7. Homework

Advanced Graph Algorithms SS 2012
Ran Duan, Jens M. Schmidt, Magnus Wahlström Tutor: Bernhard Schommer

Exercise 1: Minimum Spanning Trees
(15 points)

Assume that all edge weights are pairwise distinct. Consider an algorithm that first performs \( k \) steps of Prim’s/Jarník’s algorithm and then contracts all tree edges and applies Borůvka’s algorithm. What is the running time of this algorithm in terms of \( k, n \) and \( m \)? For which value of \( k \) is the running time minimized?

Exercise 2: Finding Quadrangle
(10 points)

Give an \( O(n^\omega) \) algorithm for finding a quadrangle (a simple cycle of 4 edges) in directed graphs.

Exercise 3: Maximum Witness
(15 points)

In the Boolean matrix product \( C \) of two Boolean matrices \( A \) and \( B \), if \( C_{i,j} = 1 \), then any index \( k \) such that \( A_{i,k} \) and \( B_{k,j} \) are both 1 is a witness for \( C_{i,j} \). The maximum witness for \( C_{i,j} \) is the largest possible witness for \( C_{i,j} \). The time complexity for computing the maximum witness for all entries of \( C \) is usually denoted by \( O(n^\mu) \), when \( A, B \) are both \( n \times n \) matrices.

Let \( \omega(1, r, 1) \) denotes the exponent of the time complexity for computing the product of an \( n \times n^r \) matrix and an \( n^r \times n \) matrix. If

\[
\omega(1, r, 1) \leq \begin{cases} 
2 & \text{if } 0 \leq r \leq \alpha \\
2 + \beta(r - \alpha) & \text{otherwise}
\end{cases}
\]

where \( \alpha = 0.294, \beta = 0.533 \), show that \( \mu < 2.575 \). (Note that by far the maximum witness has the same time complexity as the unweighted directed APSP as given in class.)