max planck institut
informatik

Seminar:
Computational Geometry and Geometric Computing

Kickoff-Meeting

Eric Berberich **Ben Galehouse** **Michael Sagraloff**

Max Planck Institute for Informatics

April 14, 2010

Outline

Computational Geometry

- **Design and analyse** algorithms and data structures for geometric settings
- Combinatorial problems
  - how to reduce the overall number of operations
  - sparse data structures
- Algebraic problems
  - fundamental layer for many algorithms in CG
  - in particular, for non-linear objects
- **Examples**: Convex hull of points (in plane/space), nearest-neighbor queries, Voronoi diagrams, arrangement computations, topology of curves and surfaces, polynomial system solving,...

Geometric Computing

- **Implement** algorithms and data structures for geometric settings
- doubles (fixed precision floating-points) are evil - usually
- sufficient for many instances in linear scenarios, but
- for non-linear objects we have to deal with larger errors in floating point computations
- "algorithm engineering": filter techniques, parallel evaluation

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CG vs. GC

Computational Geometry

- leads to theoretically best algorithms
- ignores constant factors, relies on unimplemented earlier work
- often makes assumption: General position, real-RAM (exact computation with real values)

Geometric Computing

- reformulate algorithms to handle any input
- "real-RAM" implementation
- algebraic methods in the literature are either numerical (fast, not certifying) or symbolic (slow, exact)
- combining both worlds to increase efficiency

The Group

- Department 1: Headed by Kurt Mehlhorn
- 5 subgroup leaders
- about 20 PostDocs, about 20 PhD students, 2 secretaries
- many HiWis (research assistent), B.Sc.-, M.Sc.-students
- Subgroups:
  - Foundations and Discrete Maths
  - Combinatorial Optimiziation
  - Bio-inspired computing
  - Algorithmic Game Theory
  - *Geometric Computing*

Involvement of the subgroup

- Research in Computational Geometry was always there (various number of people)
- Geometric Computing (really compute on PC)
- LEDA[1] has a geometry kernel since early 90s - for linear objects (more recent: circles)
- 1996: CGAL[2] was founded by research sites across Europe (one is D1)
- Goal: Make theory available as software!
- several EU-Research projects

[1] Library of Efficient Data Structures and Algorithms
[2] Computational Geometry Algorithms Library

Involvment of the subgroup - cont'd

- since 2001: focus on non-linear objects in GC-seeting: circles, conics,and beyond
- 2001-2004: Effective Computational Geometry (ECG)
- 2004-2007: Algorithmc for Complex Shapes (ACS)
- 2001: D1 founded EXACUS-project[3]
- since 2006: EXACUS merges into CGAL (merge not complete, but no more dev in EXACUS)

[3] Libary for Efficient and eXact Algorithms for CUrves and Surfaces

Eric Berberich

- Grew up in Saarland, joined Saarland Uni 1999
- 2001: Seminar Effective Computational Geometry
- 2002: Lecture Effective Computational Geometry
- 2002: Fopra about Arrangements of Conics
- 2004: Diplom about Quadric intersection curves
- 2008: PhD on 2.5 dimensional arrangements (of algebraic objects)
- 2009: PostDoc in Tel-Aviv on arrangements and Minkowski deconstruction
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Ben Galehouse

- Grew up in Ohio
- PhD at New York Universiy 2009 (Subdivision based meshing)
- Focus so far is on subdivision based meshing, but starting to diversify.
- Started Postdoc at MPII, November 2009

Michael Sagraloff

- Grew up in Bavaria, joined University in Bayreuth 1998 (Math/Algebraic Geometry)
- 2002-2005: PhD at Saarland University (Algebraic Geometry: classification of algebraic curves)
- 2005-now: Postdoc at MPII - heading the Geometric Computing group
  - main interest in the "algebraic stuff"
  - complexity analysis of root solvers/topology computation
  - combination of numerical/symbolic methods
  - adaptive algorithms

Now: Who are you?

- Please introduce yourself with name, studies, previous courses, interests
- What attracts you in our course? Expectations?
- Use a few sentences!

