3. Homework (Due: 21 May, 2012) Total points: 40

Advanced Graph Algorithms SS 2012

Ran Duan, Jens M. Schmidt, Magnus Wahlström Tutor: Bernhard Schommer

Exercise 1: A primal method for MWPM in bipartite graphs *(5+15=20 points)*

Balinski and Gomory [1] gave an algorithm for the maximum weighted perfect matching which is "dual to" the Hungarian method. (While the Hungarian is a dual method, this one is "primal".) The paper is given in the reference, but it is written in old style. For bipartite real-weighted graphs $G = (L \cup R, E, w)$, a simple description is given below:

- As in Hungarian method, we have a y-function on all vertices.
- Initially, find any perfect matching and choose y such that all matching edges are tight $(y(u) + y(v) = w(u, v))$. Of course there may be edges violating the domination condition.
- For every non-matching edge $(u.v)$ violating the domination condition, find an alternating cycle containing it whose vertices (u', v') all satisfy $y(u') + y(v') \leq w(u', v')$. Then augment along the alternating cycle.
- If no such cycle exists, adjust y-values to make more edges tight or make (u, v) not violating the domination condition. So we can minus Δ to the y-values of some vertices in L and plus Δ to the y-values of some vertices in R.
- Until all edges satisfy the domination condition.

Answer the following questions:

i) Explain why we need to adjust y-values when augmenting along an alternating cycle in Step 3.

ii) In Step 4, do we need to adjust the y-values of both u and v? Why? Write the set of vertices whose y-values needed to adjust.

Exercise 2: Edmonds algorithm for integer-weighted graphs *(10 points)*

In the Edmonds algorithm for general graphs, when the edge weights are positive integers at most N and initially all y-values are $N/2$, show that we can obtain a maximum weighted matching when the y-values of all free vertices are zero.

Exercise 3: Edmonds algorithm for real-weighted graphs *(10 points)*

In the Edmonds algorithm for maximum weighted perfect matching in general graphs, when the edge weights are not integers, write an equation for the amount Δ of dual-adjustment, using y-values on all the vertices and z-values on all the blossoms. Note that every vertex in G is classified as "OUT","IN" or unmarked based on the mark of its root blossom in the contracted graph G',

Literatur

[1] M. L. Balinski and R. E. Gomory. A primal method for the assignment and transportation problems. *Management Science*, 10(3):pp. 578–593, 1964.