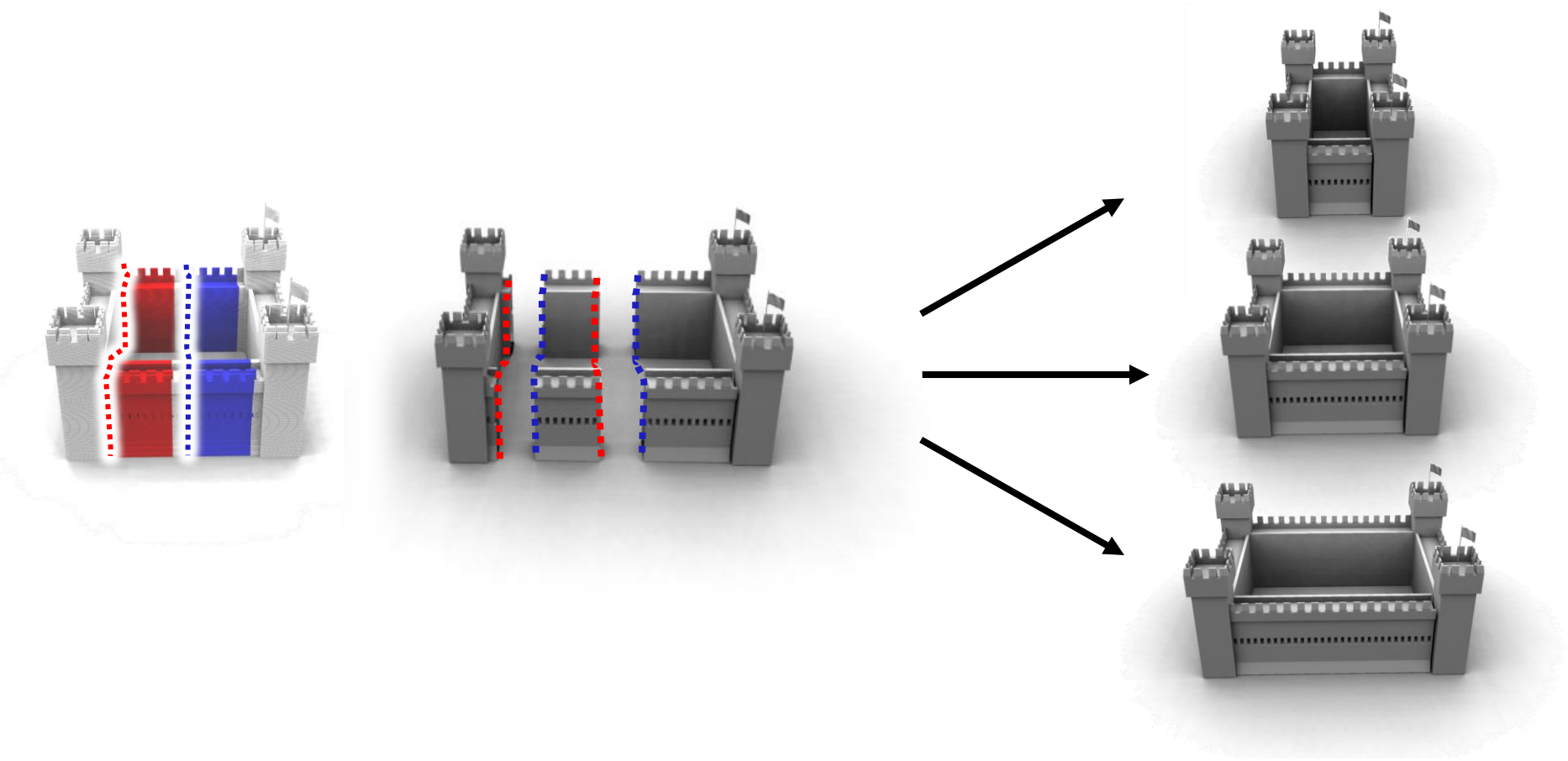
Inverse Procedural Modeling

r-Similarity

- Local neighborhoods match exemplar

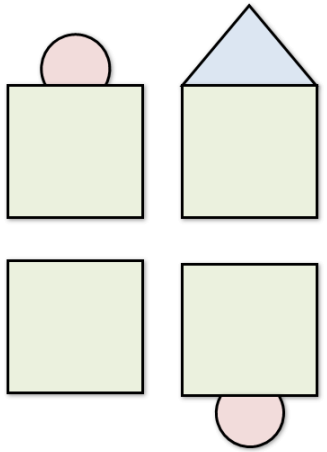


Inverse Procedural Modeling



Theoretical Results

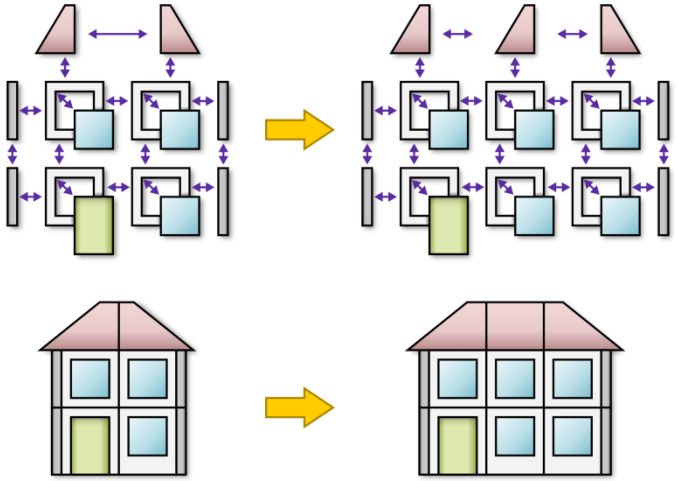
All r-similar objects are made out of $(r-\epsilon)$-microtiles



