Universität
des
Saarlandes
FR 6.2 Informatik

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Exercises for Algorithmic Game Theory

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss11/AGT/

Assignment 3 Deadline: Fr 6.5.2011

Exercise 1 *Two-player zero-sum game*

Write the linear programs that determine the optimal strategies for the row and column player in standard form. That is, the linear program for the column player should be written as $\min\{c^T x | Ax \leqslant b, x \geqslant 0\}$ (and the other linear program should be its dual). Let the number of strategies for the row player be m and for the column player be n.
Hint: how many variables do you have?

Exercise 2 *Best response criterion*

a) Give an example of a Nash equilibrium in a two-player game where one player uses a pure strategy and one player uses a mixed strategy.

b) Determine all Nash equilibria of the following game using the best response criterion. The first player (A) picks a row and the second player (B) picks a column. How many options do you need to check?

$$A = \begin{pmatrix} 3 & 3 \\ 2 & 5 \\ 0 & 6 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 4 \\ 2 & 6 \\ 3 & 1 \end{pmatrix}$$

Exercise 3 *Lemke-Howson*

Give (and prove) an upper bound for the number of vertices that the Lemke-Howson algorithm may visit before finding a Nash equilibrium. The input is an $n \times n$ matrix A.

Exercise 4 *Lemke-Howson 2*

Why is it sufficient to consider the polytope $Az \leqslant 1$ in the Lemke-Howson algorithm? Show how to reduce the general case to this case. What can you do in the special case where the most straightforward approach fails? (Think about what A looks like in this special case, and how you can modify it without changing the set of Nash equilibria (i.e. the strategies).)