

# Linear Programming – A First Example

- One kg of **gadgets**
  - requires **1 hour** of work, **1 unit** of wood, and **2 units** of metal
  - yields a **net profit of \$5**
- One kg of **gewgaws**
  - requires **2 hours** of work, **1 unit** of wood, and **1 unit** of metal
  - yields a **net profit of \$4**
- The company has **120 hours** of work, **70 units** of wood, and **100 units** of metal available

$x_1$  := amount of gadgets  
 $x_2$  := amount of gewgaws



$$\begin{array}{ll} \max z = 5x_1 + 4x_2 & \text{maximize profit} \\ \text{s.t. } x_1 + 2x_2 \leq 120 & \text{work hours constraint} \\ \quad x_1 + x_2 \leq 70 & \text{wood constraint} \\ \quad 2x_1 + x_2 \leq 100 & \text{metal constraint} \\ \quad x_1, x_2 \geq 0 & \end{array}$$

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$$\begin{aligned} \max z &= 5x_1 + 4x_2 && \text{maximize profit} \\ \text{s.t. } x_1 + 2x_2 &\leq 120 && \text{work hours constraint} \\ x_1 + x_2 &\leq 70 && \text{wood constraint} \\ 2x_1 + x_2 &\leq 100 && \text{metal constraint} \\ x_1, x_2 &\geq 0 \end{aligned}$$



$$\begin{array}{ll} \max & \begin{bmatrix} 5 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ \text{s.t. } & \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} 120 \\ 70 \\ 100 \end{bmatrix} \\ & \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{array}$$

The diagram shows the linear programming problem in matrix form. A green bracket labeled 'A' groups the coefficient matrix. A blue bracket labeled  $c^T$  groups the objective function coefficients. A red bracket labeled 'x' groups the variables. A yellow bracket labeled 'b' groups the right-hand side constants. A red bracket at the bottom groups the non-negativity constraints.

$$\begin{aligned} \max c^T x \\ \text{s.t. } Ax \leq b \\ x \geq 0 \end{aligned}$$