Universität des Saarlandes
FR 6.2 Informatik

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Exercises for Algorithmic Game Theory

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss11/AGT/

Assignment 8 Deadline: Fr. 17.6.2011

Exercise 1 *Undirected Shapley network game*

Consider the following instance of a network creation game. There are n players with n different source vertices $s_1, s_2, \ldots, s_n$ and a common target vertex t. Every source vertex s_i is connected to the target vertex t by an edge with cost $1/i$. Moreover, every source vertex s_i is connected to a vertex v by an edge with cost 0, and this vertex v is connected to the target vertex t by an edge with cost $1+\varepsilon$. As opposed to the instance discussed in the lecture, we now assume that all edges are undirected. What is the Price of Stability?

Exercise 2 *Load balancing on two identical machines*

Let G be any instance of the load balancing game with three tasks that should be placed on two identical machines. Show that any pure Nash equilibrium for G is optimal, i.e. $\mathrm{cost}(A) = \mathrm{opt}(G)$ for any equilibrium assignment A.

Exercise 3 *Load balancing on identical machines*

Show, for every $m \in \mathbb{N}$, there exists an instance G of the load balancing game with m identical machines and $2m$ tasks that has a Nash equilibrium assignment $A : [n] \to [m]$ with

$$\mathrm{cost}(A) = \left(2 - \frac{2}{m+1}\right) \cdot \mathrm{opt}(G).$$

Exercise 4 *Load balancing and the golden ratio*

Prove that the price of anarchy for pure equilibria on instances of the load balancing game with two tasks and two machines with possibly different speeds corresponds to the golden ratio $\phi = \frac{1}{2}(1+\sqrt{5})$. That is, show that

a) there is a game instance G admitting an equilibrium assignment A with $\mathrm{cost}(A) = \phi \cdot \mathrm{opt}(G)$.

b) for every game instance G and every equilibrium assignment A for this instance, it holds $\mathrm{cost}(A) \leqslant \phi \cdot \mathrm{opt}(G)$.