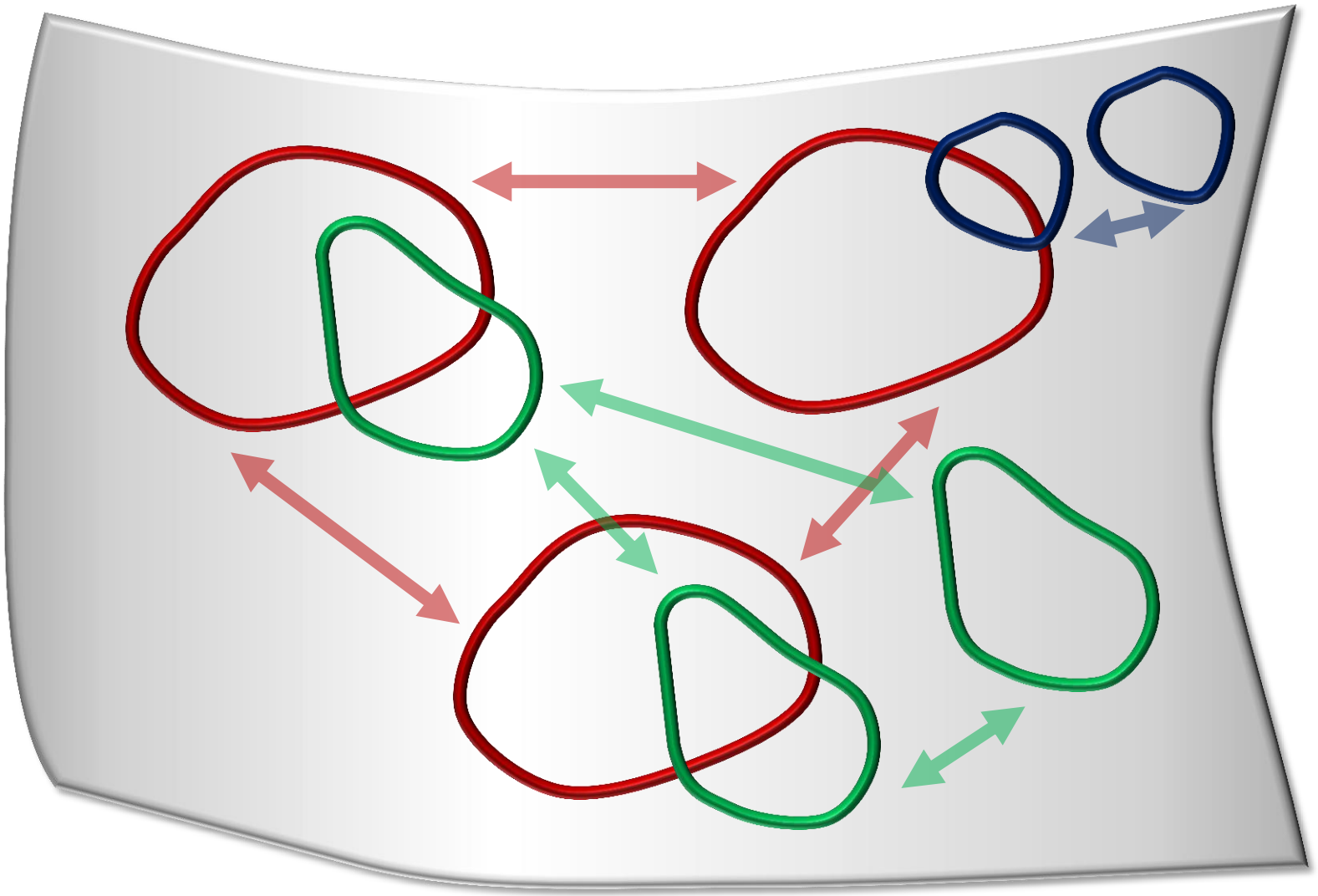
- Unique construction
- Connectivity same as in the example

Implications

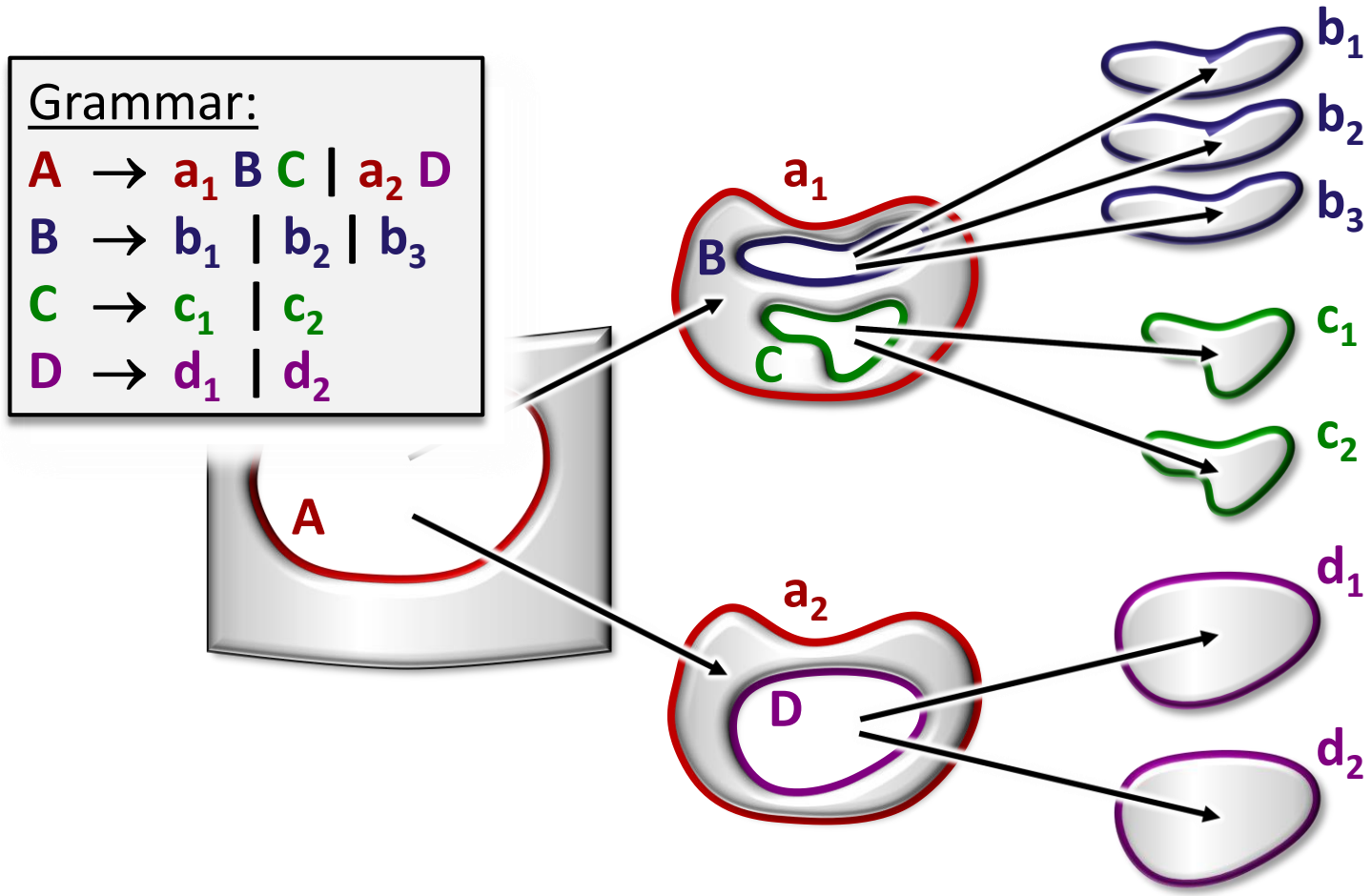
- Canonical representation
- Synthesis
  = solving jigsaw puzzles



