Homework 6

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This assignment is **due on May 27/28** in your tutorial session. You are allowed (even encouraged) to discuss these problems with your fellow classmates. All submitted work, however, must be *written individually* without consulting someone else's solutions or any other source like the web.

Problem 1 **(4 points)**
Consider an instance of the maximum assignment problem with n vertices and a cost function $c : E \to \mathbb{Z}^+$. For $\delta = \frac{1}{n+1}$ the algorithm we saw in class finds an optimal assignment in at most $O(c_{\max} n^2)$ iterations. Show that the analysis is tight by providing an instance where the algorithm runs for $\Omega(c_{\max} n^2)$ iterations.

Problem 2 **(4 points)**
Prove the following theorem.

Theorem 1 *The diameter of the perfect matching polytope of a simple graph $G = (V, E)$ is at most $1/4\,|V|$.*

Hint: Consider the symmetric difference $M \Delta N$ and construct a chain of adjacent perfect matchings from M to N. Use key lemma that we proved in class (see below). Bound the maximum total length of such a chain.

Lemma 2 *Let M and N be perfect matchings in the graph $G = (V, E)$, and let x^M and x^N be the incidence vectors of the corresponding vertices in the perfect matching polytope. Vertices x^M and x^N are adjacent if and only if $M \Delta N$ is a circuit.*

Problem 3 **(4 points)**
Show that the inverse of an upper diagonal matrix is an upper diagonal matrix and can be computed in time proportional to the product of the dimension of the matrix times the number of nonzero entries of the matrix. Also, show that the product of two upper diagonal matrices is upper diagonal.

Problem 4 **(1 point)**
Use your LP-solver to maximize $x + y$ subject to the constraints

$$4.5678923454x - 12352452.456y \leq 1.00000001 \tag{1}$$
$$4.5678923454x - 12352452.455y \geq 1\,. \tag{2}$$

Compare and discuss the solutions in the tutorial session.