This assignment is due on Apr 22/23 in your tutorial session. You are allowed (even encouraged) to discuss these problems with your fellow classmates. All submitted work, however, must be written individually without consulting someone else’s solutions or any other source like the web.

**Problem 1:** Show how to convert the polyhedron \( \{ x \in \mathbb{R}^n \mid Ax \geq b \} \) into an equivalent polyhedron where the variables cannot take negative values.  
*Hint: You’ll have to use two variables per old variable.*

**Problem 2:** Show how to convert the polyhedron \( \{ x \in \mathbb{R}^n \mid Ax \geq b, x \geq 0 \} \) following into an equivalent polyhedron in standard form.  
*Hint: You’ll have to introduce one new variable per constraint.*

**Problem 3:** Consider the following linear program

\[
\begin{align*}
\text{minimize} \quad & 7x_1 + 3x_2 \\
\text{subject to} \quad & 4x_1 + x_2 \geq 10 \\
& 4x_1 + 2x_2 \geq 16 \\
& x_1 + 2x_2 \geq 8
\end{align*}
\]

Plot the feasible region and find the optimal solution.

**Problem 4:** Propose three alternative objective functions for the above LP such that

i) The optimal solution is unique

ii) There are multiple optimal solutions

iii) There is no optimal solution (the objective function is unbounded).