



## Exercises for Limits of Computational Learning

<http://www.mpi-inf.mpg.de/departments/d1/teaching/ss12/learning/>

Assignment 8

Deadline: Thu 28.6.2012, 10am

**Exercise 1** (4pts) Show the following two statements.

- (a)  $\mathbf{GConsEx} \not\subseteq \tau(\mathbf{Rel})\mathbf{GEx}$ .
- (b)  $\tau(\mathbf{Rel})\mathbf{GEx} \not\subseteq \mathbf{GConsEx}$ .

**Exercise 2** (4pts) Show the following two statements.

- (a)  $\mathcal{T}\mathbf{Prud}(\mathbf{ItEx}) = \tau(\mathbf{T})\mathbf{GEx}$ .<sup>1</sup>
- (b)  $\mathbf{Prud}(\mathbf{ItEx}) = \mathbf{ItTEx}$ .

**Exercise 3** (4pts) Show that  $\tau(\mathbf{Rel})\mathbf{GEx}$  is closed under (binary) union.

**Exercise 4** (4pts) Show that  $\mathcal{R}\mathbf{GConsEx} \subset \mathbf{GConsEx}$ .<sup>2</sup>

For extra credit, we explore *closure under constructive infinite union*. A learning criterion  $I$  is said to be *closed under constructive infinite union* iff, for all infinite lists of (programs for) learners  $e \in \mathcal{R}$  with  $\forall i : I(\varphi_{e(i)}) \neq \emptyset$  (all learners learn *something*), we have  $\bigcup_{i \in \mathbb{N}} I(\varphi_{e(i)})$  is  $I$ -learnable.

**Exercise 5** (4pts extra credit, 2pts each)

- (a)  $\tau(\mathbf{Rel})\mathbf{GEx}$  is closed under constructive infinite union;<sup>3</sup>
- (b)  $\tau(\mathbf{Confident})\mathbf{GEx}$  is not closed under constructive infinite union.

<sup>1</sup>Recall that  $\tau(\mathbf{T})\mathbf{GEx} = \{S \subseteq \mathcal{R} \mid \exists S' \in \mathbf{UComp} : S \subseteq S'\}$ .

<sup>2</sup>Note that a learner for  $\mathcal{R}\mathbf{GConsEx}$  has to be total, that is, defined on all input, but it doesn't have to be consistent on all input, only on relevant input.

<sup>3</sup>If you successfully show this, then you do not have to submit a separate proof for Exercise 3.