Universität des Saarlandes
FR 6.2 Informatik

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Exercises for Limits of Computational Learning

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss12/learning/

Assignment 7 Deadline: Thu 21.6.2012, 10am

This assignment sheet explores certain variants of finite learning, which lie in between **GFin** and **GEx**. To this end, we make the following definitions. First, we define mind-change bounded learning. The idea is that a learner is only allowed to make up to n mind changes (not counting ?); the last conjecture has to be correct. Given a sequence p, we let $\mathrm{mc}(p)$ the number of *mind changes* of p, i.e., the number of positions n such that $p(n) \neq\ ?$ is a conjecture different from the most recent conjecture $\neq\ ?$ in p before n. In particular, any sequence of conjecture p with exactly one element in the range different from ? has $\mathrm{mc}(p) = 0$. We let

$$\mathbf{Mc}_n = \{(p, g) \in \mathbf{Ex} \mid \forall n : p(n)\downarrow \wedge\ \mathrm{mc}(p) \leqslant n\}.$$

A further generalization is that of making only *finitely many* mind changes; when applied only to successfully learned functions, this gives **GEx**. When a learner *always* has to converge, even on functions it does not learn, we get *confident* learning.

$$\mathbf{Confident} = \{(p, g) \mid \exists e \forall^{\infty} n : p(n) \in \{e, ?\}\}.$$

Note that for mind-change bounded learning, the learner has to always make an output; for confident learning, when applied to all functions, the learner has to be total.

Exercise 1 *(12pts, 4pts each) Show the following three statements.*

(a) *For all* $n \in \mathbb{N}$, $\mathbf{GMc}_n \subset \mathbf{GMc}_{n+1}$.

(b) $\bigcup_{n \in \mathbb{N}} \mathbf{GMc}_n \subset \tau(\mathbf{Confident})\mathbf{GEx}$.

(c) $\tau(\mathbf{Confident})\mathbf{GEx} \subset \mathbf{GEx}$.

Exercise 2 *(4pts) This is an* extra credit *problem. Show that* $\tau(\mathbf{Confident})\mathbf{GEx}$ *is closed under union.*

We introduce a sequence generating operator **Td** modeling *transductive* learning as follows. For all h, g, n,

$$\mathbf{Td}(h, g)(n) = h(n, g(n)).$$

That is, a learner h sees only the current datum and *nothing* else. We suppose a learner can output ? to, effectively, keep its previous conjecture. This way a **Td**-style learner can, for example, learn $\mathcal{S}_{\mathrm{SD}}$.

Exercise 3 *(4pts) Show the following three statements.*

(a) $\mathbf{TdEx} \subset \mathbf{ItEx}$.

(b) $\mathbf{TdBc} \subset \mathbf{ItBc}$.

(c) $\mathbf{TdBc} \not\subseteq \mathbf{GEx}$.