3 Goals

- *Cover* a specific exciting area of research in various *details*, i.e., with the help of (recent) scientific articles
- Make you familiar with (scientific) work: reading, understanding, discussion, presentation, writing
- Join our group as HiWi or for a thesis

Weekly Meeting

- There won't be Pizza every week.
- We meet for 2hours
  - 60-75min talk
  - 10-15min questions
  - 10-15min discussion

Issues

- Problem 1: Find a time that suits all attendees
- Problem 2: When is the first usual meeting?

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Grading rubic

7 CreditPoints

- Mandatory: Participation
- 75% Talk
- 25% Summary of up to 4 pages (LaTeX)

Supervision

We are happy to

- preview your slides (strongly encouraged)
- comment on draft versions of your topic summary

Contact us by email to schedule meetings.

Literature

General:

- Book: de Berg, Cheong, van Kreveld, Overmars: *Computational Geometry: Algorithms and Applications*
- Book: Basu, Pollack, Roy: *Algorithms in Real Algebraic Geometry* Available online at `http://www.math.purdue.edu/ sbasu/`
- Book: LaValle: *Motion Planning* Available online at `http://planning.cs.uiuc.edu/`
- Website: `www.cgal.org`
- CGGC-Lecture Winter Term 2009/2010 with a lot of material: `http://www.mpi-inf.mpg.de/departments/d1/teaching/ws09_10/CGGC`

Topic:

- Main references provided by us
- Using more is encouraged

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Minkowski Sums and Offsets

Minkowski sum

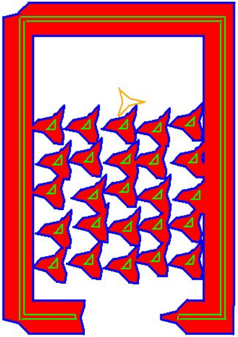
For sets X, Y: $M = X \oplus Y := \{x + y | x \in X, y \in Y\}$

X, Y polygonal

- Convex decomposition
- Convolution cycle

X polygonal, Y a disk = offset

- Exact constructions
- Approximation



- Extension: Real-time offsets of triangle soups on the GPU

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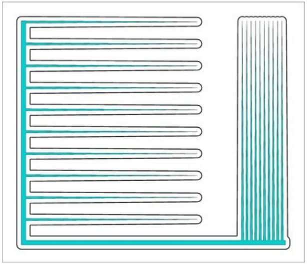

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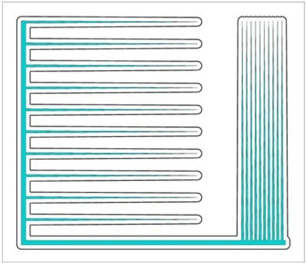

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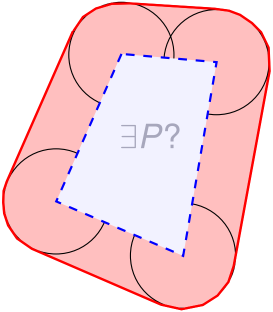
Offset Deconstruction

Reverse direction

Given a polygonal shape P, two real parameters $r, \varepsilon > 0$. Is Q a close approximation (Hausdorff-distance $\leq \varepsilon$) of the r-offset of another (unknown) polygonal shape $P^\star$.

If so, find a nice-looking P.



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- greedy construction for convex input

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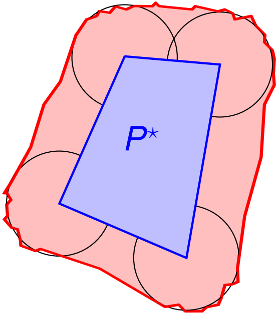

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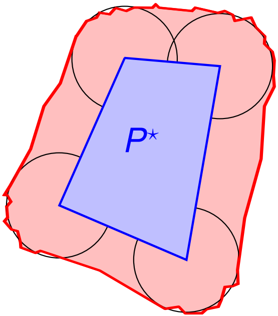

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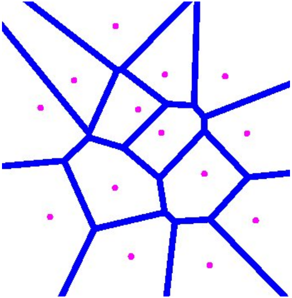
Voronoi Diagram and Medial Axis

Voronoi Diagram

Given a set of sites in the plane, compute a decomposition of the plane such that each site owns one cell and is closest site to all points of the cell it owns.



