Exercise 1: Extending \( k \)-Path \( (15 \text{ points}) \)
Recall the \( k \)-Path problem: Does \( G \) contain a path on \( k \) vertices? We define the following extension.

\textbf{\( k \)-Node Subtree}
Input: A graph \( G = (V, E) \), a tree \( T \) on \( k \) nodes.
Parameter: \( k \).
Task: Find a copy of \( T \) as a (not necessarily induced) subgraph of \( G \).

Note that \( k \)-Path is the special case where the tree \( T \) is a path. Show that \( k \)-Node Subtree is FPT. (Hint: I recommend that you start from the color coding algorithm for \( k \)-Path and try to extend it.)

Exercise 2: Chromatic Number in Polynomial Space \( (10 \text{ points}) \)
Show how the tools for Chromatic Number presented in class can be used to compute the chromatic number of a graph in time \( O^{*}(c^n) \) and polynomial space (you should be able to get \( c = 3 \)).

Exercise 3: Important Separators \( (3+3+9=15 \text{ points}) \)
(a) Which are the important \((X, Y)\)-separators (of any size) in the following graphs? (We are using edge cuts, not vertex cuts.)
1. \( G \) is a \( 3 \times 3 \) grid graph, where \( v_{i,j} \) is the vertex in row \( i \), column \( j \) (thus the edges are \( v_{i,j}v_{i+1,j} \) for \( i \in [2], j \in [3] \) and \( v_{i,j}v_{i,j+1} \) for \( i \in [3], j \in [2] \)). The sets are \( X = \{v_{1,1}\} \) and \( Y = \{v_{3,2}, v_{2,3}, v_{3,3}\} \).
2. \( G \) is a complete binary tree of height two (i.e., with four leaves). \( X \) is the root, and \( Y \) is the set of leaves.
(b) Design a quick way to decide whether a given \((s, t)\)-edge cut is an important separator. (Polynomial time is required; quicker gives more points.)