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Exercises for Algorithmic Game Theory

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss11/AGT/

Assignment 5 Deadline: Fr 20.5.2011

Exercise 1 *Public project*

a) Show that in the VCG mechanism with the Clarke pivot rule for a public project, $\sum_i p_i < C$ unless $\sum_i v_i = C$.

b) Define payments such that we still have a VCG-mechanism, but the costs of the project is guaranteed to be paid by the agents. What is the problem with these payments?

Exercise 2 *Interval auctions*

Consider an auction for items $1, \ldots, m$ where each bidder is single-minded and desires an interval of consecutive items, i.e, $S_i = \{j | k_j \leqslant j \leqslant l_j\}$ where $1 \leqslant k_j \leqslant l_j \leqslant m$. Prove that in this case the socially efficient allocation can be determined in polynomial time.
Hint: use dynamic programming.

Exercise 3 *An efficient auction*

Consider a combinatorial auction for m items among n bidders, where each valuation is represented as a vector of $2^m - 1$ numbers (a value for each possible subset). Prove that the optimal allocation can be computed in polynomial time. What is the size of the input?

Exercise 4 *Single-minded bidders*

Consider an auction with single-minded bidders who are all interested in at most two items. Show that in this case an optimal allocation can be found in polynomial time.
You need to reduce this problem to a weighted matching problem in general nonbipartite graphs. In the matching problem, you select a set of edges so that no node occurs in more than one edge. The value of the goal function is then the total value of all the selected edges.

a) What should the value of an edge between items A and B be?

b) How do you represent bids for one specific item?

c) How many nodes will there be in the graph? How many edges?

d) What is the running time of your algorithm?