I N F O R M A T I K

Universität
des
Saarlandes
FR 6.2 Informatik

Dr. Ernst Althaus, Dr. Benjamin Doerr, David Steurer **SS 2005**

Exercises for Optimization

8. Assignment Due 24.06.2005

Exercise 1 (3 *points*)
Show that there are bounded (feasible) integer linear programs which do not have an optimal solution.
Hint. Two variables are sufficient. You can use the fact that for an irrational number a there is a sequence $(a_n)_{n\in\mathbb{N}}$ of rational numbers such that $a_n < a$ for all $n \in \mathbb{N}$ and $\lim_{n\to\infty} a_n = a$.

Exercise 2 (3 *points*)
Consider the integer linear program

$$\text{maximize } c^T x \text{ subject to } Ax \leq b, x \geq 0, x \in \mathbb{Z}^n$$

where all entries of A, b and c are *non-negative*. Show how to compute a feasible solution $\hat{x}$ in polynomial time such that its objective function value is no farther from optimal than

$$\sum_{i=1}^{n} c_i$$

Hint. Consider an optimal solution to the relaxation of the integer linear program.

Exercise 3 (4 *points*)
Formulate the satifiability problem for a boolean expression as an integer linear program.
For an arbitrary boolean expression ϕ with n boolean variables and m junctors, your program should have $\mathcal{O}(n+m)$ variables and constraints. Hence, you cannot assume that ϕ is e.g. in conjunctive normal form.

Exercise 4 $(1+1+1+2+2+2$ *points*)
Let A be a $n \times m$ matrix with entires $0, +1$, or -1. Show that the following are equivalent.

a) A is totally unimodular;

b) A^T is totally unimodular;

c) $\begin{pmatrix} A \\ I_m \end{pmatrix}$ is totally unimodular;

d) a matrix obtained from A through multiplication of rows and columns by -1 or permutation of rows and columns ;

e) $\begin{pmatrix} A \\ I_m \end{pmatrix}$ is unimodular;

f) all matrices obtained from A through pivot operations are totally unimodular;

g) A^{-1} is totally unimodular.

For showing a) $\Leftrightarrow$ b), a) $\Leftrightarrow$ c) and a) $\Leftrightarrow$ d) you get one point each. For showing a) $\Leftrightarrow$ e), a) $\Leftrightarrow$ f) and a) $\Leftrightarrow$ g) you get two points each.

Exercise 5 (1 *points*)
Find a matrix which is unimodular but not totally unimodular.