8. Homework

Advanced Graph Algorithms  SS 2012
Ran Duan, Jens M. Schmidt, Magnus Wahlström  Tutor: Bernhard Schommer

Exercise 1: Small maximal matchings  (5 points)
Sometimes, a small maximal matching can be useful (see for example the crown reduction for VERTEX COVER from Thursday’s lecture, which gives a better result if a smaller maximal matching is found). How small can a maximal matching be, compared to a maximum matching of a graph? (Prove your claim.)

Exercise 2: Variants of the Feedback Vertex Set problem  (5+7=12 points)
This week’s lectures covered the Feedback Vertex Set problem (which is NP-hard). Consider instead the Feedback Edge Set problem: Given a graph G, we want to find at most k edges which intersect all cycles.

i) Show that Feedback Edge Set can be solved in polynomial time.

ii) Consider now the problems on directed graphs: the input is a directed graph G and an integer k, and the problem is to hit all directed cycles, using k vertices resp. edges. Show that the vertex-deletion variant reduces to the edge-deletion variant (that is, show how the vertex-deletion problem for a directed graph G and an integer k can be translated into an edge-deletion question (G', k) on a modified graph G').

Exercise 3: Cluster graphs  (8+15=23 points)
A cluster graph is a graph where every connected component is a clique.

1. Prove that a graph G = (V, E) is a cluster graph if and only if it contains no induced copy of P_3 (i.e., there are no three vertices u, v, w ∈ V such that uv, vw ∈ E but uw /∈ E).

2. Because of noisy data, it can happen that a graph which is supposed to be a cluster graph has some extra edges (or some edges missing) – for example, think of data clustering applications, where you create an edge between two objects (vertices) if they are deemed sufficiently similar. For this reason, we define a cluster graph cleaning problem as follows:

   Cluster Graph Edge Modification

   **Input:** A graph G, an integer k
   **Task:** Remove or insert at most k edges in G so that the result is a cluster graph.

Find a 2^{O(k)}-time FPT algorithm for Cluster Graph Edge Modification. **Hint:** Use the characterization from the previous step. Partial credit if you only solve the edge deletion variant (where you are asked to remove at most k edges, but not allowed to add any edges).