Universität des Saarlandes
FR 6.2 Informatik

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Exercises for Graph Theory

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss11/graph_theory/

Assignment 9 Deadline: Thursday, June 16, 2011

Rules: The first problem serves as a preparation for the test conducted in the exercise class. You should solve it, but you do not need to hand it in. The test yields 8 points. The remaining problems have to be handed in *nicely written up as you would do in a thesis, as the Diestel does, ...* in the Thursday lecture. These homework problems yield 8 points in total.

You need to collect at least 50% of all these points (tests and written homework) from (i) the first three exercise sessions, (ii) the first six sessions, and (iii) the whole term.

Occasionally, there might be bonus problems, which yield additional bonus points. They are, typically, more difficult or not that closely related to that week's content of the lecture.

There is no exercise class on Monday, June 13. **The test will therefore take place in the lecture on Tuesday, June 14.**

Exercise 1 (*oral homework, in total 8 points via test*)

(Re-)read carefully and understand the material in Sections 5.0–5.2 (up to and including 5.2.4.) of the Diestel book. (See also Exercise 4 below for Section 5.1.)

Exercise 2 (*written homework, 3 points*)

In this exercise you will give an alternative proof of Proposition 5.3.1 in the Diestel book. Recall that by $\chi'(G)$ we denote the chromatic index (also called the edge-chromatic number) of G.

a) Prove that every d-regular bipartite graph G satisfies $\chi'(G) = d$. [1.5P.]
(Hint: Recall what we know about matchings in bipartite graphs.)

b) Use (a) to show that every bipartite graph G satisfies $\chi'(G) = \Delta(G)$. [1.5P.]

Exercise 3 (*written homework, 3 Points*)

Determine the chromatic index of the complete graph on ℓ vertices. You may want to have a look at the statement of Vizing's Theorem (Theorem 5.3.2), but you are not allowed to use it in your arguments. (Hint: Deal first with the case where ℓ is odd.)

Exercise 4 (*written homework, 2 Points*)

a) Read the proof of the Five Color Theorem (Proposition 5.1.2) in the Diestel book. You can ignore the detailed topological argument in the third and fourth paragraph and simply take for granted that every v_1–v_3 path $P \subseteq H$ separates v_2 from v_4 in H. [0P.]

b) Imagine you want to use the same inductive approach to prove the Four Color Theorem, i.e., that every planar graph is 4-colorable. As in the proof you just read, let $v \in G$ be a vertex of degree at most 5 in G, and assume that G is not 4-colorable (but $H := G - v$ is by induction). Consider a fixed proper 4-coloring of H. What can you deduce about the colors appearing in the neighborhood of v? [1P.]

c) At first glance, it perhaps looks as if the proof can be completed as before. Explain why this does not work. [1P.]

If you cannot answer c), feel free to submit a short proof of the Four Color Theorem and become famous! ;-)

Exercise Bonus 1 (*4 Points*)

a) Prove that at any party with at least six people there are three people that pairwise know each other, or three people that pairwise do *not* know each other. (We assume that 'know each other' is a symmetric relation here.) [1P.]

b) Prove that for all positive integers k and ℓ there exists $P = P(k, \ell)$ such that at any party with at least P people there are k people that pairwise know each other, or ℓ people that pairwise do *not* know each other. [3P.]
(Hint: Use induction on $k + \ell$.)