max planck institut informatik

SAARLAND UNIVERSITY
COMPUTER SCIENCE

Reto Spöhel, Rob van Stee **Summer 2012**

Exercises for Optimization

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss12/OPT

Exercise sheet 7 Due: **Tuesday, June 12, 2012**

You need to collect at least 50% of all points on the first six exercise sheets, and at least 50% of all points on the remaining exercise sheets. You are allowed to hand in homework in teams of two.

Exercise 1 (*10 points (adapted from BT 4.25)*)

Applying the dual simplex method is not the same as taking the dual, putting it into standard form and applying the primal simplex method. This can be seen by considering the following problem and its dual.

$$\begin{array}{lrcrcl} \text{minimize} & x_1 & + & x_2 & & \\ \text{such that} & x_1 & & & = & 1 \\ & & & x_2 & = & 1 \\ & x_1 & & & \geq & 0 \\ & & & x_2 & \geq & 0 \end{array} \qquad \begin{array}{lrcrcl} \text{maximize} & y_1 & + & y_2 & & \\ \text{such that} & y_1 & & & \leq & 1 \\ & & & y_2 & \leq & 1 \end{array}$$

(a) Solve this LP using the dual simplex method.

(b) Convert the dual into standard form.

(c) Apply the primal simplex to the standard form problem from (b). What changes compared to (a)?

Exercise 2 (*10 points (Sensitivity analysis)*)

(a) Suppose we **remove** a variable from a problem. What can we say about the solution to the original LP? How can we solve the new problem (if needed)?

(b) Suppose we **remove** a constraint from a problem. What can we say about the solution to the original LP? How can we solve the new problem (if needed)?

Exercise 3 (*20 points*)

Consider again the linear program

$$\begin{aligned} \text{minimize} \quad & -5x_1 - 4x_2 - 3x_3 \\ \text{s.t.} \quad & 2x_1 + 3x_2 + x_3 \leq 5 \\ & 4x_1 + x_2 + 2x_3 \leq 8 \\ & 3x_1 + 4x_2 + 2x_3 \leq 11 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

that you have repeatedly considered before. As you will recall, the final (optimal) matrix B^{-1} was

$$\begin{pmatrix} 0.4 & -0.2 & 0 \\ -0.2 & 0.6 & 0 \\ -1.2 & -0.4 & 1 \end{pmatrix}$$

(a) Let $b_3 = 11 + \delta$. For what values of δ is the current optimal solution still optimal? What is the optimal value if δ is in this range?

(b) Solve this linear program for $b_3 = 8$ using the dual simplex algorithm.

(c) Let $c_1 = -5 + \delta$ (and $b_3 = 11$). For what values of δ is the current optimal solution still optimal? What is the optimal value if δ is in this range?

(d) Solve this linear program for $c_1 = -7$ (and $b_3 = 11$) using the primal simplex algorithm.