4. Homework (Due: 31 May, 2012) Total points: 40

Advanced Graph Algorithms SS 2012

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Exercise 1: Sparsification of dynamic minimum spanning tree *(9+6=15 points)*

Suppose we have a dynamic minimum spanning tree structure with $\tilde{O}(\sqrt{m})$ worst-case update time. We can also use the technique of sparsification to improve the update time to $\tilde{O}(\sqrt{n})$, which needs to prove:

i) Let F_1 and F_2 be the minimum spanning forests of G_1 and G_2, respectively. Show that there is a minimum spanning forest of $G_1 \cup G_2$ which does not use any edge in $G_1 - F_1$ and $G_2 - F_2$.

ii) Suppose we have already constructed a binary tree for sparsification, where each node in the binary tree represents a graph with $O(n)$ edges, and the graph of a non-leaf node consists of edges of the minimum spanning forests in its two children. When inserting an edge e to some leaf node G_i, show that we only need to update a constant number of edges in G_i's ancestors.

Exercise 2: d-edge-failure connectivity *(15 points)*

Let G be any n-vertex connected graph with a spanning tree T. By the ET-tree, design a data structure of $\tilde{O}(n^2)$ space which can reconnect the spanning tree in $O(d^2 \log^2 n)$ time after deleting d edges.

Exercise 3: Planar separator *(10 points)*

Let G be any n-vertex connected planar graph having non-negative vertex costs summing to no more than one. Suppose that the vertices of G are partitioned into levels according to their distance from some vertex v, and that $L(l)$ denotes the number of vertices on level l. Given any two levels l_1 and l_2 such that levels less than l_1 have total cost at most $2/3$ and levels more than l_2 have total cost at most $2/3$, prove that we can find a partition A, B, C of the vertices of G such that no edge joins a vertex in A with a vertex in B, neither A nor B has total cost exceeding $2/3$, and $|C|$ is at most $L(l_1) + L(l_2) + \max\{0, 2(l_2 - l_1 - 1)\}$.