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| Optimization<br>Summer 2011 | Homework 7 | Khaled Elbassioni<br>Anke van Zuylen |

This assignment is **due on July 7** in class. You are allowed (even encouraged) to discuss these problems with your fellow classmates. All submitted work, however, must be written individually without consulting someone else's solutions or any other source like the web.

**Problem 1:** Let $A \in \mathbb{R}^{m \times n}$ have full row-rank, and assume that $c \in \mathbb{R}^n$ is not a linear combination of the rows of $A$. Let $y > 0$ satisfy $Ay = b$, $Y = \text{diag}(y_1, \ldots, y_n)$, and $\beta \in (0, 1)$ be a given number.

(i) The Karush-Kuhn-Tucker (KKT) conditions for optimality for the problem:

$$\min\{c'x \mid Ax = b,\ (x - y)'Y^{-2}(x - y) \leq \beta^2\}$$

assert that the negative gradient of the objective function should be contained in the cone spanned by the gradients of the binding constraints. Use this fact to derive a closed form expression for the optimizer $x^*$ of the above quadratic programming problem.

(ii) Consider the problem of minimizing $\|Yc - YA'\lambda\|^2$ over $w \in \mathbb{R}^m$. Show by setting the gradient of the objective function equal to zero that $\lambda = (AY^2A')^{-1}AY^2c$ solves this problem.

**Problem 2:** Starting from the point $(x_1, x_2, x_3, x_4) = (1, 1, 2, 2)$, apply the potential reduction algorithm towards solving the following LP.

$$\begin{array}{ll} \text{minimize} & x_1 + x_2 \\ \text{subject to} & 2x_1 + x_2 - x_3 = 1 \\ & x_1 + 2x_2 - x_4 = 1 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{array}$$

Start with $(s_1, s_2, s_3, s_4) = (\frac{1}{2}, \frac{1}{2}, \frac{1}{6}, \frac{1}{6})$ for the slacks of the dual problem. Do only the first 3 iterations. Show the reduction in the potential function in each iteration.