Exercise 1: Combinatorial Embeddings (8 points)

Let $G$ be a planar embedding. Let $A$ be a set of lists, one for each face of $G$, such that each list contains all the edges of its face in clockwise order. Show that $A$ and a combinatorial embedding are equivalent in the sense that they define each other.

Exercise 2: Reducing Planarity (10 points)

Show that a graph is planar if and only if its 2-connected components are planar.

Exercise 3: Dual Graphs (10 points)

Is $G^*$ connected for every planar (not necessarily connected) graph $G$? Find a counterexample or proof.

Exercise 4: Colorings (12 points)

Let $G$ be a graph with maximal vertex degree $k$. Find an efficient algorithm that colors $G$ with $k + 1$ colors. Faster running time ⇒ more points.