Universität des Saarlandes
FR 6.2 Informatik

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Exercises for Graph Theory

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss11/graph_theory/

Assignment 2 Deadline: *Thursday*, April 28, 2011

Rules: There are 4 regular exercise problems. The first two serve as preparation for the test conducted usually in the exercise class (*but this time in the first 20 minutes of the lecture on April 26*). You should solve them, therefore, but you do not need to hand them in. The remaining two problems have to be handed in *nicely written up as you would do in a thesis* in the lecture on Thursday, April 28. Each problem can yield up to 4 points. You need to collect at least 50% of all these points (tests and written homework) from (i) the first three exercise sessions, (ii) the first six sessions, and (iii) the whole term.

General assumption: Whenever not specified otherwise, $G = (V, E)$ is a graph.

Exercise 1 (*oral homework, 4 points via test*)

Read, learn by heart, and understand all definitions in Sections 1.3 and 1.4 of the Diestel book. Be able to answer simple questions concerning them, give examples or counterexamples.

Exercise 2 (*oral homework, 4 points via test*)

(a) Show that if G contains a *walk* of length ℓ between the vertices u and v, then it also contains a *path* between u and v that has length at most ℓ.

(b) Prove that the graph distance $d_G : V \times V \to \mathbb{N}$ is a metric, that is, it satisfies (i) $\forall u, v \in V : d_G(u, v) = 0 \iff u = v$, (ii) $\forall u, v \in V : d_G(u, v) = d_G(v, u)$, and (iii) $\forall u, v, w \in V : d_G(u, w) \leq d_G(u, v) + d_G(v, w)$.

Exercise 3 (*written homework, 4 points*)

(a) Prove that if $\delta(G) \geq 2$, then G contains a cycle. [1P]

(b) Prove, without using Proposition 1.4.1, that each connected graph G contains a vertex v such that $G - v$ is connected. [1P]

(c) Prove the weak version of Theorem 1.3.4 stated right before it in the Diestel book, that is, the one where $d(G)$ is replaced by $\delta(G)$. [2P]

Exercise 4 (*written homework, 4 points*)

This exercise will show that every connected graph G contains a path of length at least $\min\{2\delta(G), |V(G)| - 1\}$. To this aim, let $P = x_0 x_1 \dots x_k$ be a longest path in G. Assume that $k < \min\{2\delta(G), |V(G)| - 1\}$.

(a) If there is an edge connecting start and end vertex of the path, then either the path contains all vertices or it can be made longer (both leads to contradiction).

(b) If there is a $j \in \{1, \dots, k\}$ such that $\{x_0, x_j\} \in E$ and $\{x_{j-1}, x_k\} \in E$, then again we find a cycle on the vertices $x_0, \dots, x_k$ and obtain a contradiction.

(c) There always is a j as in (b).

Happy Easter!