

---

**Problem Set 8**  
**Topological Methods in Geometry**

SS 2011

---

**Problem 1.** Prove that the Radon point is unique.

**Problem 2.** Given  $d + 2$  points  $P$  in  $\mathbb{R}^d$ , prove that there exists a plane spanned by  $d$  points of  $P$  such that the remaining two points are on different sides of this plane.

**Problem 3.** Complete the proof of topological Radon's theorem for any dimension by induction.

**Problem 4.** Given a continuous function  $f : \mathbb{S}^1 \rightarrow \mathbb{R}$  and any two points  $p, q \in \mathbb{S}^1$ , prove that one can always rotate the two points  $p$  and  $q$  around  $\mathbb{S}^1$  (without changing their position relative to each other) to get the points  $p'$  and  $q'$  such that  $f(p') = f(q')$ .

**Problem 5.** Compute  $\text{Ind}_{\mathbb{Z}_2}((\mathbb{R}^d \times \mathbb{R}^d) \setminus \{(y, y) \mid y \in \mathbb{R}^d\})$ .