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Summer 2011

Graph Theory: Test 12 (Monday, July 4, 2011)

Time: 20 Minutes

Name: _____

Exercise 1+2 (*total 8 points*)

Answer each of the following questions. If proofs are needed, a short sketch of the main argument is sufficient. If counterexamples are needed, it suffices to give the example (unless it is not obvious why this is a counterexample). All questions can be answered in about two lines. Each item is worth one point.

- a) State Ramsey's Theorem.

- b) Give the Ramsey number of 3.

- c) What would you need to show in order to prove that the Ramsey number of 6 is at least 120? Give your answer in the *coloring terminology* (as opposed to the clique/independent set terminology).

- d) What would you need to show in order to prove that the Ramsey number of 6 is at most 150? Give your answer in the *clique/independent set terminology* (as opposed to the coloring terminology).
- e) Explain in 1-2 sentences how we inferred a deterministic statement from a probabilistic one (via the so-called *probabilistic method*) when we proved lower bounds for Ramsey numbers.
- f) In the proof of upper upper bounds for Ramsey numbers given in the sample solutions for Assignment 9, we derived a simple recursive inequality for the asymmetric Ramsey numbers $R(k, \ell)$. State that recursive inequality.
- g) How many different graphs are there on the vertex set $V := \{a, b, c, d\}$? (Two graphs $G = (V, E)$ and $G' = (V, E')$ are different if and only if $E \neq E'$. Two different graphs may still be isomorphic to each other.)
- h) What is the probability that a random graph G_n (a graph drawn uniformly at random from all graphs on vertex set $V_n = \{0, \dots, n-1\}$) contains no edge?

Feedback:

How many hours did you spend working on the last assignment sheet?

The material covered last week was [] easy, [] fine, [] difficult, [] very difficult.

Comments?