6. Homework Total points: 40

Advanced Graph Algorithms SS 2012

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Exercise 1: Black and White *(8 points)*

Draw an arbitrary closed curve on a paper. The curve may intersect itself; however, every self-intersection must be a point. It is also possible that more than two parts of the curve intersect at one point.

Show that the resulting regions on the paper can be colored with the colors black and white such that no neighbored regions have the same color (two regions are *neighbored* if their intersection contains a point that is no self-intersection).

Exercise 2: Planar Graphs *(7+7=14 points)*

- If a graph G contains a subdivision of a graph H (i. e., a H*-subdivision*), G has also a H-minor. Prove the following inverse statement: If G has a $K_{3,3}$-minor or a K_5-minor, G contains a subdivision of $K_{3,3}$ or of K_5.
- Prove the following statement or show a contradiction: A simple planar graph with $n \geq 2$ contains at least 2 vertices that have degree at most 5, respectively.

Exercise 3: Orientations *(8+5+5=18 points)*

- Prove that every planar graph has a 4-orientation.
- Give an efficient algorithm that computes a 4-orientation in a planar graph.
- Show that there are planar graphs not having any 2-orientation.