Exercise 1 Return of the double-minded bidders
Let us consider a seemingly easier case of double-minded bidders. Consider a setting where each player $i$ wants either a specific set $S_i^*$ at value $v_i^*$ or all the items at value $\tilde v_i$. Modify the greedy mechanism for single-minded bidders as follows.
First, hold a Vickrey auction for the whole set of items. Compare this to the output of the greedy mechanism with subsets $S_i^*$. Take the output which gives the highest social welfare and use the payments of the corresponding mechanism.
Show that this mechanism is not truthful.

Exercise 2 Pigou’s example
Pigou’s example is the selfish routing game discussed in class, with two parallel links. Modify Pigou’s example so that the lower edge has cost function $c(x) = x^d$ for some $d \geq 1$.
What is the price of anarchy of the resulting selfish routing network, as a function of $d$?

Exercise 3 Machine covering
Consider machine scheduling where the objective is to \textit{maximize} the minimum load. Jobs are selfish (i.e. controlled by selfish agents) and want to be on a machine with low load. The load of a machine is the total size of the jobs that are on this machine divided by its speed.

a) Show that the price of anarchy is unbounded already on two machines if one machine is at least twice as fast as the other.

b) The price of stability is the ratio of the cost of the best Nash equilibrium to the optimal cost. When is the price of stability unbounded for two machines?