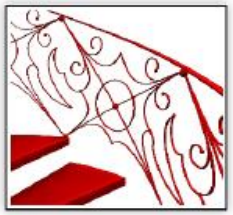
Shape Grammar

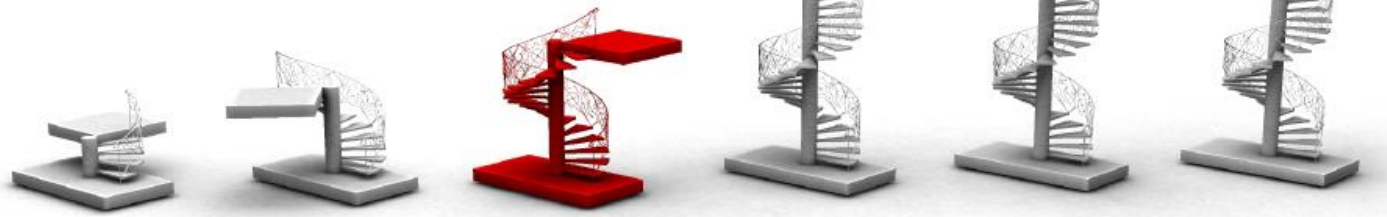


Practice: Context Free Grammar

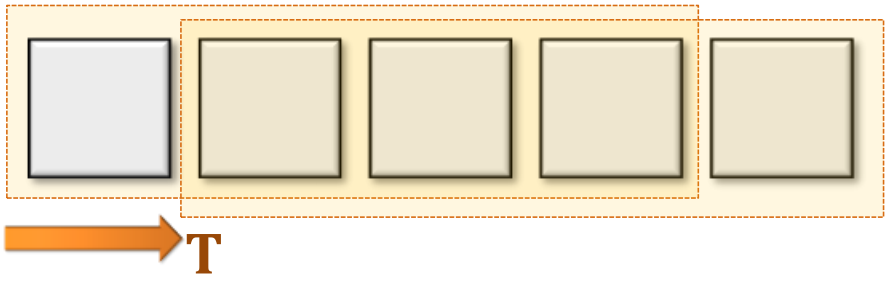


