UNIVERSITÄT DES SAARLANDES

Problem Set 1

Topological Methods in Geometry

SS 2011

Problem 1 (Equipartition by L-Shapes)**.**
An L-shape consists of a point in the plane and two rays emanating from it - one horizontal and one vertical. The vertical ray could be upwards or downwards and similarly the horizontal ray could be to the right or to the left. This way, there are four different types of L-shapes. Show that given n red and n blue points in the plane, there is an L-shape so that both the closed regions of the plane defined by it contain at least $\lceil n/2 \rceil$ red and at least $\lceil n/2 \rceil$ blue points.

Problem 2 (Equipartition by orthogonal lines)**.**
Show that for any set of n points in the plane there exists a pair of intersecting lines so that each of the (closed) quadrants defined by them contains at least $\lceil n/4 \rceil$ of the points. Can we always find a pair of lines which are perpendicular to each other?

Problem 3 (1D Helly's theorem via Brouwer's Fixed Point Theorem)**.**
Let I be a finite set of closed intervals on the real line such that each pair of them intersect. Prove that all of them intersect at a point. Can you prove this using Brouwer's fixed point theorem? *Hint*: For any point $x \in \mathbb{R}$ and any interval $a \in I$, let $p_a(x)$ be the point on a that is the closest to x. Define a function f which maps x to $\frac{1}{|I|} \sum_{a \in I} p_a(x)$.

Problem 4 (Matching Points)**.**
Show that given any set of n red points and n blue points in the plane in general position (no three points on a line), it is possible to join each red point to a distinct blue point with a straight segment such that the n segments matching the red and blue points do not intersect.