- New algorithm that relates to edges of a VD to the medial axis of an augmented planar domain
- Can be also used for computing offsets

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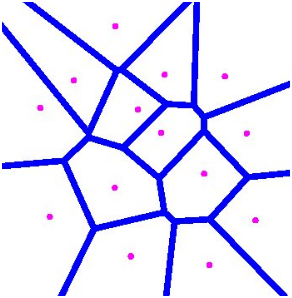

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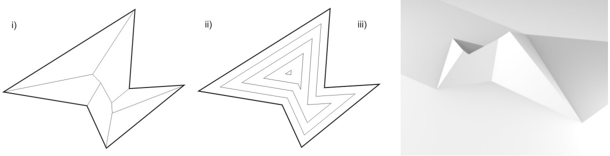
2D Straight Skeleton

Straight Skeleton (simplified definition)

The straight skeleton of a polygon is given by moving the edges at a fixed rate, and watching where the vertices go.

- similar to *medial axis*
- not a voronoi diagram!
- interesing applications: “roofing”, terrain reconstructions, origami folding, planar motion planning, offset construction

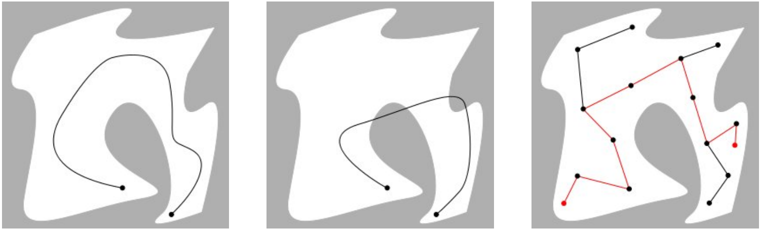


Robot Motion Planning (Sampling)

Moving pieces

Is is possible to move a piece in a space from position *A* to posititition *B* without colliding with any obstacle?

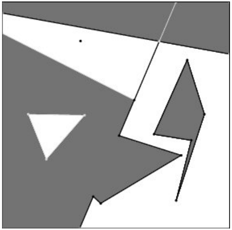
- Definition of Configuration Space
- Techniques: Rapid Exploring Dense Tree, Visibility Roadmap

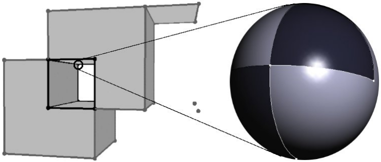


Nef Polyhedra (in 2d and 3d)

Nef-Polyhedra

A Nef-polyhedron in dimension d is a point set $X \subset \mathbb{R}^d$ generated from a finite number of open halfspaces by set complement and set intersection operations.



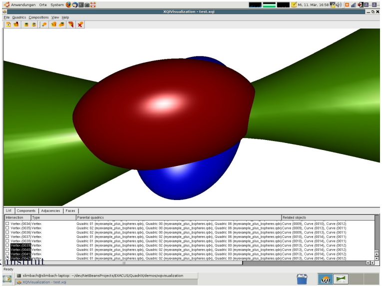


Towards “Nef”-Quadrics

Quadric

Algebraic surface of degree 2: Ellipsoids, Paraboloid, Hypberboloid,. . .

- Given a set of intersecting quadrics: Compute how they are connected, and which volumes they induced (bounded by faces, edges, vertices)
- Two steps: (1) Adjacency graph (2) Neighborhood maps



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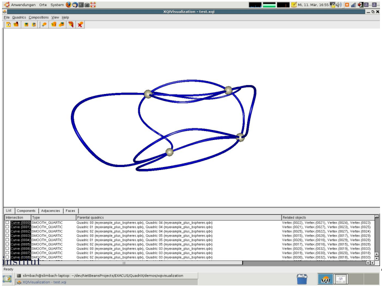

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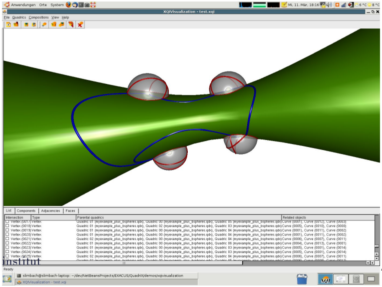


Subdivision

- One approach to analyzing implicit surfaces is to subdivide a finite domain into regions until each region is “simple”.
- Using interval arithmetic system, it is possible to handle non-singular problems in an essentially numeric way.
- Literature:
  - Simon Plantinga and Gert Vegter “Isotopic Meshing of Implicit Surfaces”, 2005 — A classic and very well studied algorithm.
  - Long Lin and Chee Yap “Adaptive Isotopic Approximation of Nonsingular Curves: the Parametrizability and Nonlocal Isotopy Approach”, 2009 — An example of recent work extending the Plantinga-Vegter algorithm.

