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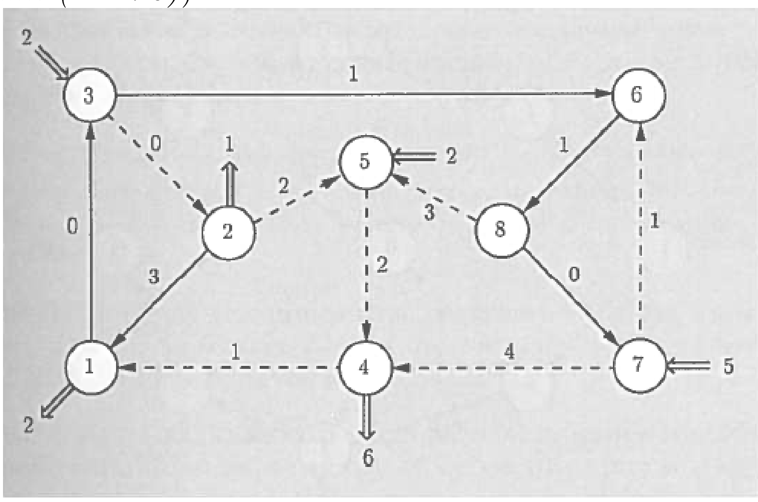
Exercises for Optimization

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss12/OPT

Exercise sheet 9 Due: **Tuesday, June 26, 2012**

You need to collect at least 50% of all points on the first six exercise sheets, and at least 50% of all points on the remaining exercise sheets. You are allowed to hand in homework in teams of two.

Exercise 1 (*10 points (BT 7.9)*)



Consider the uncapacitated network flow problem shown above. The label next to each arc is the cost. Solve this problem using the network simplex algorithm. Start with the tree indicated by the dashed arcs in the figure.

Exercise 2 (*10 points (BT 7.21)*)

Show that a feasible solution to a capacitated network problem (if one exists) can be found by solving a maximum flow problem.

Exercise 3 (*10 points (BT 7.19)*)

Give an algorithm with $O(|A|)$ running time that determines whether the maximum flow in a given network is unbounded (infinitely high).

Exercise 4 (*10 points (adapted from BT 7.37)*)

Consider a directed graph in which each arc is associated with a cost c_{ij}. For any directed cycle, we define its mean cost as the sum of the costs of its arcs, divided by the number of arcs. To guarantee termination of the negative cost cycle algorithm, we are interested in a directed cycle whose mean cost is minimal. We assume that there exists at least one directed cycle.

a) Consider the algorithm

$$p_i(t+1) = \min_{j \in O(i)} \{c_{ij} + p_j(t)\} \qquad \text{for } i = 1, \dots, n,$$

initialized with $p_i(0) = 0$ for all i. Show that $p_i(t)$ is equal to the length of a shortest walk that starts at I and traverses t arcs.

b) Prove that the optimal mean cycle cost λ satisfies

$$\lambda = \min_{i=1,\dots,n} \max_{k=0,\dots,n-1} \frac{p_i(n) - p_i(k)}{n-k},$$

where n is the number of nodes in the graph.

c) Give an algorithm for finding a negative cost cycle with minimal mean cost, as well as an estimate of its running time.