Practical Results

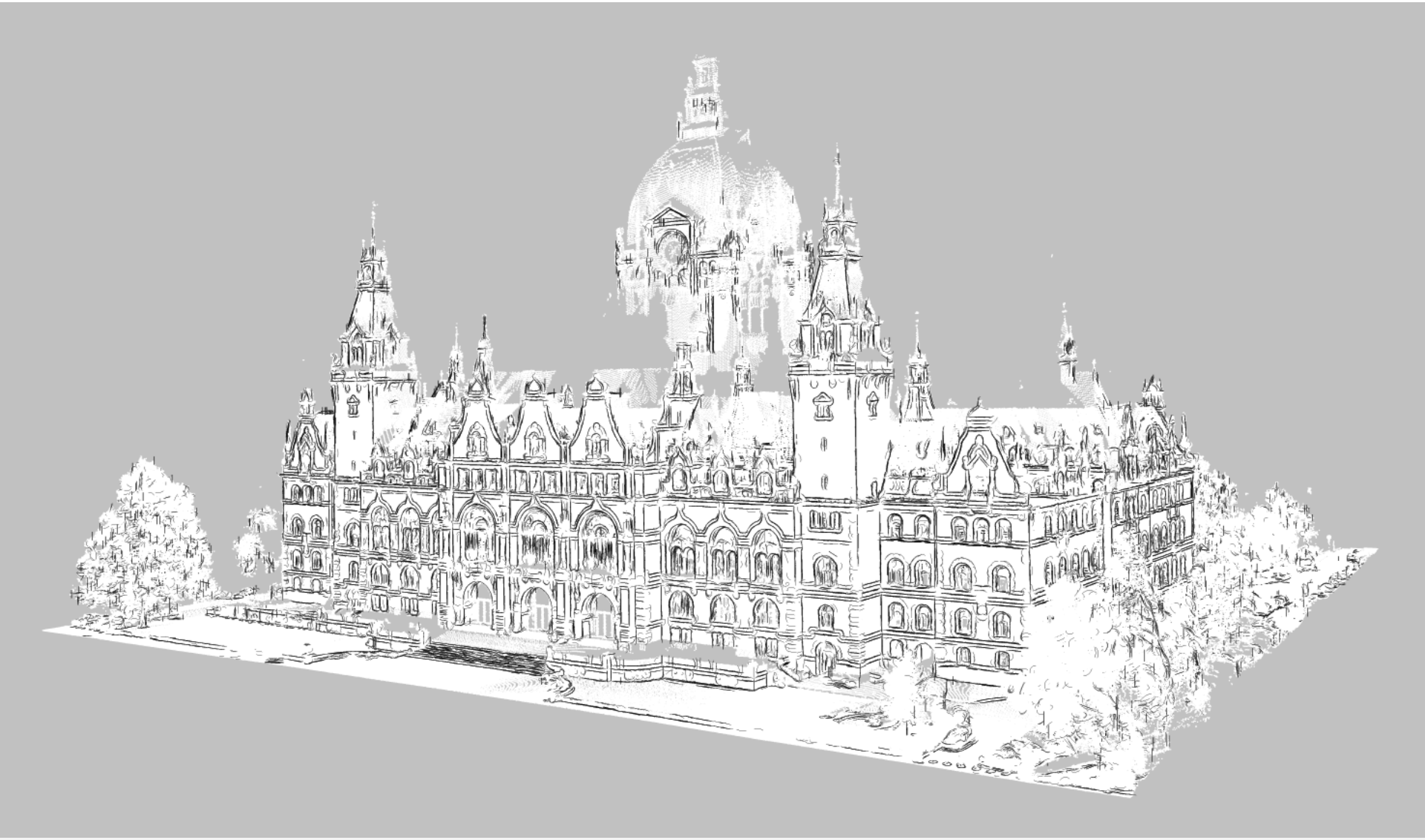




Fast Pairwise Matches

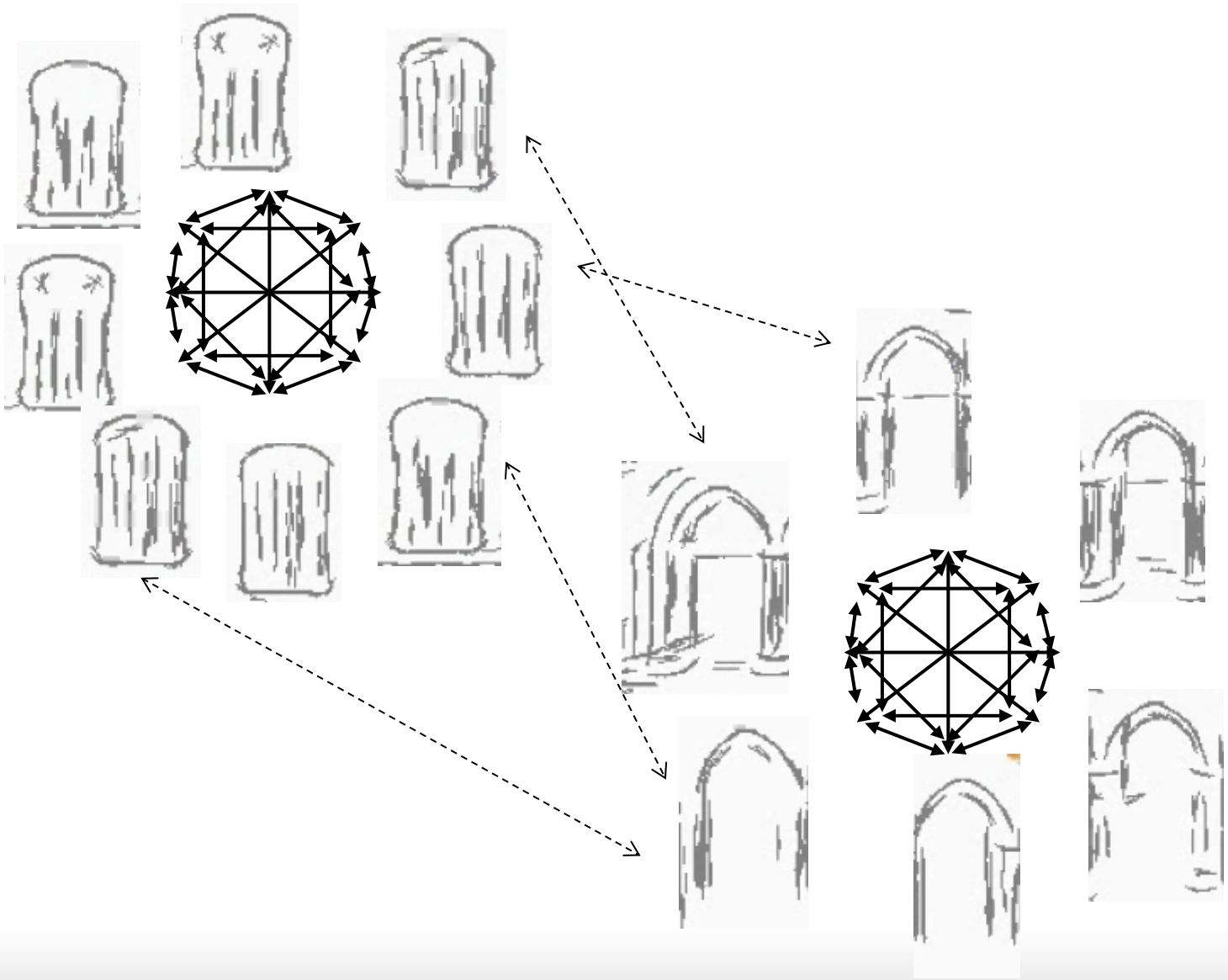


Quadratic Complexity?