Numerical solvers

- Finding roots of polynomials using numerical (floating point) computation is one of the oldest problems in computer science literature.
- Typically, these methods combine some variant of Newton iteration with various theoretic results and guarantee convergence when all roots are simple.
- Literature:
  - Dario Andrea Bini "Numerical computation of polynomial zeros by means of Aberth's method", 1995

Shortest Path among Discs

- The problem is to find the shortest path between two points in the plane which avoids a given collection of discs.
- Problem is interesting because exact solutions can be found, even though the problem is in a sense transcendental.
- Literature:
  - Yap, et al. “Shortest Path amidst Disc Obstacles is Computable”, 2005

Snap Rounding

- One approach to the analysis of exact geometric structures is to find and compute using a low precision approximation of them.
- The result is a solution to an approximated problem, but certain properties can be preserved.
- Literature:
  - Dan Halperin and Eli Packer “Iterated snap Rounding”, 2001
  - Arno Eigenwillig et al., “Snap Rounding of B’ezier Curves”, 2007

Controlled Perturbation

Problem

Exact computation is often very challenging: you have to deal with degeneracies and costly symbolic computations. But: Can we compute the exact result for nearby input, at least, without this additional effort?

- framework for CP.
- general analysis: how much precision is needed for certain geometric predicates.
- application to well-known algorithms
- many research problems: so far, CP has only been applied to “linear problems”.

Eval - a real root solver

Problem

Given a polynomial $f \in \mathbb{R}[x]$, determine a set of disjoint intervals (boxes) that contain all real (complex) roots of f.

- "simplest" approach checks whether an interval contains a root of f or its derivative f'.
- if an interval I contains no root of f', then it suffices to check endpoints of I.
- testing for roots with interval arithmetic
- efficiency in comparison to more sophisticated methods (Sturm/Descartes)
- generalization to complex roots and polynomials with approximate coefficients.

Quadratic interval refinement

Problem

Given a polynomial $f \in \mathbb{R}[x]$ and an isolating interval for a root of f, how can we get arbitrary good approximations of the root in an efficient way.

- simple binary research is not good enough (only linear convergence)
- Newton like methods (approximation of the polynomial by a linear function) lead to quadratic convergence.
- complexity of approximating a root up to a certain precision.

Algebraic Curve Analysis via RUR

Problem

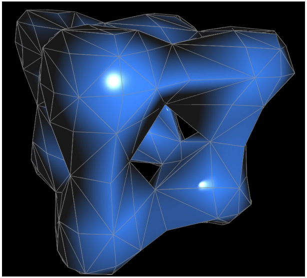
Given a polynomial $f \in \mathbb{Z}[x, y]$, describe the set C of all points $(x, y) \in \mathbb{R}^2$ with $f(x, y) = 0$. Compute a linear isotopic approximation of C.

- CAD approach vs. RUR approach
- determine critical points and connect them in appropriate manner

Surface Analysis/Triangulation

Problem

Given a polynomial $f \in \mathbb{Z}[x, y, z]$, describe the set S of all points $(x, y, z) \in \mathbb{R}^3$ with $f(x, y, z) = 0$. Compute a triangular mesh which is isotopic to S.



- $f = z^4 + (-5) \cdot z^2 + (y^4 + (-5) \cdot y^2 + (x^4 + (-5) \cdot x^2 + 10))$
- "generalization" of topology computation for curves
- new ideas for adjacency computation/vertical lines
- open problems: arrangements of surfaces, complexity of triangulation

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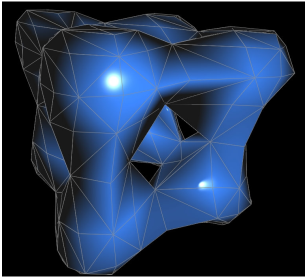

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