



## Exercises for Limits of Computational Learning

<http://www.mpi-inf.mpg.de/departments/d1/teaching/ss12/learning/>

Assignment 2

Deadline: Wed 9.5.2012, 10am

We define the following sets of natural numbers.

$$\begin{aligned} A_1 &= \{x \in \mathbb{N} \mid \varphi_x(0) \downarrow\}; \\ A_2 &= \{x \in \mathbb{N} \mid \varphi_x \text{ is total}\}; \\ A_3 &= \{x \in \mathbb{N} \mid \text{dom}(\varphi_x) \text{ is infinite}\}; \\ A_4 &= \{x \in \mathbb{N} \mid \text{dom}(\varphi_x) \neq \emptyset\}; \\ A_5 &= \{x \in \mathbb{N} \mid \text{range}(\varphi_x) \text{ is infinite}\}; \\ A_6 &= \{x \in \mathbb{N} \mid \exists a : \forall^\infty t : \varphi_x(t) = a\}. \end{aligned}$$

**Exercise 1** (8pts, 2pts each) Show the following relations.

(a)  $A_3 \leq_1 A_2$ ; (b)  $A_2 \leq_1 A_6$ ; (c)  $A_5 \leq_1 A_2$ ; (d)  $A_2 \leq_1 A_5$ .

**Exercise 2** (2pts) Show that there is a  $p$  such that  $\varphi_p = \varphi_{p+1}$ .

**Exercise 3** (2pts) For each  $p$ , let

$$\Theta(\varphi_p) = \begin{cases} \lambda x. 0, & \text{if range}(\varphi_p) \text{ is infinite;} \\ \lambda x. \uparrow, & \text{otherwise.} \end{cases}$$

Show that  $\Theta$  is not an effective operator.

**Exercise 4** (2pts) Show that, for all total computable  $f$ , there is an  $e$  such that the complement of  $\text{dom}(\varphi_e)$  contains exactly one element, but  $\varphi_{f(e)}$  is not a total computable extension of  $\varphi_e$ .

**Exercise 5** (6pts, 2pts each) Show the following relations.

(a)  $\overline{A_2} \not\leq_1 A_2$  and  $A_2 \not\leq_1 \overline{A_2}$  (b)  $A_6 \not\leq_1 A_5$  and  $A_6 \not\leq_1 \overline{A_5}$  (c)  $A_1 \leq_1 \overline{A_2}$  and  $\overline{A_2} \not\leq_1 A_1$ .

**Exercise 6** (2pts) Give a set  $A$  such that  $A \equiv_1 \overline{A}$ .