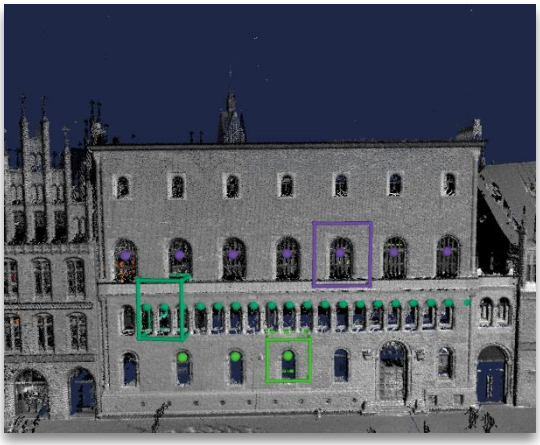
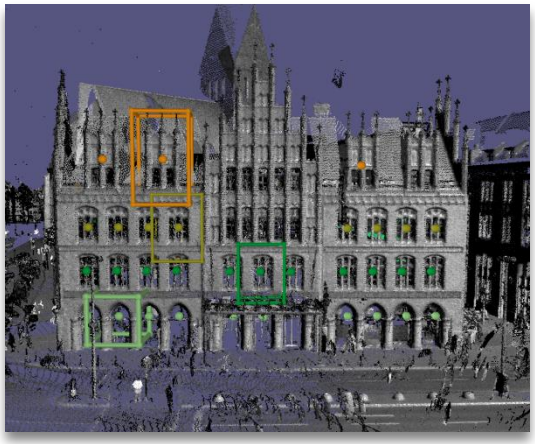


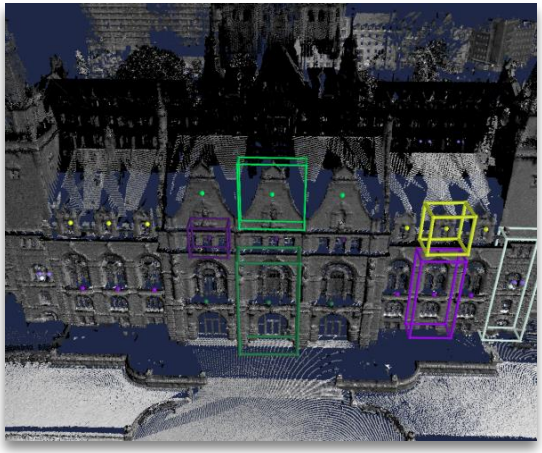
[data set: C. Brenner, IKG Univ. Hannover]

Cliques / Equivalence Classes



Scalable Symmetry Detection

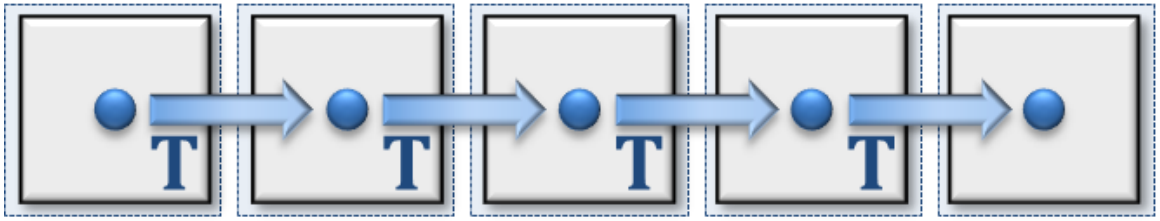
Hannover scans:
128M points / 14GB

detection: 23 min
preproc.: 43 min

[Kerber et al. 2013]

[data set: C. Brenner, IKG Univ. Hannover]

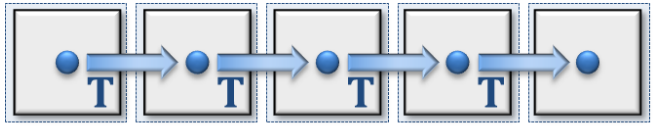
Regular Transformations



Applications

Symmetry: regularity (transformations)

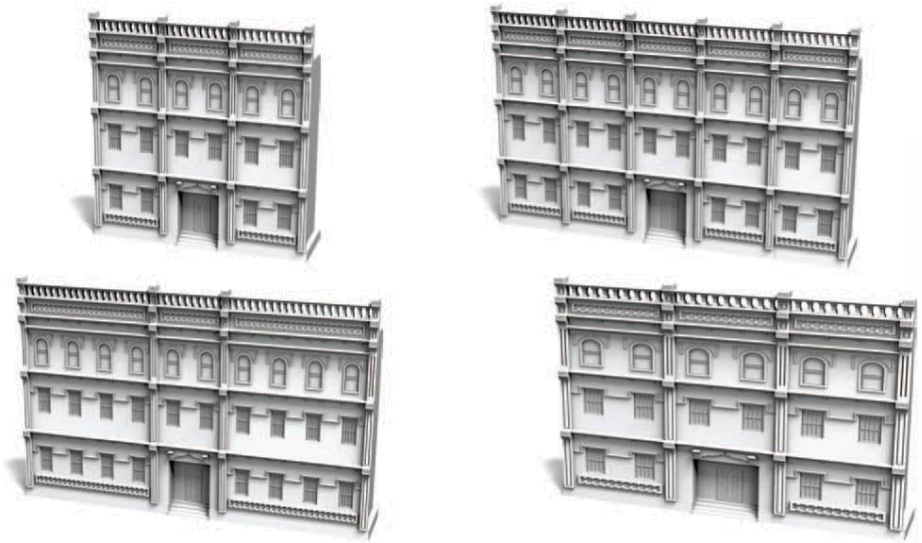
- Inverse procedural modeling
- Regularity preserving editing
- Shape recognition
- Shape understanding



