10. Homework Total points: 40

Advanced Graph Algorithms SS 2012

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Recall that a *dominating set* in a graph $G = (V, E)$ is a set X of vertices such that for every $v \in V$, either $v \in X$ or v has a neighbour in X. The Dominating Set problem is, given G and k, to decide whether G has a dominating set of size at most k.

Exercise 1: Problems on d-degenerate graphs *(5+5+5=15 points)*

A graph is *d-degenerate* if every induced subgraph contains a vertex of degree at most d. This is quite a general notion of sparseness, but still allows for some algorithmic conclusions.

(a) Show that the "degeneracy number", i.e., the smallest d such that G is d-degenerate, can be computed in polynomial time.

(b) Show that Independent Set and Dominating Set have constant-factor approximations on d-degenerate graphs.

(c) Show that for Independent Set, we also get a linear kernel.

Exercise 2: Dominating Set by bounded treewidth *(10 points)*

Show that Dominating Set can be solved in $2^{O(k)}n^{O(1)}$ time for graphs of treewidth k.

Exercise 3: More treewidth *(5+10=15 points)*

Show the following.

(a) Let u and v be two vertices of a graph G, connected by a flow of more than $k+1$ vertex-disjoint paths. Then any tree decomposition of G of width at most k must contain a bag X with $u, v \in X$.

(b) Let $G = (V, E)$ be a planar graph, $v \in V$, and $N_d[v]$ be the set of vertices at distance at most d of v (including v itself). Then $G[N_d[v]]$ has treewidth bounded by $O(d)$. (Hint: Go via outerplanarity.)