Optimization
Summer 2010

Homework 9

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This assignment is **due on June 24/25** in your tutorial session. You are allowed (even encouraged) to discuss these problems with your fellow classmates. All submitted work, however, must be *written individually* without consulting someone else's solutions or any other source like the web.

Problem 1 **(4 points)**

Give an example of an integral matrix A and an integer vector b such that the polyhedron $P := \{x \mid Ax \leq b\}$ is integral, while A is not unimodular.

Problem 2 **(2 points)**

Consider the *knapsack problem*: Given is a weight bound K and a set of n items, each with a nonnegative weight w_j and *nonnegative* profit c_j, $j = 1, \ldots, n$. Determine a subset of items $S \subseteq \{1, \ldots, n\}$ of maximum total profit, $\sum_{j \in S} c_j$ such that $\sum_{j \in S} w_j \leq K$.

Give an ILP formulation for the knapsack problem and explain it.

Problem 3 **(4 points)**

Consider the *shortest path problem*: Given is a directed graph $G = (V, A)$ with two distinct nodes $s, t \in V$. For each $(i, j) \in A$, we are given a nonnegative length, $c(i, j)$. The length of a path is defined as the sum of the lengths of its arcs. The task is to find a shortest path, that is a path of minimum length, from s to t.

Give an ILP formulation for the shortest path problem and explain it.

Problem 4 **(4 points)**

Show that the following polyhedron has Chvatal rank 2.

$$\begin{aligned} -2x_1 + x_2 &\leq 0 \\ 2x_1 + x_2 &\leq 6 \\ -x_2 &\leq -1 \end{aligned}$$