Advanced Graph Algorithms SS 2012

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Exercise 1: Extending k-Path *(15 points)*

Recall the k-Path problem: Does G contain a path on k vertices? We define the following extension.

k-Node Subtree
Input: A graph $G = (V, E)$, a tree T on k nodes.
Parameter: k.
Task: Find a copy of T as a (not necessarily induced) subgraph of G.

Note that k-Path is the special case where the tree T is a path. Show that k-Node Subtree is FPT. (Hint: I recommend that you start from the color coding algorithm for k-Path and try to extend it.)

Exercise 2: Chromatic Number in Polynomial Space *(10 points)*

Show how the tools for Chromatic Number presented in class can be used to compute the chromatic number of a graph in time $O^*(c^n)$ and polynomial space (you should be able to get $c = 3$).

Exercise 3: Important Separators *(3+3+9=15 points)*

(a) Which are the important (X, Y)-separators (of any size) in the following graphs? (We are using edge cuts, not vertex cuts.)

1. G is a 3×3 grid graph, where $v_{i,j}$ is the vertex in row i, column j (thus the edges are $v_{i,j}v_{i+1,j}$ for $i \in [2]$, $j \in [3]$ and $v_{i,j}v_{i,j+1}$ for $i \in [3]$, $j \in [2]$). The sets are $X = \{v_{1,1}\}$ and $Y = \{v_{3,2}, v_{2,3}, v_{3,3}\}$.
2. G is a complete binary tree of height two (i.e., with four leaves). X is the root, and Y is the set of leaves.

(b) Design a quick way to decide whether a given (s, t)-edge cut is an important separator. (Polynomial time is required; quicker gives more points.)