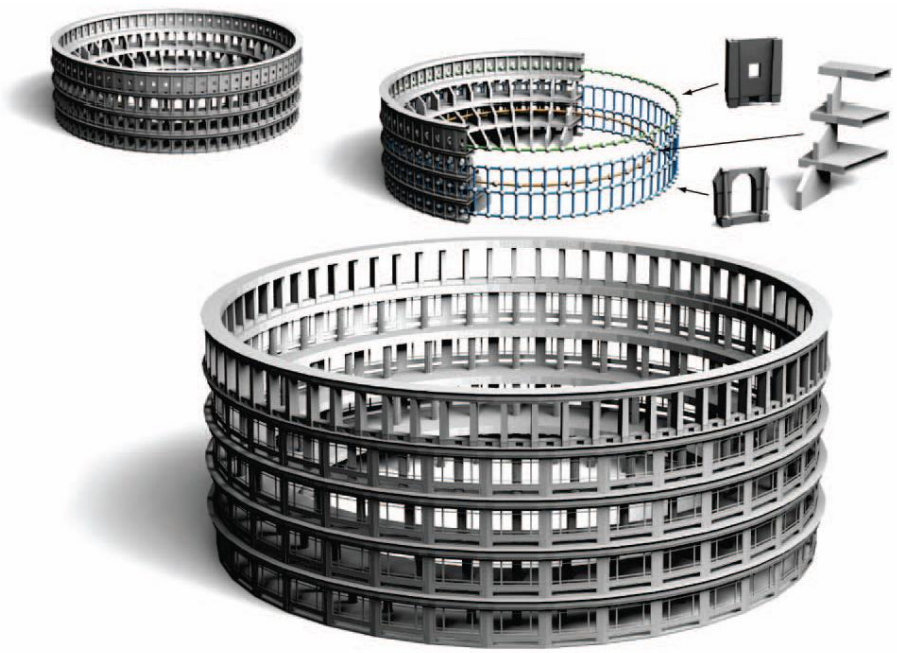
Techniques

- Transformation groups characterize shapes
- Transformation group structure as invariants

Inverse Procedural Modeling

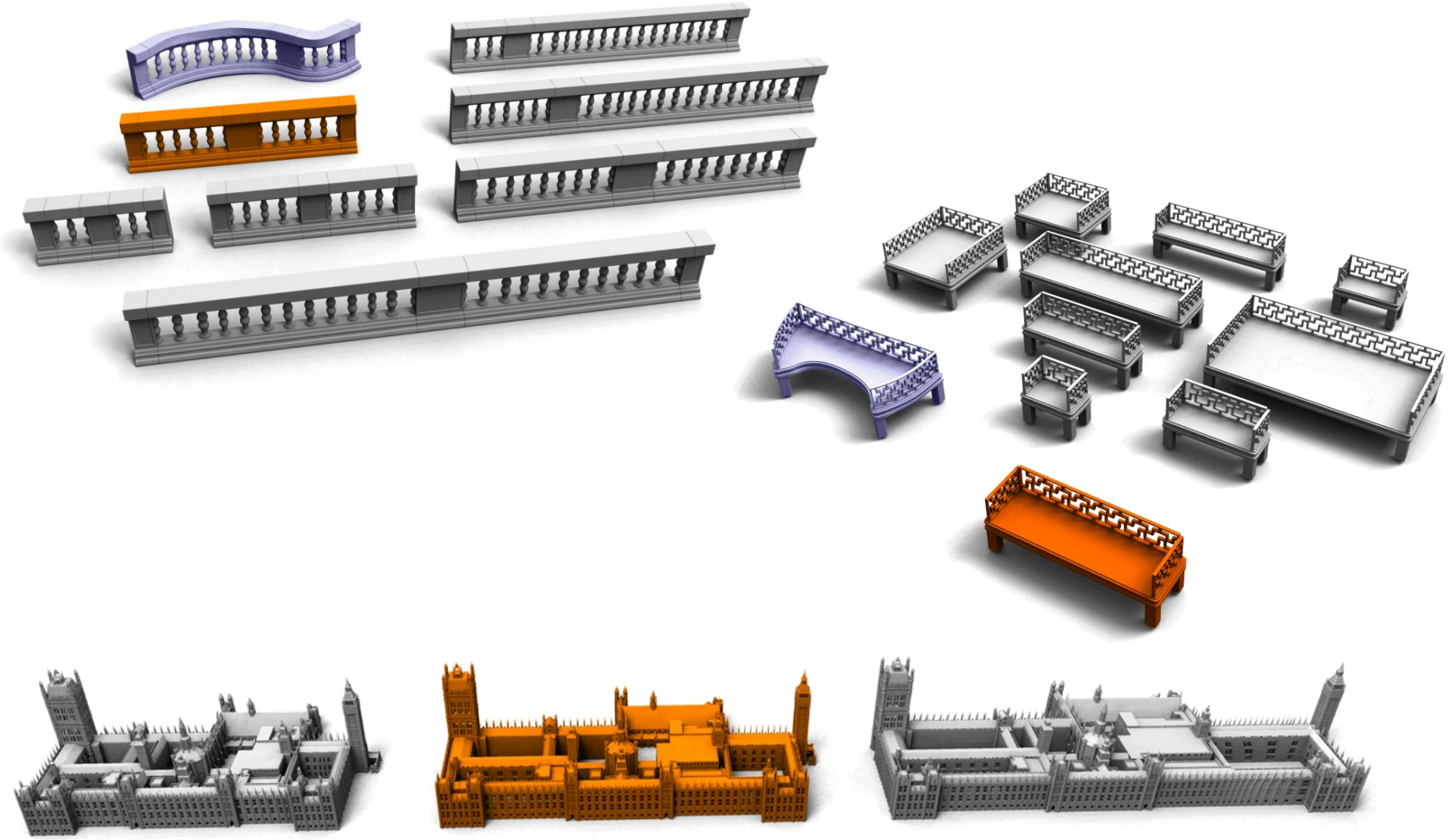


[Mitra et al. 2008]



