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Exercises for Optimization

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss12/OPT

Exercise sheet 5 Due: **Tuesday, May 29, 2012**

You need to collect at least 50% of all points on the first six exercise sheets, and at least 50% of all points on the remaining exercise sheets. You are allowed to hand in homework in teams of two.

Exercise 1 (*8 points*) (adapted from BT, Exercise 3.18)

Consider the simplex method applied to a standard form problem and assume that the rows of the matrix A are linearly independent. Decide whether the following two statements are true, and justify your answers.

a) An iteration of the simplex method may move the feasible solution by a positive distance while leaving the value of the objective function unchanged.

b) If x is an optimal solution of the problem and moreover a *non-degenerate* bfs, then it is the *unique* optimal solution of the problem.

Exercise 2 (*12 points*) (BT, Exercise 3.19)

While solving a problem in standard form, we arrive at the following tableau, with x_3, x_4 and x_5 being the basic variables:

-10	δ	-2	0	0	0
4	-1	η	1	0	0
1	α	-4	0	1	0
β	γ	3	0	0	1

The entries $\alpha, \beta, \gamma, \delta, \eta$ in the tableau are unknown parameters. For each one of the following statements, find some parameter values that will make the statement true.

a) The current solution is optimal and there are multiple optimal solutions.

b) The optimal cost is $-\infty$.

c) The current solution is feasible but not optimal.

Exercise 3 (*10 points*)

When discussing the revised simplex method, we said that in a given iteration, the matrix $\bar{B}^{-1}$ (the inverse of the new basis matrix) is obtained by a sequence of elementary row operations from B^{-1}, the inverse of the old basis matix. Furthermore, we argued that this sequence of elementary row operations can be represented by a transformation matrix T, i.e., we have $\bar{B}^{-1} = TB^{-1}$.

Give the matrix T explicitly, as a function of the entries $u_1, \dots, u_m$ of the pivot column $u := B^{-1}A_j$. (As usual, j denotes the entering variable here; and you may assume that the entry u_ℓ corresponds to the leaving variable $x_{B(\ell)}$.)

*(Recall that in actual implementations of the revised simplex or full tableau implementation, we do **not** compute or store T explicitly.)*

Exercise 4 (*12 points*)

We go back to the very first example of an LP we discussed in class, the one with the gadgets and the gewgaws. Here are the numbers again: One kg of gadgets requires 1 hour of work, 1 unit of wood, 2 units of metal, and yields a net profit of \$5. One kg of gewgaws requires 2 hours of work, 1 unit of wood, 1 unit of metal, and yields a net profit of \$4. The company has 120 hours of work, 70 units of wood, and 100 units of metal available.

a) Suppose a businessman approached the company and offered to purchase their resources at \$1.50 per hour of work, \$1 per unit of wood, and \$1 per unit of metal. He is willing to buy whatever amount of each resource the company is willing to sell. What should the company do to maximize its profit?

   Note that by simply selling all their resources, they will make a profit of \$350, which is more than the \$310 they can make from producing gadgets and gewgaws. They can do even better than that. Think about whether it makes sense for the company to produce gadgets, and whether it makes sense for the company to produce gewgaws.

b) Suppose now the company is offered \$3 per unit of wood and \$1.01 per unit of metal (and nothing for hours of work). What should the company do now?

c) Imagine now you are a businessperson, and you want to make an offer that induces the company to sell all of its resources. What conditions does your offer need to satisfy? What offer should you make to get all the company's resources for as little money as possible?

Exercise 5 (*10 points, extra credit*)

Read pages 117-119 in the book, and solve the LP from HW4, Exercise 2 using the big-M method.

For your convenience, here's the LP again: "Minimize $c^T x$ s.t. $Ax = b$ and $x \geq 0$", where

$$A := \begin{bmatrix} 0 & 2 & 1 & 1 \\ -4 & 0 & 2 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}, \qquad b = \begin{bmatrix} 3 \\ 0 \\ 3/2 \end{bmatrix}, \qquad \text{and} \qquad c^T = \begin{bmatrix} 2 & 1 & 3 & 1 \end{bmatrix}.$$