This assignment is due on Apr 29/30 in your tutorial session. You are allowed (even encouraged) to discuss these problems with your fellow classmates. All submitted work, however, must be written individually without consulting someone else’s solutions or any other source like the web.

Problem 1: Let $P \subseteq \mathbb{R}^n$ be a polyhedron and be $\pi_k(P)$ the projection of $P$ onto its first $k$ coordinates. Assume $x \in P$. For each of the following statements either disprove with a counter example or provide a short proof.

i) If $x$ is an extreme point of $P$ then $\pi_k(x)$ is an extreme point of $\pi_k(P)$.

ii) If $\pi_k(x)$ is an extreme point of $\pi_k(P)$ then $x$ is an extreme point of $P$.

Problem 2: Is it the case that every non-empty polyhedron has at least one extreme point? Justify your answer.

Problem 3: Consider the polyhedron \( \{x \in \mathbb{R}^3 \mid Ax = b, \ x \geq 0\} \) where

\[
A = \begin{bmatrix}
1 & 1 & 2 \\
2 & 0 & 4 \\
\end{bmatrix} \quad b = (0, -2)
\]

List all its bases and their associated basic solutions specifying which are feasible.

Problem 4 (extra credit): Let $A_1, \ldots, A_m$ be vectors in $\mathbb{R}^n$ and assume $m > n$. The convex hull of these vectors is

\[
C = \left\{ \lambda_1 A_1 + \cdots + \lambda_m A_m \mid \sum_{i=1}^{m} \lambda_i = 1 \text{ and } \lambda_i \geq 0 \text{ for } i = 1, \ldots, m \right\}.
\]

Show that each $y \in C$ can be written as a convex combination of just $n + 1$ of these vectors.