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SAARLAND UNIVERSITY

COMPUTER SCIENCE

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Exercises for Optimization

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss12/OPT

Exercise sheet 8 Due: **Tuesday, June 19, 2012**

You need to collect at least 50% of all points on the first six exercise sheets, and at least 50% of all points on the remaining exercise sheets. You are allowed to hand in homework in teams of two.

Exercise 1 (*10 points (BT 7.3)*) Each of n teams plays against every other team a total of k games. Assume that every game ends in a win or a loss (no draws) and let x_i be the number of wins of team i. Let X be the set of all possible outcome vectors $(x_1, \ldots, x_n)$. Given an arbitrary vector $(x_1, \ldots, x_n)$, we would like to determine whether it belongs to X, that is, whether it is a possible tournament outcome vector. Provide a network flow formulation of this problem.

Hint: if team n wins x_i times, it loses $(n-1)k - x_i$ times.

Exercise 2 (*10 points (BT 7.5)*)

Consider an uncapacitated network flow problem and assume that $c_{ij} \geq 0$ for all arcs. Let S_+ and S_- be the sets of source and sink nodes, respectively. Let d_{ij} be the length of the shortest directed path from node $i \in S_+$ to node $j \in S_-$. Let $d_{ij} = \infty$ if no path exists. We construct a transportation problem with the same source and sink nodes and the same values for the supplies and the demands. For every source node i and every sink node j. we introduce a direct link with cost d_{ij}. Show that the two problems have the same optimal cost.

Exercise 3 (*10 points (BT 7.7)*)

Consider a network flow problem in which we impose an additional constraint $f_{ij} \geq \ell_{ij}$ for every arc (i, j). Construct an equivalent network flow problem in which there are no nonzero lower bounds on the arc costs.

Hint: let $\bar{f_{ij}} = f_{ij} - \ell_{ij}$ and construct a new network for the arc flows $\bar{\ell_{ij}}$. How should b_i be changed?

Exercise 4 (*10 points (adapted from BT 7.8)*) Consider a transportation problem in which all cost coefficients c_{ij} are positive. Suppose that we increase the supply at some source nodes and the demand at some sink nodes (in a consistent way). Is it true that the value of the optimal cost will also increase? Prove or give a counterexample. What happens if some c_{ij} values may be negative?