Universität des Saarlandes
FR 6.2 Informatik

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Exercises for Limits of Computational Learning

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss12/learning/

Assignment 3 Deadline: Wed 16.5.2012, 10am

Exercise 1 *(2pts) Let $f \in \mathcal{R}$ be strictly monotone increasing. Show that* range(f) *is decidable.*

Exercise 2 *(2pts) Give an $f \in \mathcal{R}$ such that* range(f) *is* not *decidable.*

Exercise 3 *(4pts) Let $r \in \mathcal{R}$ be such that, for all n, $\varphi_{r(n)}$ is total. Let $\mathcal{S} = \{\varphi_{r(n)} \mid n \in \mathbb{N}\}$. Show that there is $h \in \mathcal{R}$ such that, for all $g \in \mathcal{S}$,*

$$\forall^\infty t : h(g[t]) = g(t).^1$$

Exercise 4 *(4pts) We define the operator $\mathbf{X}$ on any two functions f, f' as the unique pair of functions (p, q) such that, for all i,*

$$p(i) = f(q[i]); \quad (1)$$
$$q(i) = f'(p[i]). \quad (2)$$

We denote the pair (p, q) as $\mathbf{X}(f, f')$ and talk about crossfeeding *f and f'.*

*Let $g \in \mathcal{R}$. Use **ORT** to show that there is an infinite family $(h_i)_{i\in\mathbb{N}}$ of computable functions such that (a), (b) and (c) below hold. For all i, let with $(p_i, q_i) = \mathbf{X}(h_i, g)$.*

(a) For all i, $h_i(\emptyset) = i$.[2]

(b) For all i, $\forall^\infty t : \varphi_{p_i(t)} = q_i$;

(c) For all i and j we have not *$\forall^\infty t : q_i(t) = p_j(t)$.*

[1] For a function g and $i \in \mathbb{N}$, we use $g[i]$ to denote the sequence $g(0), \ldots, g(i-1)$; in particular, $g[0]$ is the empty sequence.

[2] We use $\emptyset$ to denote the empty sequence.