7. Homework | Total points: 40

Advanced Graph Algorithms | SS 2012

Ran Duan, Jens M. Schmidt, Magnus Wahlström | Tutor: Bernhard Schommer

Exercise 1: Minimum Spanning Trees *(15 points)*

Assume that all edge weights are pairwise distinct. Consider an algorithm that first performs k steps of Prim's/Jarník's algorithm and then contracts all tree edges and applies Borůvka's algorithm. What is the running time of this algorithm in terms of k, n and m? For which value of k is the running time minimized?

Exercise 2: Finding Quadrangle *(10 points)*

Give an $O(n^\omega)$ algorithm for finding a quadrangle (a simple cycle of 4 edges) in directed graphs.

Exercise 3: Maximum Witness *(15 points)*

In the Boolean matrix product C of two Boolean matrices A and B, if $C_{i,j} = 1$, then any index k such that $A_{i,k}$ and $B_{k,j}$ are both 1 is a *witness* for $C_{i,j}$. The *maximum witness* for $C_{i,j}$ is the largest possible witness for $C_{i,j}$. The time complexity for computing the maximum witness for all entries of C is usually denoted by $O(n^\mu)$, when A, B are both $n \times n$ matrices.

Let $\omega(1, r, 1)$ denotes the exponent of the time complexity for computing the product of an $n \times n^r$ matrix and an $n^r \times n$ matrix. If

$$\omega(1, r, 1) \leq \begin{cases} 2 & \text{if } 0 \leq r \leq \alpha \\ 2 + \beta(r - \alpha) & \text{otherwise} \end{cases}$$

where $\alpha = 0.294, \beta = 0.533$, show that $\mu < 2.575$. (Note that by far the maximum witness has the same time complexity as the unweighted directed APSP as given in class.)