Optimization	**Summer 2010**

Lecture 5 — April 26

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5.1 Tableau implementation of the simplex algorithm

The basic idea is to keep a table containing all the necessary information to facilitate the execution of an iteration of the simplex algorithm. Rather than computing the table from scratch in each iteration it is more efficient to update the table at the end of each iteration. Here is the information we keep in the table

$-\mathbf{c}'_\mathbf{B}\mathbf{A}_\mathbf{B}^{-1}\mathbf{b}$	$\mathbf{c}' - \mathbf{c}'_\mathbf{B}\mathbf{A}_\mathbf{B}^{-1}\mathbf{A}$
$\mathbf{A}_\mathbf{B}^{-1}\mathbf{b}$	$\mathbf{A}_\mathbf{B}^{-1}\mathbf{A}$

(5.1)

Here are some observations about this table to keep in mind:

1. The table has $m+1$ rows and $n+1$ columns. Let us index the entries so that $(0,0)$ is the left-most top-most entry.
2. The entry $(0,0)$ stores minus the cost of the current solution
3. Entries $(1,0)$ through $(m,0)$ store the values of the basic variables.
4. Entries $(0,1)$ through $(0,n)$ store the reduced costs $\bar{\mathbf{c}}$.
5. Suppose the ith row corresponds to the basic index b_i, then:
   i) The entry $(0,i)$ equals zero, since $\bar{c}_{b_i} = 0$.
   ii) The vector defined by entries $(1,i)$ through (m,i) equals $\mathbf{e_i}$, since $\mathbf{A}_\mathbf{B}^{-1}\mathbf{A}_{\mathbf{b_i}} = \mathbf{e_i}$.

5.2 A typical iteration

In one iteration of the simplex algorithm we do the following steps:

1. Find column j such that $\bar{c}_j < 0$. If no such index exists then we are at an optimal solution.
2. Let $\mathbf{u}$ be the jth column of the matrix in the bottom right cell of (5.1). If $\mathbf{u} \leq \mathbf{0}$ then the objective is unbounded.
3. Find row i such that $u_i > 0$ minimizing x_{b_i}/u_i.
4. Pivot around the entry (i,j) of our table.

In this last step we derive a new table using the following row operations:

$$\text{new row } k = (\text{old row } k) + \alpha_k\,(\text{old row } i),$$

where the coefficients α_k are chosen so that the jth column in the new table equals $\begin{bmatrix} 0 \\ \mathbf{e_i} \end{bmatrix}$.