Universität des Saarlandes
FR 6.2 Informatik

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Exercises for Graph Theory

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss11/graph_theory/

Assignment 11 Deadline: Thursday, June 30, 2011

Rules: The first problem serves as a preparation for the test conducted in the exercise class. You should solve it, but you do not need to hand it in. The test yields 8 points. The remaining problems have to be handed in *nicely written up as you would do in a thesis, as the Diestel does, ...* in the Thursday lecture. These homework problems yield 8 points in total.

You need to collect at least 50% of all these points (tests and written homework) from (i) the first three exercise sessions, (ii) the first six sessions, and (iii) the whole term.

Occasionally, there might be bonus problems, which yield additional bonus points. They are, typically, more difficult or not that closely related to that week's content of the lecture.

Exercise 1 (*oral homework, in total 8 points via test*)

Read carefully and understand the proofs of Theorem 5.5.4 and Lemma 5.5.5 in the Diestel. Also read Section 8.1 of Chapter 8.

Exercise 2 (*written homework, 2 points*)

Prove (directly, without using any lemmas or theorems from the Diestel) that if a graph G is perfect then $|V(H)| \leq \alpha(H) \cdot \omega(H)$ for every induced subgraph $H \subseteq G$.

Exercise 3 (*written homework, 4 points*)

The following exercise is intended to give an alternative proof of Proposition 5.5.2 in the Diestel. A graph is called *chordal* if it has no induced cycles of length greater than 3.

(a) Show that every minimal separator of a chordal graph G induces a complete subgraph in G. [2P]

(b) A *simplicial* vertex of a graph G is a vertex whose neighbors induce a complete subgraph in G. Use (a) to prove that every chordal graph which is not complete has at least two non-adjacent simplicial vertices. [1P]

(c) A simplicial ordering of a graph G is an ordering $(v_1, \ldots, v_n)$ of the vertices of G such that v_i is a simplicial vertex of $G[\{v_1, \ldots, v_i\}]$, for all $i \in \{1 \ldots, n\}$. Use (b) to prove that every chordal graph has a simplicial ordering. [0.5P]

(d) Use (c) to prove that chordal graphs are perfect. [0.5P]

Exercise 4 (*written homework, 2 points*)

Let G be an infinite graph, and let $A, B \subseteq V(G)$. Show that if no finite set of vertices separates A from B in G, then G contains an infinite set of disjoint A-B paths.