I N F O R M A T I K

Universität des Saarlandes
FR 6.2 Informatik

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Exercises for Optimization

9. Assignment Due 1.07.2005

Exercise 1 ($3+3+3$ *points*)

a) Show that the incidence matrix of a directed graph $D=(V,A)$ is totally unimodular.

b) A consecutive-ones matrix is a 0,1-matrix such that the ones of each row are consecutive.

Show that consecutive-ones matrices are totally unimodular.

c) An interval hypergraph is a hypergraph $\mathcal{H}=(V,\mathcal{E})$ on vertices $V=\{1,\ldots,n\}$ such that each edge in $\mathcal{E}$ consists of consecutive natural numbers.

A stable set in a hypergraph $\mathcal{H}=(V,\mathcal{E})$ is a subset S of V such that each edge $E\in\mathcal{E}$ contains at most one vertex in S.

Show that the maximum weight stable set problem on a hypergraph $\mathcal{H}=(V,\mathcal{E})$ with weights $w: V\to\mathbb{R}$ can be solved in polynomial time if $\mathcal{H}$ is an interval hypergraph.

Hint. For a) and b) you can either use theorems from the lecture or induction on the size of the considered submatrix.

Exercise 2 ($3+1$ *points*)
Given a directed graph $D=(V,A)$ and a flow $f: A\to\mathbb{R}_{\geq 0}$ such that

$$\sum_{A\ni a \text{ outgoing of } v} f(a) \quad = \sum_{A\ni a \text{ incoming to } v} f(a) \text{ for all vertices } v\in V, \tag{1}$$

can you find a representation of f as a non-negative linear combination of at most $|A|$ functions χ^C for simple cycles $C\subseteq A$ and

$$\chi^C(a)=\begin{cases}1 & a\in C\\ 0 & a\notin C\end{cases}$$

in time $\mathcal{O}(|V|\cdot|A|)$?
Show that a flow $f: A\to\mathbb{N}_0$ with (1) can be represented as a non-negative *integer* linear combination of simple cycles.
Hint. Try induction on the number of arcs which carry positive flow.

Exercise 3 (4 *points*)
Let $A \in \mathbb{R}^{m\times n}$. Show

$$\operatorname{herlindisc}(A) = \sup_{x\in[0,1]^n} \min_{y} \|A(x-y)\|_\infty$$

where the minimum is over all roundings y of x.
Hint. You might want to prove the following fact beforehand,

$$\sup_{x'\in[0,1]^k} \min_{y'\in\{0,1\}^k} \|A_0(x'-y')\|_\infty = \sup_{x'\in(0,1)^k} \min_{y'} \|A_0(x'-y')\|_\infty,$$

where A_0 is a $m \times k$-submatrix of A and the second minimum is over all roundings y' of x'.

Exercise 4 (3 *points*)
Compute the linear and hereditary linear discrepancy of the matrix

$$A = (1\ 2\ 4\ \ldots\ 2^{n-1}).$$