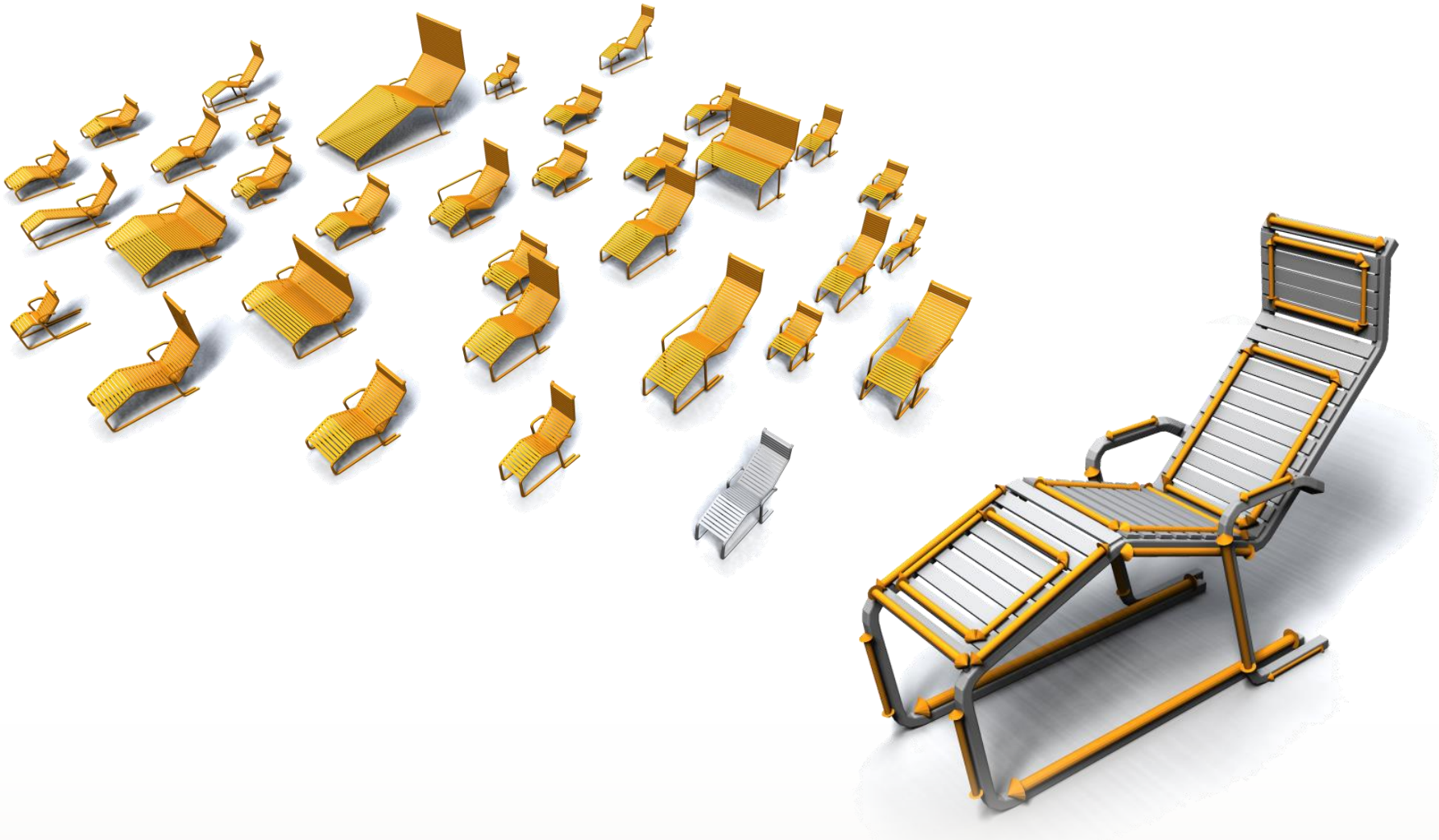
[Pauly et al. 2008]

Regularity Aware Deformation



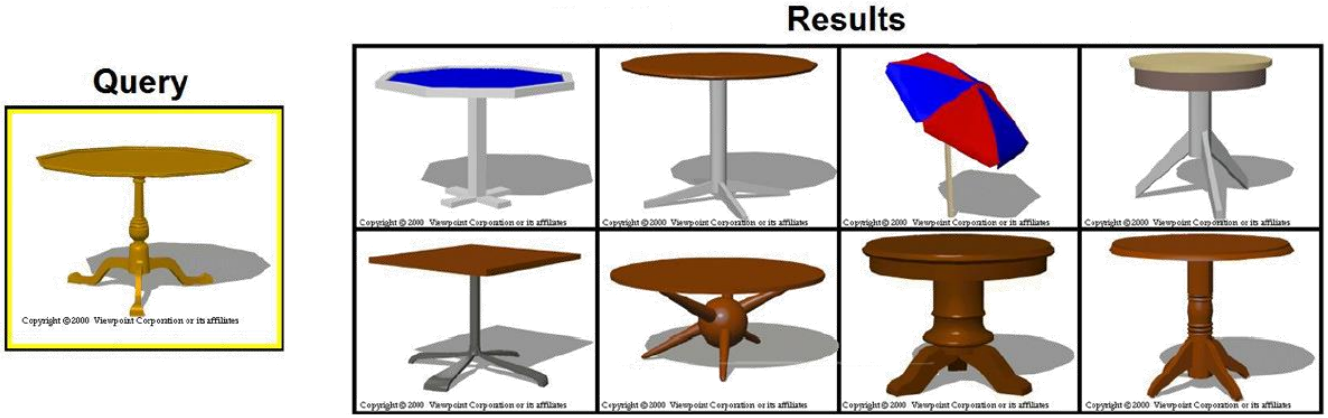
[Bokeloh et al. 2011]

