8. Homework Total points: 40

Advanced Graph Algorithms SS 2012

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Exercise 1: Small maximal matchings *(5 points)*

Sometimes, a *small* maximal matching can be useful (see for example the crown reduction for VERTEX COVER from Thursday's lecture, which gives a better result if a smaller maximal matching is found). How small can a maximal matching be, compared to a maximum matching of a graph? (Prove your claim.)

Exercise 2: Variants of the FEEDBACK VERTEX SET problem *(5+7=12 points)*

This week's lectures covered the FEEDBACK VERTEX SET problem (which is NP-hard). Consider instead the FEEDBACK EDGE SET problem: Given a graph G, we want to find at most k edges which intersect all cycles.

i) Show that FEEDBACK EDGE SET can be solved in polynomial time.

ii) Consider now the problems on *directed* graphs: the input is a directed graph G and an integer k, and the problem is to hit all directed cycles, using k vertices resp. edges. Show that the vertex-deletion variant reduces to the edge-deletion variant (that is, show how the vertex-deletion problem for a directed graph G and an integer k can be translated into an edge-deletion question (G', k) on a modified graph G').

Exercise 3: Cluster graphs *(8+15=23 points)*

A cluster graph is a graph where every connected component is a clique.

1. Prove that a graph $G = (V, E)$ is a cluster graph if and only if it contains no induced copy of P_3 (i.e., there are no three vertices $u, v, w \in V$ such that $uv, vw \in E$ but $uw \notin E$).
2. Because of noisy data, it can happen that a graph which is supposed to be a cluster graph has some extra edges (or some edges missing) – for example, think of data clustering applications, where you create an edge between two objects (vertices) if they are deemed sufficiently similar. For this reason, we define a cluster graph cleaning problem as follows:

   CLUSTER GRAPH EDGE MODIFICATION
   Input: A graph G, an integer k
   Task: Remove or insert at most k edges in G so that the result is a cluster graph.

   Find a $2^{O(k)}$-time FPT algorithm for CLUSTER GRAPH EDGE MODIFICATION. *Hint:* Use the characterization from the previous step. Partial credit if you only solve the edge *deletion* variant (where you are asked to remove at most k edges, but not allowed to add any edges).