Algorithmic Game Theory
Summer 2010

Homework 1

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This assignment is **due on May 17** in the Tutorial.

Problem 1: (Pareto optimality) Let f be a strategyproof (actually it suffices to assume that f is monotone) voting rule that is onto. If $u \in \mathcal{U}$ is a preference profile and $a, b \in A, a \neq b$, two alternatives such that $u_i(a) > u_i(b)$ for any player i, then $f(u) \neq b$.

Problem 2: Find two different randomized voting rules that are strategyproof (both different from the deterministic dictatorship).

Problem 3: Design a game with 2 players and three strategies for each player: "truthtelling", "lie A" and "lie B" for each player, where "truthtelling" is a dominant strategy for both players.

Problem 4: Let a, b be two different outcomes. We say that a voting rule satisfies Independence of irrelevant alternatives (IIA) if whenever the outcome changes from a to b there has to be some player who previously prefered a to be b and then in his preference profile.

Consider the following voting system: The alternative with the highest weigthed sum of votes wins. Suppose that the weights for each one of the positions $1, \ldots, 4$ are $w_1 = 4, w_2 = 3, w_3 = 2, w_4 = 1$.

Show that this voting rule does not satisfy IIA.

Hint: Suppose that the preference of profile of the society is at first $\begin{pmatrix} a & a & c \\ b & b & d \\ c & c & a \\ d & d & b \end{pmatrix}$ and then candidate b leaves, while the relative preferences of the other candidates remain the same: $\begin{pmatrix} a & a & c \\ c & c & d \\ d & d & a \end{pmatrix}$.