Algebraic Shape Editing

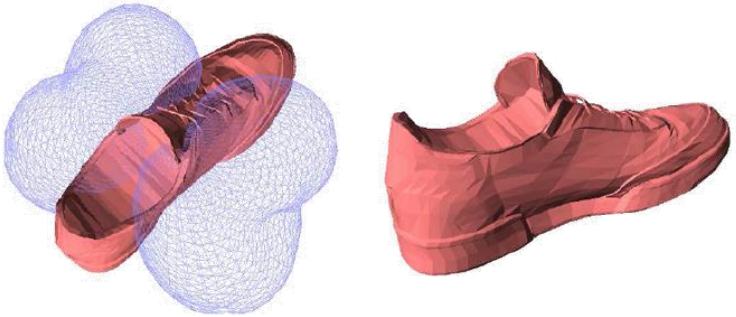


[Bokeloh et al. 2012]

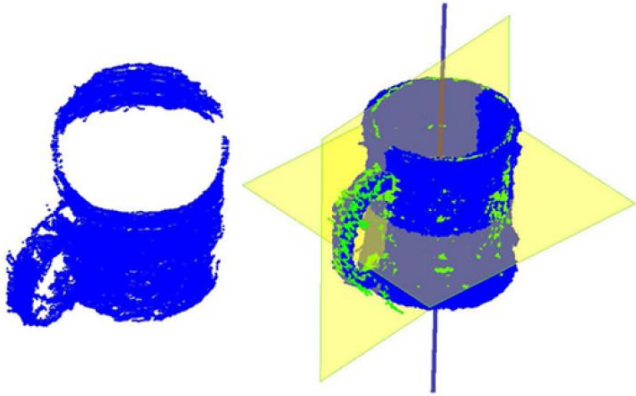
Shape Recognition



[Kazhdan et al. 2004]

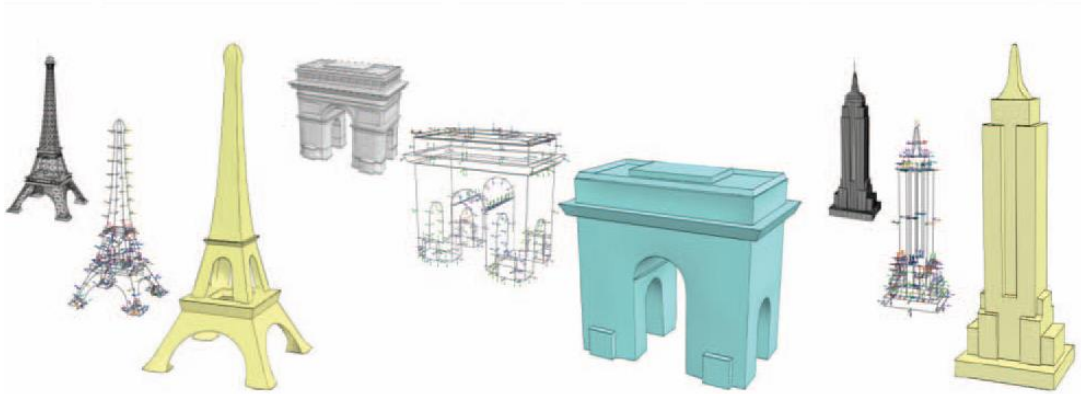


[Podolak et al. 2006]

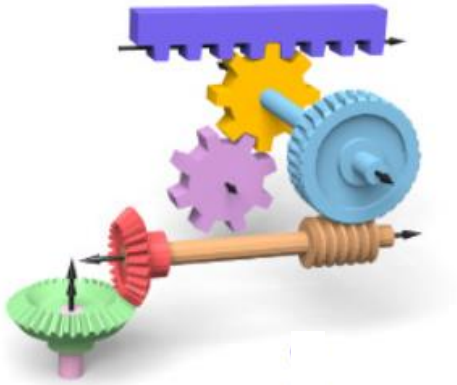


[Thrun et al. 2005]

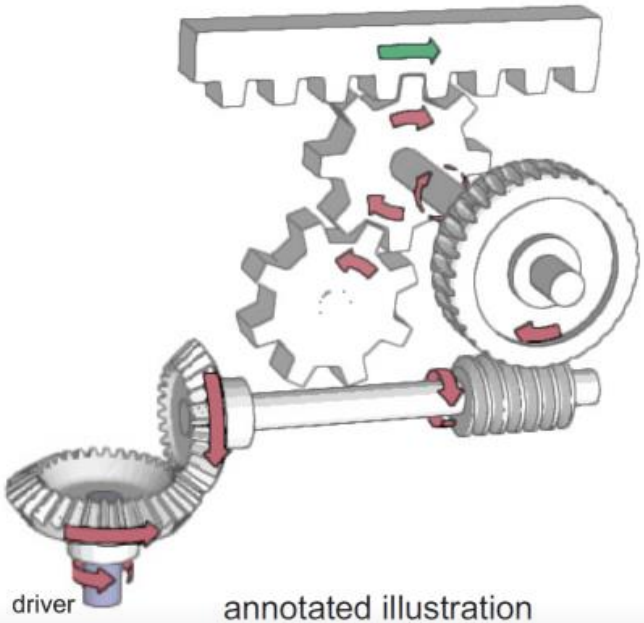
Shape Understanding



[Mehra et al. 2009]



[Mitra et al. 2010]



Conclusions

Symmetry

Principle

- Absence of information
- Invariance under operations

Structure

- **Global Symmetry:** transformation groups
- **Partial Symmetry:** permutations of building blocks

Detection

- Pairwise matching (efficient pruning, segmentation)
- Regular transformations: estimate generators
- Intrinsic formulations

Applications

Different structural insights

- Correspondence
  - Equivalence
  - Pairwise relations
- Permutations
  - Building blocks
  - Shape grammar
  - Hierarchical encoding
- Regularity
  - Structural invariant
  - Regularity relations

⇒ Different Applications

Open Problems

Open Problems

Future Work & Open Problems

- Detection
  - Scalability
  - Partial intrinsic symmetry detection
  - Approximate (deformable) symmetry
- Modeling
  - More general, semantic symmetry
  - Equivalence of chairs, cars, houses? Avoid overfitting?
- Theoretical framework
  - Approximate group theory?